ARAB REPUBLIC OF EGYPT

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo.No. 1493

The Optimum Production Rate in "Vehicle Fleet Scheduling" Problems

By

Mohamed Yehia Abdel Rahman

Jan . 1989

CAIRO
SALAH SALEM St-NASR CTTY

Introduction

This paper was presented and accepted at the 5-th International congress for statistics, computer science, social and demographic research (29 March -3 April 1980).

Summary :

The well known V.F.S problems became one of the classical problems under the assumption that the quantities which were found at the production pointsare fixed and do not depend on time.

This paper aims at remodifying the problem in a more realistic and applicable mode by assuming that the quantities will increase in a given rate. The quesion which faces us is :
what is the production rate, which satisfies the best collection cost between two given rate bounds ?

In addition through the presented formulas, the calculation of the collected quantities and the corresponding time which is necessary for a tour. Also the formula which gives the difference between the collected quantity in a direction and the collected quantity in its inverse by a given tour.

Formulation of the problem:

Assuming that :

1) a geographic zone represented by the graph $G = \{X, U\}$, where $X = \{1, 2, \ldots, r\}$ represents the production points with a center 0, and $U = \{(x, y) \in X \times X \,/\, x \neq y\}$; where every link (x,y) represent the

distance (time , cost , ... etc.) between the points x and y ;

2) at every production point i, an initial quantity q_i is found and increases with the time by a given rate A, such that $q_i \leqslant C$ the maximum capacity of the vehicle, and $A_{min} \leqslant A \leqslant A_{max}$

We have to find the ideal production rate A which lies between A_{min} , A_{max} and optimise the cost of the unit of the collected item. That problem represents one of the V.F.S. problems, because the condition $q_i < C$ implies that the vehicle must pass by more than one of the production points aiming that the cost for every tour must be minimum. If we denote for the tour j by W^j, and the distance (time, cost, ... , etc.) between x and y by d(x,y), we have to calculate firstly :

$$\min \; L \; (W^j) = \sum_{j = i}^{P} \; \sum_{xy \in W^j} d(x,y)$$

where P is the number of the tours which passes by all the production points and satisfies the conditions of the problem . To find an optimal tour, we can use Fletcher-Clark and Wright algorithm.

Now, condider a tour W^j , (Fig. 1) at which :

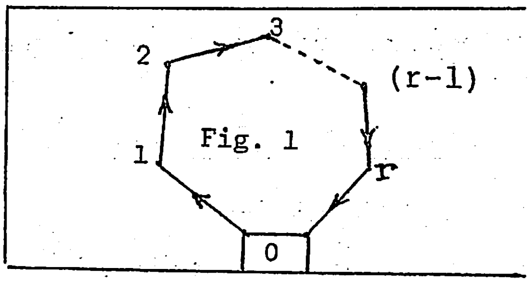


Fig. 1

q_i^j : the initial quantity at the production source i on the tour W^j ;

t(i,k): the time between the source i and the source k, and t(i,k) = t(k,i);

A : The production rate at the points (number of units produced at a unit of time) ;

B : the charging rate at the points (number of units which could be charged in a unit of time);

q_i^j: total quantity which can be charged at the point i on the tour W_i^j ,

Q^j: the collected quantity by the tour W^j ;

T_o : a dead time at every production point ;

T^j: the total time of the tour W^j (the time occupied since the vehicle starts from the depot till it returns back) ;

C : Capacity of the vehicle (to be considered as fixed) ;

r : number of production points on the tour W^j.

Number of added units to the initial quantity q_i^j , when the vehicle arrive at point 1 is : $|t(o,1).\ A|$, where $|x|$ is the integer value of x.

$$Q_1^j = q_1^j + |t(o,1) \cdot A|$$

The time for charging $Q_1^j = \gamma_1 = (q_1^j + |t(0,1).A|) \cdot B$
$= Q_1^j \cdot B$

The vehicle arrive to the 2nd point after its starting

by $\vartheta_1 = t(o,1) + (q_1^j + |t(o,1).A|).\ B + t(1,2)$

$= t(o,1) + Q_1^j .\ B + t(1,2)$

$$Q_2^j = q_2^j + \left[\left| \{ t(o,1) + (q_1^j + |t(o,1).\ A|) .\ B + t(1,2) \} .A \right| \right]$$
$$= q_2^j + |\vartheta_1 .\ A|$$

Repeating the same above steps, till arriving the point r, we can deduce that :

The total quantity collected by the tour W^j is :

$$Q^j = \sum_{i=1}^{r} Q^j_i = \sum_{i=1}^{r} \left[q^j_i + \left\| \left\{ \sum_{h=1}^{i} (t(h-1),h) + B. \sum_{k=1}^{i-1} Q^j_k \right\} . A \right\| \right] \qquad (1)$$

and

$$T^j = \left(\sum_{i=1}^{r} t(i-1, i) \right) + t(r,o) + B. Q^j + r. T_o \qquad (2)$$

putting A = o in (1), we obtain the classical case :

$$Q^j = \sum_{i=1}^{r} q^j_i \qquad (3)$$

also , assuming that Q^j_D is the total quantity collected on the directed direction of the tour W^j :

$$0 \longrightarrow 1 \longrightarrow 2 \longrightarrow \ldots \longrightarrow (r-1) \longrightarrow r \longrightarrow 0$$

and Q^j_{INV} is the total quantity collected on the inversed direction of the tour W^j :

$$0 \longrightarrow r \longrightarrow (r-1) \longrightarrow \ldots \longrightarrow 2 \longrightarrow 1 \longrightarrow 0$$

then

$$Q^j_D - Q^j_{INV} = \sum_{k=o}^{r-1} \left\{ t(k,k+1) - t(k,k+1) - t(r-k+1,r-k) \right\} \sum_{s=1}^{r-k} \binom{r-k}{s} A^s . B^{s-1}$$

$$+ \sum_{k=1}^{r-1} \left(Q^j_k - Q^j_{r-k+1} \right) \sum_{m=1}^{r-k} \binom{r-k}{m} A^m . B^m \qquad (4)$$

where $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$Q^j_D - Q^j_{INV} = A. \left[\sum_{k=o}^{r-1} \left\{ t(k,K+1) - t(r-k+1, r-k) \right\} . \sum_{m=1}^{r-k} \binom{r-k}{m} A^{m-1} . B^{m-1} \right.$$

$$\left. + B. \sum_{k=1}^{r-1} \left(Q^j_k - Q^j_{r-k+1} \right) \sum_{h=1}^{r-k} \binom{r-k}{h} A^{h-1} . B^{h-1} \right] \qquad (5)$$

it is clear that if A = o, then :

$$Q_D^j - Q_{INV}^j = 0$$

and if $A > 0$, then $(Q_D^j - Q_{INV}^j)$ augment with A, also the two quantities :

$$\sum_{h=1}^{r-k} \binom{r-k}{h} A^{h-1} . B^{h-1} \quad \text{and} \quad \sum_{m=1}^{r-k} \binom{r-k}{m} A^{m-1} . B^{m-1} .$$

but with a less rate because :

$$A^{s-1} . B^{s-1} \leq A.B < A \qquad (A,B < 1 \text{ and } s \geq 2)$$

From equation (1), it is clear that the collected quantity over a tour augment with A , so the total quantity colleced over a group of tours W^h (h = 1, 2, ..., p) also augment with A. With the continuous increase in A, we can find that ~~about~~ both Q_D and Q_{INV} over a tour does not verify the constraints of the problem , at this stage we have to recalculate another group of tours W^m (m = 1, 2, ..., m′) where $m' \geq P$.

In continuing the increase of A, we will attain at a situation in which every production point forms an individual tour, and the collected quantity by every tour must be firstly less than or equal the capacity of the vehicle C, till the collected quantity over each tour (by every production point) equals C.

In plotting the relation between the total quantities collected over all the tours at the various values of A, we can determine the optimal value of A which gives the maximum collected quantity at a lower possible cost under condition in which we can change its values within the limits A min., and A max.

Example:

Let us consider the following lower half matrix (fig 2) in which the upper number in each cell represent the time, while the lower number represent the cost. The capacity of the vehicle is 25 units, B = 0.1, and $T_o = 0$.

q_i	0					
7	30 20	1				
8	32 21	54 34	2			
5	45 30	45 30	36 24	3		
8	15 10	42 28	39 26	45 30	4	
6	36 24	48 32	48 32	30 20	38 25	5

(fig. 2)

The results at different values of A, can be summarised in the following table :

A	The tours W^j	Q^j	Cost of the W^j	$Q = \sum Q^j$	Total Cost (R)	R/Q
0.03	0→2→3→5→0	23	89	25+17 = 42	147	3.5
	0→5→3→2→0	25				
	0→1→4→0	17				
	0→4→1→0	16	58			
0.04	0→2→3→5→0	25	89	25+18 = 43	147	3.42
	0→5→3→2→0	26 > C				
	0→1→4→0	18				
	0→4→1→0	17	58			
0.05	0→4→5→3→0	25	85	25+20 = 45	160	3.56
	0→3→5→4→0	29 > C				
	0→2→1→0	20				
	0→1→2→0	20	75			
⋮	⋮	⋮	⋮	⋮	⋮	⋮

A	The tours W^j	Q^J	Cost of the j tour	Q = QJ	Total Cost (R)	R/Q
	0→1→0	11	40			
	0→4→2→0	25	57			
0.14	0→2→4→0	30 > C		11+25+25=61	171	2.8
	0→5→3→0	25	74			
	0→3→5→0	27 > C				
	0→1→0	11	40			
0.15	0→2→0	12	42	11+12+24+11 = 58	200	3.45
	0→4→3→0	24	70			
	0→3→4→0	32 > C				
	0→5→0	11	48			
⋮	⋮	⋮	⋮	⋮	⋮	⋮
	0→1→0	12	40			
	0→2→0	14	42			
0.19	0→3→0	13	60			
	0→4→0	10	20	61	210	3.44
	0→5→0	12	48			
⋮	⋮	⋮	⋮	⋮	⋮	⋮
	0→1→0	20	40			
	0→2→0	22	42			
0.44	0→3→0	24	60	101	210	2.08
	0→4→0	14	20			
	0→5→0	21	48			

These results were plotted (fig. 3) in taking the rate of production A on the X-Co-ordinate, while the total cost, the total quantity, the cost of collecting one unit of the item, on the set of tours which corresponds to the gives value of A, on the Y-Co-ordinate.

Let us consider that, the value of A can be changed within the bounds:

$$A_{min} = 0.06 \text{ (point G)} \leqslant A \leqslant A_{max} = 0.11 \text{ (point H)}$$

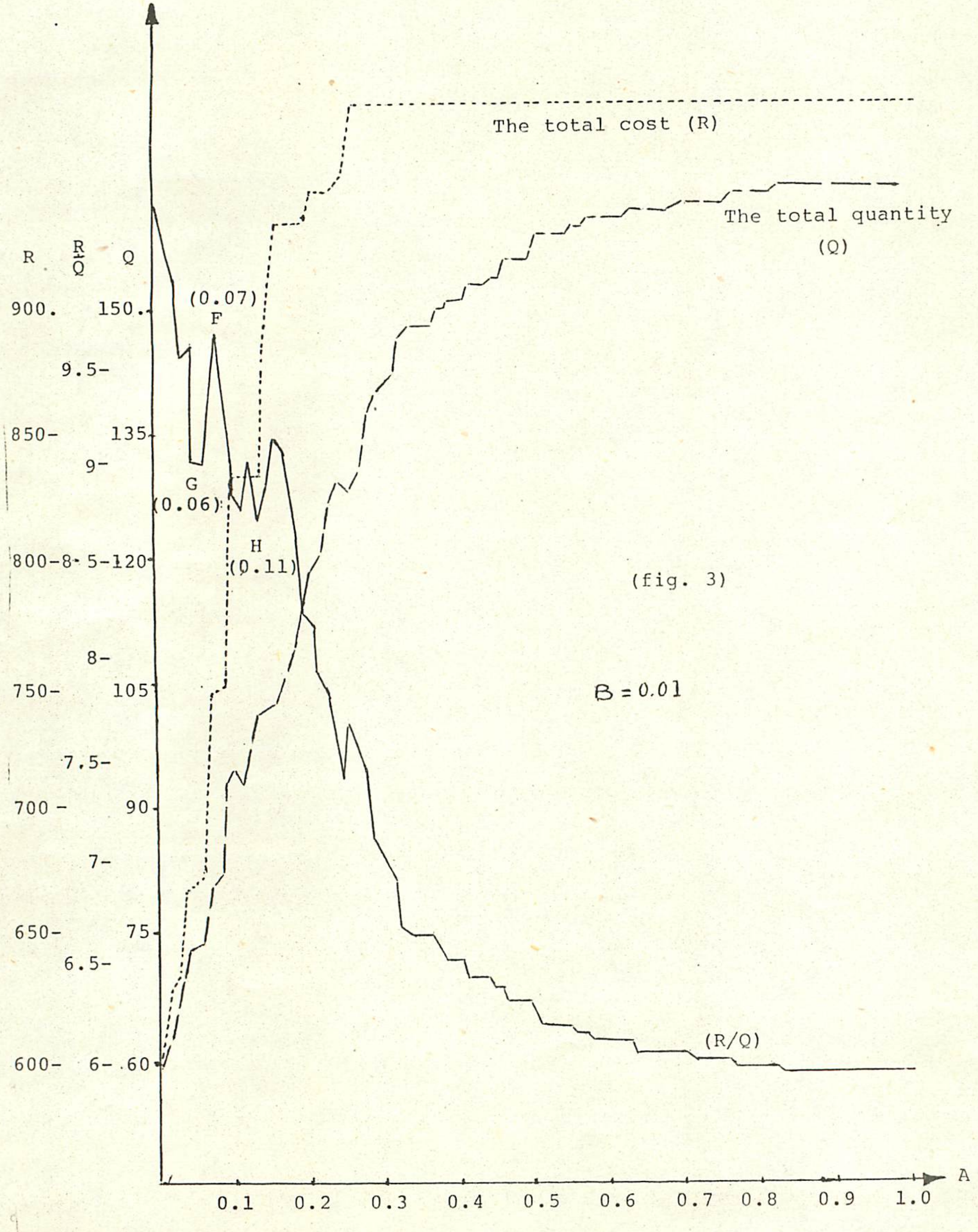
then, it is better to choose the rate of production A as one of these limits instead of a production rate A = 0.07 (point F) for example, at which the cost of collecting one unit is more higher than the cost at the two other bounds .

These results were plotted (fig. 3) in taking the rate of production A on the X-Co-ordinate, while the total cost, the total quantity, the cost of collecting one unit of the item, on the set of tours which corresponds to the gives value of A, on the Y-Co-ordinate.

Let us consider that, the value of A can be changed within the bounds:

$$A_{min} = 0.06 \text{ (point G)} \leqslant A \leqslant A_{max} = 0.11 \text{ (point H)}$$

then, it is better to choose the rate of production A as one of these limits instead of a production rate A = 0.07 (point F) for example, at which the cost of collecting one unit is more higher than the cost at the two other bounds .


The total cost (R)
The total quantity (Q)
(R/Q)
R
R/Q
Q
900.
150.
9.5-
850-
135
9-
800-8.5-120
8-
750-
105
7.5-
700 -
90
7-
650-
75
6.5-
600-
6-
60
(0.07)
F
G
(0.06)
H
(0.11)
B = 0.01
A
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

(fig. 3)

AIN SHAMS UNIVERSITY

Proceeding of
The Fifth International Congress

For
Statistics, Computer Science,
Social, and Demographic Research
29 March — 3 April 1980

Under the Auspices
of
PRESIDENT ANWAR EL-SADAT

In Memory of
Prof. Dr. A.M.N. El-Shafie

Vol. IV
Statistical Studies
Faculty of Commerce

INDEX

مطبعة معهد التخطيط القومى