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THE TWO - LEVEL PLANNING MODEL

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Introduction

Economic planning is a task that can be approached by different method and idea. As an example by the use of input-output table. But such input-output tables are not suited for optimiziation and is merely designed to achieve correct proportions between sectors.
That mean that optimization methods are used only for project analysis but not for the whole national economy some proposals and trouls were made in the direction of optimization to optimize the whole economy such as those done by Ragnar Frish,Johonson Rodalf and others, but those trails were complicated to be applied and solved, due to the large compution needed for such approaches. Now we have a big computers and highly information systems.

The solution of optimization problem can now by done without no diffcults.
The planning processe can be dealt as a single linear programming problem. called (main problem or center problem).
This is the global problem or the overwhole problem of national economy. After that (the problem of formulating the whole econommy) one must move to other level of which is the sectoral level of industry.

The sectors of production thus are connected to the center by some sort of constraints. Thus it can be said that planning as an optimization task is done on two-levels, which are the center level (center model) and the sector level(sector model) i.e. two types of model are needed for two-level planning process. This paper will deal then with two-level planning models.

Two-level Planning Model(General):

The two-level planning model provides some organizational principles concering the division of functions between the center and sectors, and with that the flow of information between the two levels.

The optumum allocation of resource can be achieved only if the information sent up by sectors is frank and objective, because this will effect the objective function and the constraints of both levels.

The two-level planning model contain elements of competition, such as the home production and import are compting with one anther, also there is competition between direct satisfaction of domestic demand by production and its indirect satisfaction by imports paid for by export. Also the center of the two-level planning put and study the critera to reallocate the resources.

At each level, all units will have to construct a mathematical programming model embarcing their activities and constraint prescribed. In the two-level planning models, the results and information output of calculation carried out with one model provides the basic data, the information input for one or more of the other models. These informations will be as

final output obligations-material quatas-resource quatas-shadow prices-plan estimates-etc. On the basis of information, recived by the unit will continuously correct the individual parameters of their models and carry out improvment of their plan.

Level I.

The General Model:

This model gives certain defenitions and provides a basis for the economic application of the idea of two-level planning. In that level the planning board formulate the planning proposal for the plan targets and figures for the sector due to the general information about the sectors. The center due to its information about the sectors begin to distribute its avialable resources (material, manpower, etc) among sectors as a frist run, meaniwhile demand from the sectors some of output target are required from the sectors.

The sectors begins after reciving the proposal from the center to propose changes to the center due to their ability. Accordingly and on theat base the center planning board begins to modifies its original targets and again sends this new proposals to the sectors.
Now to formulate the center model and the sector model, a briet elaboration about primal and dual linear programming problems, since the two-level planning will be derived us linear programming.

In such treatment one must take into account that the original linear programming problem (center in formation problem) which is decomposed into sectorial problems which is related to the center.

The mathematical formulation of the overall problem is given by:

Given a vector X which represents a primal variable for the center program then the primal and dual problem will be.

primal problem	dual problem
$C'X$ - - - max	$y'b$ — — — mim

subject to

$AX \leq b$	$y'A \leq C$....(I)
$X \geqslant 0$	$y \geqslant 0$

The dual problem will be the center shadow price system.

An optimal solution for such system will be

$$\max C'X = \min y'b = C'X^* = y^{*\prime}b =$$

where $x^* \in X$, $y^* \in Y$

which means that a feasible solution of the primal and dual exist. Now taking the above illustration in mind the center model will be formulated as follows

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} Z_{jit} + d_{it} = V_{it} \leq V_{it} \qquad (1)$$

$$\sum_{i=1}^{n} W_{it} = W_t \qquad (2)$$

$$V_{it} \geqslant 0$$

$$Z_{jit} \geqslant 0$$

$$W_{it} \geqslant 0$$

For $i=1,\dots n,$; $t=1,\dots,T$

where.

V_{it} = is what to be produced by the sector i in the interval t (supply task).

Z_{ijt} = is the quantity to be produced from the sector i of the project j in the interval to (material quota).

W_{it} = is the available manpower for the sector i in the internal t (manpower quota).

d_{it} = the consumption from product i in the time interval t.

all the above information is given to the sector i by the center.

Level II

The primal sector model:

First we deal with the general linear programming problem(I).

If $A=\left[A_1, A_2, \dots A_n\right]$

i.e. the Matrix A in the general model is divided to sub-matrices $(A_1, A_2, \dots A_n)$

then problem (I) will be

Primal problem

$C_1'X_1 + C_2'X_2 + \ldots\ldots + C_n'X_n \ldots\ldots \max !$

$A_1X_1 + \ldots\ldots + A_nX_n \leqslant b$

$X_1 \geqslant 0$

$X_2 \geqslant 0$

$x_n \geqslant 0$

dual problem

$y' b - - - \min !$

$y'A \geqslant C_1'$

$y'A_n \geqslant 0$

$y \geqslant 0$

Now let a vector u be used such that

$$u_1 + u_2 + \ldots\ldots u_n = b$$

i.e. u have the some size as the constraint vector b

it follows that

$$U = \left[u_1', \ldots, u_n' \right]$$

u will be centeral program and u_i will be the sector componant. i.e. the problem can be formulated.

The primal problem

$C_i' X_1 --- \max !$

$A_iX_i \leqslant u_i$

$X_i \geqslant 0$

and $i = 1,2,\ldots\ldots,n$

The dual problem

$y_i' u_i --- \min !$

$y_i' A_i \geqslant C_i'$

$y_i \geqslant 0$

Mathematical Formulation of the Sector Model:-

In order to get a formulation of this model we frist define the following:

X_{ikt} = denote the k th output of the sector i during the period t (t = 1,....,T).

X_{ik} = the volume of investment activity in the i^{th} sector

X_{ikt} = the k^{th} export activity of the i^{th} product in the t^{th} period , k = exp (t = 1,...,T)

X_{ikt} = the volum of the k^{th} bounded import activity, importing the i^{th} product in the t^{th} period , k = impo (t = 1,...,T).

The above variables are given according to their economic nature The formulation of the primal problem will be.

Under the following set of constraints

The frist set of contrants

$$V_{it} \leq \sum_{k \neq imp} f_{ikt} X_{ikt} + \sum_{k = inv} f_{ikt} X_{ikt} \leq V_{it}$$

k = pro,ex (t=1,...T)

imp,o

where

where

f_{ikt} = is the output cofficient

$$= \begin{cases} = 1 & \text{for production} \\ \geqslant 0 & \text{for investment} \\ = -1 & \text{for export} \\ = \ 1 & \text{for import} \end{cases}$$

k in the frist part is for production ; for export, for import only, and k in the second part for investment the above set of constraint are of the type which shows that the sector complies with the plan figuers received from the center.

The second set of constraints are

$$\sum_{\substack{k=all \\ k \neq inv}} g_{ijkt} \, x_{ikt} + \sum_{k=inv} g_{ijkt} \, x_{ikt} \leqslant Z_{ik}$$

$$j = 1,\ldots,n \quad , \quad j \neq i \quad , \quad t = 1,\ldots,T$$

where

g_{ijkt} = material input coefficient

$$= \begin{cases} \geqslant 0 & \text{for production and also for investment} \\ = 0 & \text{for foreign trad activities} \end{cases}$$

The thrid set of constraints

$$\sum_{\substack{k = all \\ k \neq inv}} h_{ikt}\, x_{ikt} + \sum_{k=inv} h_{ikt}\, x_{ik} \qquad W_{it}$$

where

h_{ikt} = labour force coefficient

$$= \begin{cases} > 0 & \text{for production} \\ \geq 0 & \text{for investment} \\ = 0 & \text{for foreign trade} \end{cases}$$

those above three types of constraints are related directly to the center.

Also some types of constraints can be set up for special sectors which have certain characteristic. These types of constraints may be written in the general form.

$$\sum_{t=1}^{T} \sum_{\substack{k = all \\ k \neq inv}} a^{o}_{ilkt}\, x_{ikt} + \sum a^{o}_{ilkt}\, x_{ik} \leq b^{o}_{il}$$

where

b^{o}_{il} is the upper limit of production in the sector

The objective function of level two will be

$$\sum_{t=1}^{T} \sum_{\substack{k=all,o \\ k \neq inv}} S_{ikt}\, x_{ikt} + \sum_{k=inv} S_{ik}\, X_{ik} \; .... \quad \max !$$

where S_{ikt} , S_{ik} are the fore n currency return.

It is also assumed that

$$\max_{k=exp} S_{ikt} \leq \min (-S_{ikt}) \qquad t=1,...T$$

Here it is clear that the maximization of sector objective function will leads to the maximization of the objective function on the national scale.

<u>The dual sector model</u>:

The dual problem of the above formulation of the sector model is as follow and the set of constraints

$$f_{ikt}\,(x_{it} - \nu_{it}) + \sum_{\substack{j=1 \\ j=i}}^{n} g_{ijkt}\;\lambda_{ijt} + h_{ikt}\;W_{it}$$

$$+ \sum_{i=spec} a^{o}_{ilkt}\;\sigma_{ie} \geq S_{ikt}$$

where

where

k = production, export, import, 0

$t = 1,\ldots,T$

$$f_{ikt}(x_{it} - \lambda_{it}) + \sum_{j=1}^{n} g_{ijkt} \quad {}_{ijt} + h_{ikt} W_{it}$$

$$+ \sum_{l=spe} a^{o}_{iekt} \quad \sigma_{il} \geq S_{ik}$$

k = investment only, $t = 1,\ldots T$.

and

$$\nu_{it} \geq 0 \quad (\quad \lambda_{ijt} \geq 0 \quad (\quad W_{it} \geq 0$$

$$, \quad x_{it} \geq 0 \quad , \quad \sigma_{il} \geq 0$$

$j = 1,\ldots n$, $j \neq i$, $t = 1,\ldots T$

l = spec.

The dual objective function is

$$\sum_{t=1}^{T} (V_{it} \gamma_{it} - \nu_{it} v_{it}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} Z_{ijt} \quad \lambda_{ik} + W_{it} W_{it})$$

$$+ \sum_{l=spec} b^{o}_{il} \sigma_{il} , \ldots\ldots \text{ min } !$$

In the above dual sector model ν_{it}, is the shadow price for the supply, task v_{it} , λ_{ijt} is the shadow price for material quota Z_{ijt} and also w_{it} is that of manpower quota. Let γ_{it} to gives the shadow price of the upper boundary V_{it} of the i^{th} supply task and $\sigma_{i\ell}$ that of the boundary $b^{o}_{i\ell}$ in the ℓ^{th} special costrant.

Economic Interpretion And Calculation Process:

The aim of this study is to give an idea of the two-level planning models and how it can be used for planning process. The programms of sectors are not our subject here and it can be left for a detailed study. Were will be an illustration of the main relations of the two-level planning model and the iteration process to get the optimal solution.

The calculation process consist of a number of phases, these numbers are not mor than four phases, each phase consist of a number of steps. The phase mainly concentrate on a given objective function. The calculations are done through an iterative procedure which depend on upper and lower optima. In our problem the value of the objective function will certainly fall between the upper and lower optima in any step of iteration.

Now let U^*_{n-1}, L^*_{n-1} be the uper and lower optima in the $|n-1|$ iteration ant let δ be any small incremental value. Now-the four phases are as follows.

Phase I.

This phase is a centeral level phase (i.e. the calculation in done in the center of plan.) and as a test for calculations is that.

Case I

if $U^{*}_{n-1} - L_{n-1} \leq \delta$

then the calculation is stoped and we have on optimum,

Case II

$U^{*}_{n-1} - L_{n-1} \geqslant \delta$

then the calculation is continued untell we get an optimum for the problem

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left\{ \alpha^{*}_{it} (n-1)\, V_{it} + \sum_{j=1}^{n} \beta^{*}_{ijt} (n-1)\, Z_{ijt} + \gamma^{*}_{it}(n-1)\, W_{it} \right\} \ldots \max$$

subject to

$$\sum_{j=1}^{n} Z_{ijt} + d_{it} = V_{it} \leq V_{it} \qquad (i=1,\ldots.n, t=1,\ldots T)$$

$$\sum_{i=1}^{n} W_{it} = W_{it} \qquad (t=1,\ldots.T)$$

$V_{it} \geqslant 0$, $Z_{ijt} \geqslant 0$ $W_{it} \geqslant 0$

$(i=1,\ldots n$, $i \neq j$ $j=1,\ldots n,$ $t=1,\ldots T)$

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where α_{it} is the supply shadow price of the production V_{it}, β_{it} is the shadow price for the material quota Z_{ijt} (demand price) and γ_{it} is the shadow price for the manpower quota. all these shadow prices are send.

In the frist phase there must be a decision about the distribution of resources in this case we have

a- if the supply price are higher than any of the demand prices, in this case the center dose not supply the sector with any raw material

$Z_{ijt} = 0$ and consumption requirment only is to be produced i.e.

$$Z_{ijt}(n) = 0 \quad , \quad V_{it}(n) = d_{it}$$

b- if the demand price is higher than the supply price , then the all quantity will be given to the sector which will give higher price. This mean that the actual distrbution will be between both extrems The frist phase in general ended with formulating partial programs.

Phase II

This phase begin after the frist distribution taken in phase I. This phase is also centeral level.

Let $\overset{*}{\Phi}_{(n)}$ be the summtion of optimals for the partial programms obtained.

$$\overset{*}{\Phi}(n) = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{n} \left[\max_{j=i} \beta^{*}_{jit}(n-1) - \alpha^{*}_{it}(n-1)\, V_{it} - \alpha^{*}_{it}(n-1)\, d_{it} + \max \gamma^{*}_{it}(n-1)\, W_{it} \right] \right\}$$

Aspecial optimum component is added to this summation, This summation is $\sum_{i=1}^{n} \overset{o}{\Phi}_{i}(N-1)$. Now the calculation of the upper bound of the optimal value in the n^{th} run will take place. i.e.

$$\overset{*}{\Phi}(N) = \overset{*}{\Phi}(N) + \sum_{i=1}^{n} \overset{o}{\Phi}_{i}(n-1)$$

As done in the frist phase in the continuation of calculation or not will be in this phase according to the condition.

$$\overset{*}{\Phi}(N) - L^{*}(n-1) \leq \delta$$

or

$$\overset{*}{\Phi}(N) - L^{*}(N-1) > \delta$$

which means that we must have real distribution instead of extrem distribution in an side.

The centeral program send to the productive sectors will be according to the following basic formulas.

$$V^*_{it}(N) = \frac{N-1}{N} V^*_{it}(N-1) + \frac{1}{N} V_{it}(N)$$

$$Z^*_{ijt}(N) = \frac{N-1}{N} Z^*_{ijt}(N-1) + \frac{1}{N} Z_{ijt}(N)$$

$$W^*_{it}(N) = \frac{N-1}{N} W^*_{it}(N-1) + \frac{1}{N} W_{it}(N)$$

where

$(i=1,\ldots,n \ , \ j=1,\ldots-n)$

$, \ j \neq i \ ; \quad t=1,\ldots T$

this is in real simple arithmatic mean for the partial programs obtained

After phase one and two were done the role of the center is ended.

Phase III:

This phase is done on the sectorial level of planning (Level II). The sectors begins to prepare there programms according to plan directive

$V^*_{it}(N)$, $Z^*_{ijt}(N)$, $W^*_{it}(N)$

given to the sectors by the center.

The primal and dual problem now can be solved to get.

a) The shadow prices at this stage for the sectors. $\alpha_{it}(N)$, $\beta_{it}(N)$ and $\gamma_{it}(N)$.

b) The value of the objective function of the optimal programm. Now the summation of optimal programms of the sectors gives the lower value of the planning problem as whole in the N^{th} stage, i.e.

$$L^{*}(N) = \sum_{i=1}^{n} L_i(N).$$

c) Provisional special sector optimum component

$$\Phi_i^{o}(N) = \sum_{t=1}^{T} V_{it} \cdot \Phi_{it} + \sum_{L=spec} \beta_{iL}^{o} \, \gamma_{iL} \, (N)$$

This value contains the calculated limit for V_{it} and the limt of the constraint (special) for the sector calculated by shadow prices.

Phase IV:

The shadow prices obtained in phase III must be smited with the plane directives given to the planning center for this reason the must be a real evaluation for thise shadow prices. this is done in phase IV as follows.

$$\overset{*}{\alpha}_{it}(N) = \frac{N-1}{N} \overset{*}{\alpha}_{it}(N-1) + \frac{1}{N} \alpha_{it}(N)$$

$$\overset{*}{\beta}_{it}(N) = \frac{N-1}{N} \overset{*}{\beta}_{it}(N-1) + \frac{1}{N} \beta_{it}(N)$$

$$\overset{*}{\gamma}_{it}(N) = \frac{N-1}{N} \overset{*}{\gamma}_{it}(N-1) + \frac{1}{N} \gamma_{it}(N)$$

where

$(i=1,\ldots,n \quad ; \quad j=1,\ldots n \quad ; \quad j \neq i$

$t = 1,\ldots,T)$

Also the special optimal component of the sector are finally obtained by the same way in the above stages.

$$\overset{o}{\Phi}_{i}(N) = \frac{N-1}{N} \overset{o}{\Phi}_{i}(N-1) + \frac{1}{N} \overset{o}{\Phi}(N)$$

$, (i = 1,\ldots.n)$

At the end of phase IV, the shadow prices and also the optimum value of the sectors are send to the center with also the mixed special optimum component.

As a feature of the two level planning is that (in case of optimal programm the shadow prices are equal and this is done as follows

a) as an example $\gamma_{it} = \gamma_{2t} = \ldots = \gamma_{nt}$

b) the spply price for certain product is equal to that of demand price. i.e. $\alpha_{it} = \beta_{ijt}$.

Two-LEVEL PLANNING MODEL : (Illustrative Prototype Examples)

The purpose of this chapter is to show how some large-scale LP models of special angular structure can arise through the combination of smaller LP models and how such large-scale LP models prove to be more powerful as decision tools than solving the individual smaller models from which they are constructed.
Such large models are called MULTIDIVISIONAL LP MODELS WITH SPECIAL ANGULAR STRUCTURE .

Introduction :

The model will be re-illustrated here through a very small demonstrative example.

Multi-Plant Model :

Suppose we have a company consists of 2 factories A and B . Each factory can produce 2 types of a product (normal and lux). The profit per unit of the normal type is 10 pounds ; while it is 15 for the lux type .

Each factory has 2 processes of production for producing its products.

Factory A has a maximum of 80 hours per week in process-I and 60 hours per week in process-II .

Factory B has 60 and 75 hours per week in process-I and process-II .

Each unit of normal and lux types needs a time for production as follows :

Production process	Company					
	Factory A			Factory B		
	Products		Available time per week	Products		Available time per week
	Normal	Lux		Normal	Lux	
I	4	2	80	5	3	60
II	2	5	60	5	6	75
Prof./Unit	10	15		10	15	

In addition to the information above , each unit of each product uses 4 Kg of material and the company has only 120 Kg of material per week.

Suppose that Factory A is allocated only 75 Kg of material and the remaining 45 Kg is allocated to Factory B .

Each Factory can build its LP model to maximize its profit as follows :

Factory A :

$$\text{Max. } Z_1 = 10\,x_1 + 15\,x_2$$

s.to:

$$4\,x_1 + 4\,x_2 \le 75$$

$$4\,x_1 + 2\,x_2 \le 80$$

$$2\,x_1 + 5\,x_2 \le 60$$

&

$$x_1 \; , \; x_2 \ge 0$$

{ x_1 represents the desired quantity of normal product and x_2 represents the desired quantity of lux product }

Factory B :

$$\text{Max. } Z_2 = 10x_3 + 15x_4$$

s.to:

$$4x_3 + 4x_4 \le 45$$

$$5x_3 + 3x_4 \le 60$$

$$5x_3 + 6x_4 \le 75$$

&

$$x_3, x_4 \ge 0$$

The optimal solutions of these individual models are :

For Factory A :

Z_1 = 225 pounds ,

x_1 = 11.25 units , and

x_2 = 7.5 units .

For Factory B :

Z_2 = 168.75

x_3 = 0.0

x_4 = 11.25

Notice that :

At Factory A : There is a surplus capacity at Process I = 20 hours

At Plant B : There is a surplus capacity at Process I = 26.25 hours
There is a surplus capacity at Process II = 7.5 hours

Suppose now that an overall Company model is built in order to maximize the total profit of this company .

Assume also that the Factories remain distinct and geographically separated . Moreover , no longer allocation of 75 Kg of material to A and 45 Kg to B . Instead , the model will decide this allocation . So , there will be a single material constraint limiting the Company to 120 Kg only per week .

The Company Model can be illustrated as follows :

$$\text{Max.} \quad Z = 10\,x_1 + 15\,x_2 + 10\,x_3 + 15\,x_4$$

s.to:

$$4\,x_1 + 4\,x_2 + 4\,x_3 + 4\,x_4 \le 120 \quad \text{(Available Material)}$$

$$4\,x_1 + 2\,x_2 \le 80$$

$$2\,x_1 + 5\,x_2 \le 60$$

$$5\,x_3 + 3\,x_4 \le 60$$

$$5\,x_3 + 6\,x_4 \le 75$$

&

$$x_j \ge 0 \quad ; \; j = 1,2,3,4 \; .$$

<u>We want this model to split the material optimally between the Factories A and B since it would be expected that a more efficient split would be happen which maximize the overall Company profit .</u>

The optimal solution of this model is :

$Z = 404.15$ pounds

$x_1 = 9.17$

$x_2 = 8.33$

$x_4 = 12.5$

Note also that :

1. There is a surplus capacity of 26.67 hours at Factory A.

2. There is a surplus capacity of 22.5 hours at Factory B.

3. The total profit is 404.15 pounds which is > the combined profit from Factory A and Factory B acting independently (225 + 168.75 = 393.75).

4. Factory A alone contributes 187.50 pounds to the Company total profit as a whole , whereas before , it contributes 225 pounds only.

5. Factory B contributes 216.65 pounds to the Company profit whereas before , it only contributes 168.75.

6. Factory A now uses 70 Kg of material and Factory B uses 50 Kg. It is clear that the Company model as a whole has biased production more towards Factory B than before . This has been done by allocating Factory B 50 Kg of material instead of 45 Kg and so decreased what allocated to Factory A by 5 Kg.

The above example shows how a Multi-palnt model can arise . It is a method of using LP Models in COORDINATION between Plants in a large Organizations with many Plants as well as in helping of Decision-Making within Plants.

The same situation is valid for the COORDINATION between the different Sectors in a NATIONAL PLANNING.

This Model was a very simple example of a common sort of structure which arises in Multi-Plant Models.
The structure of such models is Known as Block-Angular-Structure :

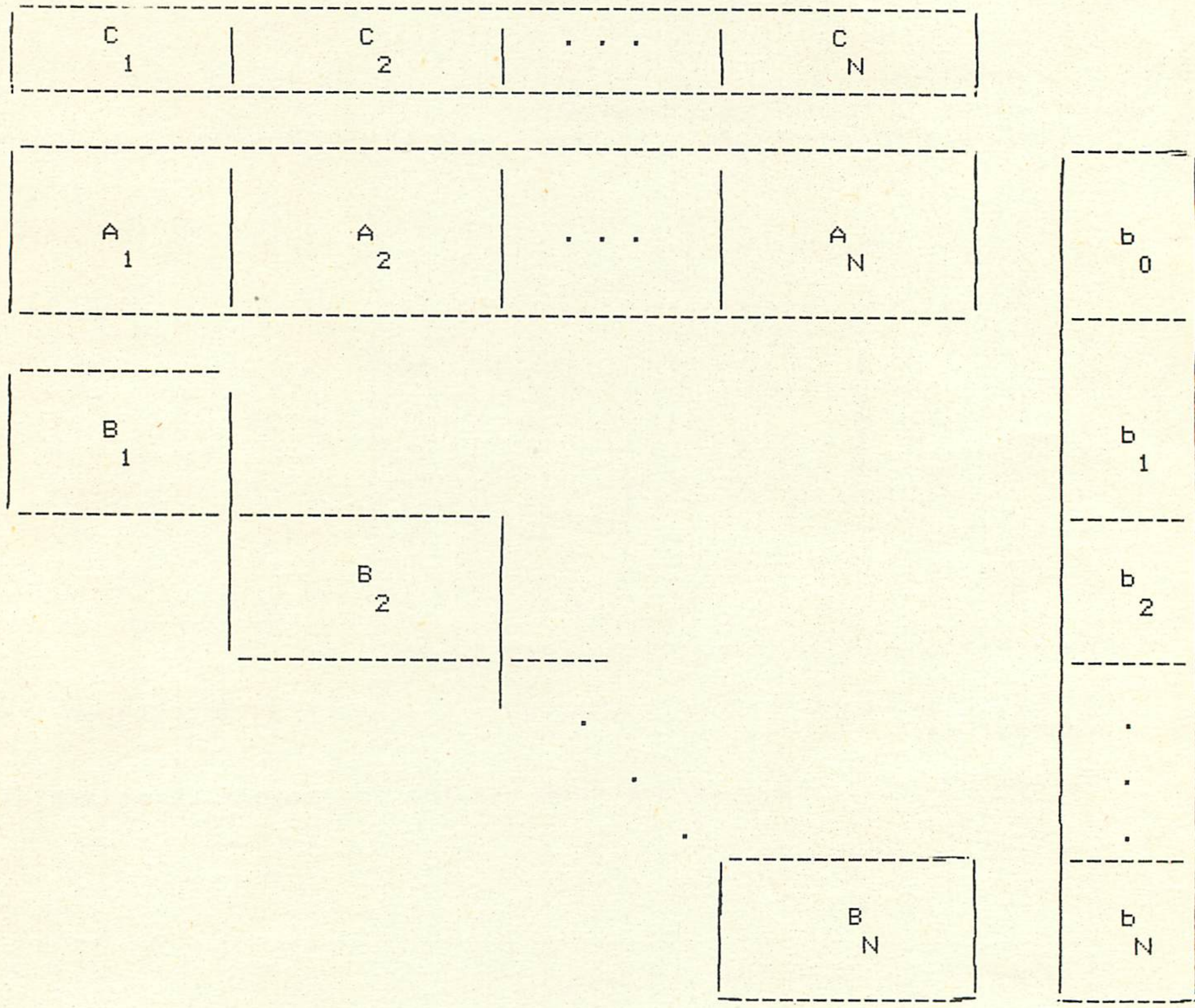
10	15	10	15		
4	4	4	4	<=	120
4	2			<=	80
2	5			<=	60
		5	3	<=	60
		5	6	<=	75

. The first row is the Objective function row,

. The next rows (here is only one row) are Known as the Common constraints.

. The two diagonally blocks are Known as the Sub-models constraints.

For a more general problem with a number of rarely shared resources and N Plants, the following General Block-Angular-Structure could be obtained :



where :

$C_1 , C_2 , . . . , C_N$ are row vectors representing the coefficients of the Objective function.

$A_1 , A_2 , . . . , A_N$, and

$B_1 , B_2 , . . .$ B_N are Blocks of constant coefficients.

$b_0 , b_1 , . . . , b_N$ are column vectors of coefficients forming the right-hand-side.

$A_1 , A_2 , . . . , A_N$ represent the common (shared) constraints in Multi-plant Models and usually involve allocating the scarce resources (materials , processing capacities , manpower , . . . etc.) across Plants.

Note that :

- If the problem of Block-Angular-Structure has no common constraints, it should be clear that optimizing it simply be equal to optimizing each problem with its appropriate portion of the Objective function. For the present example, if there is no material constraint, each Plant's model could be solved independently and an overall company optimum would be obtained.

- Sometimes there are common constraints, but this will be no longer the case as in the small example illustrated here. However, the more common constraints there are, the more interconnected the separate Plants must be.

In the next chapter, a discussion of how a Knowledge of the optimal solutions of the Sub-models might be used to obtain an optimal solution to the whole model. This can be quite important computation ally since such structured models are often of very large-scale type and takes a long time and a very large computer memory to solve if treated as one large-scale model.

The DECOMPOSITION ALGORITHM :

The importance of decomposing a large multi-divisional model is not only computational, but also economic. Usually a Decomposition algorithm applies to LP models of Block-Angular-Structured practical problems. If the structured model represents, for example, a multi plant model, the decomposition procedure reflects the idea of DECENTRALIZATION IN PLANNING.

Consider again the small demonstrative example in the previous chapter :

$$\text{Max.} \quad Z = 10\,x_1 + 15\,x_2 + 10\,x_3 + 15\,x_4$$

s.to :

$$4\,x_1 + 4\,x_2 + 4\,x_3 + 4\,x_4 \le 120 \qquad \{\text{ material }\}$$

$$4\,x_1 + 2\,x_2 \le 80$$

$$2\,x_1 + 5\,x_2 \le 60$$

$$5\,x_2 + 3\,x_4 \le 60$$

$$5\,x_3 + 6\,x_4 \le 75$$

&

$$x_j \ge 0 \quad ; \; j = 1, 2, 3, 4$$

It was seen that splitting the 120 Kg of material between Plant A and Plant B in the ratio 75 / 45 led to a non-optimal overall solution. The optimal overall solution showed that this ratio should be 70 / 50. Unfortunately, the overall model will be solved in order to find this optimal split. If a method of predetermining this optimal split, it could be able to solve the individual models for Plant A and Plant B and then combining the solutions to give an optimal solution for the overall model.

For a general Block-Angular-Structure Model, we would need to find optimal splits in all the right-hand-side coefficients (b_0) for the common constraints.

Decomposition Algorithms of a Block-Angular-Structure based on this principle do exist. Such Algorithms are Known as Decomposition By Allocation. One such Algorithm is the algorithm of Rosen (1964).

An alternative approach is Decomposition be Pricing. In a Block- ,such as the one above, where the common constraints represent constraints on material availability, we could try to seek for a fixed-values of that limited resources ,material, . These fixed-values could be used as internal (shadow) prices to be directed to the sub-models. If accurate fixed-values valuations could be obtained, we might hope to get each sub-model optimizing to the overall benefit of the overall model. One such approach is the Dantzig-Wolfe Decomposition Algorithm. A full description of the algorithm is given in Dantzig (1963).

This paper dealt with a less rigorous description, paying attention to the Economic anology with the Decentralized Planning.

Now ,

If the material was not in limited supply , we would have the following sub-models for Plant A and Plant B as follows :

For Plant A :

$$\text{Max. } Z_1 = 10\, x_1 + 15\, x_2$$

s.to:

$$4\, x_1 + 2\, x_2 <= 80$$

$$2\, x_1 + 5\, x_2 <= 60$$

&

$$x_1 \, , \, x_2 >= 0$$

For Plant B :

$$\text{Max. } Z_2 = 10\, x_3 + 15\, x_4$$

s.to:

$$5\, x_3 + 3\, x_4 <= 60$$

$$5\, x_3 + 6\, x_4 <= 75$$

&

$$x_3 \, , \, x_4 >= 0$$

These sub-models should not be confused with the sub-models for the same problem mentioned in the previous chapter except that the material availability constraints were included there in both sub-models with a guessing allocation of material between them. Here, such constraints are not included. Instead, an attempt is made to find a suitable internal (shadow) price for the material and to incorporate this internal price into the sub-models.

Suppose that the material were to be internally priced at p pounds per Kg. If we have taken the material availability constraints multiplied by p and then subtracted them from the Objective functions, then, the above Objective functions of the sub-models become :

$$Z_1 = (10-4p)\, x_1 + (15-4p)\, x_2 \qquad (1)$$

$$Z_2 = (10-4p)\, x_3 + (15-4p)\, x_4 \qquad (2)$$

Now, if p has a very small value, then the combined solutions to the sub-models use more material than is available in which case p should be increased.

Example :

If p = 0 , (i.e., there is no internal price for the material), the following optimal solution could be obtained :

For Plant A :

$Z_1 = 250$

$x_1 = 17.5$

$x_2 = 5$

For Plant B :

$Z_2 = 187.5$

$x_3 = 0$

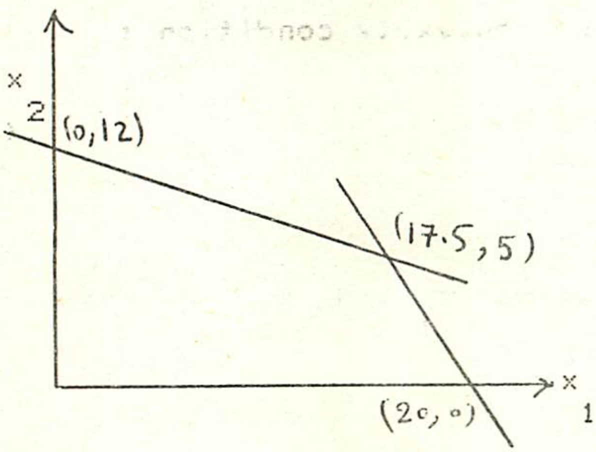
$x_4 = 12.5$

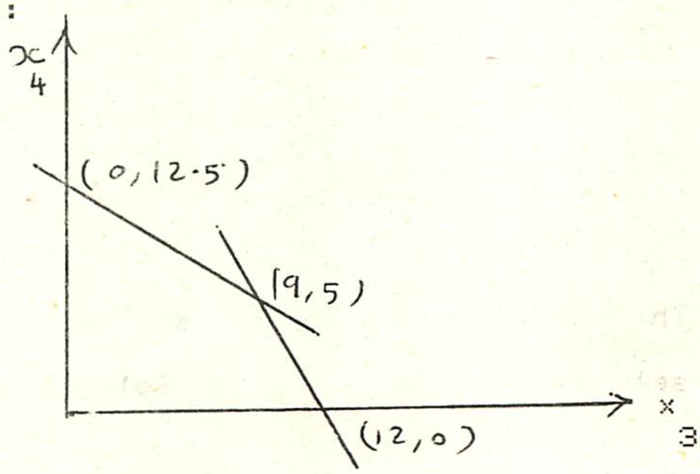
These solutions are clearly unacceptable to the Company as a whole since they demand 140 Kg ($4x_1 + 4x_2 + 4x_3 + 4x_4$) of material which is more than the 120 Kg available.

Therefore, it is necessary to seek for some way of estimating a more realistic value for the internal price p.

Whatever the value of p, Plant A and Plant B will have optimal solutions which are Extreme-point solutions of the sub-models presented above.

Since the sub-models only involve 2 variables each, they can be represented graphically as follows :





With p = 0, it can be easily verified that the optimal solutions above are (17.5 , 5) and (0 , 12.5) respectively.

Any feasible solution to the overall problem must be feasible to both sub-problems (as well as additionally satisfying the material availability limitation).

The values of x_1 and x_2 in any feasible solution to the overall problem must therefore be a convex linear combination of the vertices (extreme-points) of the feasible-region shown in the first figure above.

i.e.,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \sigma_{11} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \sigma_{12} \begin{bmatrix} 20 \\ 0 \end{bmatrix} + \sigma_{13} \begin{bmatrix} 17.5 \\ 5 \end{bmatrix} + \sigma_{14} \begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad (3)$$

where,

σ_{11} , σ_{12} , σ_{13} , σ_{14} are the weights attached to the vertices.

They must be non-negative and satisfy the convexity condition :

$$\sigma_{11} + \sigma_{12} + \sigma_{13} + \sigma_{14} = 1 \quad (4)$$

The vector equation (3) is a way of relating x_1 and x_2 to a new set of variables σ_{ij} as follows :

$$x_1 = 0\,\sigma_{11} + 20\,\sigma_{12} + 17.5\,\sigma_{13} + 0\,\sigma_{14} \quad (5)$$

$$x_2 = 0\,\sigma_{11} + 0\,\sigma_{12} + 5\,\sigma_{13} + 12\,\sigma_{14} \quad (6)$$

In the same manner , x_3 and x_4 from the second figure can be related to more variables σ_{21} , σ_{22} , σ_{23} , and σ_{24} by the following equations :

$$x_3 = 0\,\sigma_{21} + 12\,\sigma_{22} + 9\,\sigma_{23} + 0\,\sigma_{24} \qquad (7)$$

$$x_4 = 0\,\sigma_{21} + 0\,\sigma_{22} + 5\,\sigma_{23} + 12.5\,\sigma_{24} \qquad (6)$$

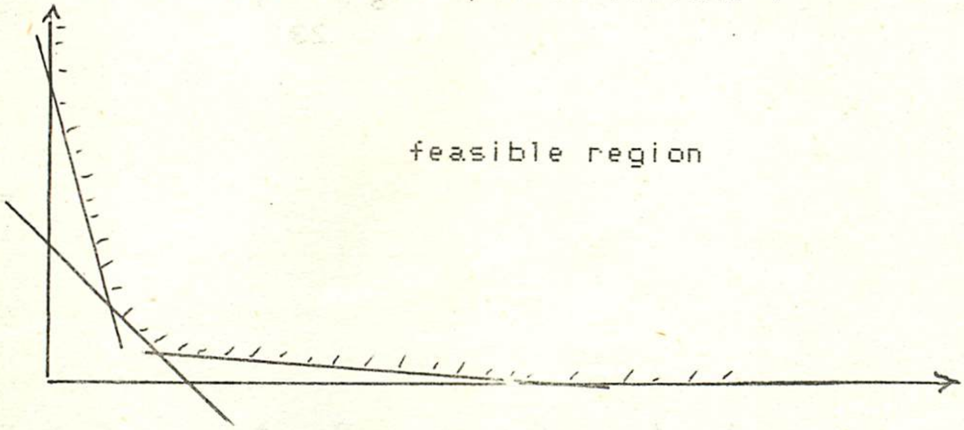
&

$$\sigma_{21} + \sigma_{22} + \sigma_{23} + \sigma_{24} = 1 \qquad (9)$$

where σ_{2j} are the weight for vertices in the second sub-model.

Note that :

A slight complication arises when the feasible regions of some of the sub-models are open as follows :



Anyhow, this complication is easily dealt with and fully explained in Dantzig (1963).

We can use equations (5) , (6) , (7) , and (8) to substitute for x_1 , x_2 , x_3 , and x_4 in the objective function and the single common constraint of the overall model.

The process I and process II constraints of the two sub-models will be satisfied as long as the σ_{ij} are non-negative and satisfy the convexity constraints (4) and (9) .

In this way , the multi-plant model can be reformulated as follows :

Max.

$$f(\sigma) = 200\,\sigma_{12} + 250\,\sigma_{13} + 180\,\sigma_{14} + 120\,\sigma_{22} + 165\,\sigma_{23} + 187.5\,\sigma_{24}$$

s.to:

Material :

$$80\,\sigma_{12} + 90\,\sigma_{13} + 48\,\sigma_{14} + 48\,\sigma_{22} + 56\,\sigma_{23} + 80\,\sigma_{24} <=$$

Convexity 1 :

$$\sigma_{11} + \sigma_{12} + \sigma_{13} + \sigma_{14} =$$

Convexity 2 :

$$\sigma_{21} + \sigma_{22} + \sigma_{23} + \sigma_{24} =$$

This model is known as the MASTER MODEL. It can be interpreted as a model to find the optimum mix of extreme-point solutions of each of the sub-models.

At any internal price (shadow price) p directed to Plants A and B, each of these plants produces an extreme-point solution. Such solutions are known as PROPOSALS since they represent PROPOSEd solutions from the sub-models given the provisional internal price p for the material.

The PROPOSALs are the columns of the coefficients in the MASTER Model corresponding to a particular vertex of a sub-model. For example, the PROPOSAL from the third vertex of the first sub-model is the column :

$$\begin{bmatrix} 250 \\ 90 \\ 1 \\ 0 \end{bmatrix}$$

This PROPOSAL is given a weight of σ_{13} in the MASTER Model.

The role of the MASTER Model is to choose the best combination of all the PROPOSALs which have been obtained.

Note that :

In practice, the MASTER Model would be of impractical use. It would generally have far fewer constraints than the original model. There will be the same number of common constraints as the overall model. Each sub-model, however, has been condensed down into a single convexity constraint such as convexity 1 in the example above.

Unfortunately, the saving in constraints will generally be more than offset by a vast expansion in the number of variables. We will have a σ_{ij} variable for each vertex of each sub-model.

In practice, the great majority of PROPOSALs corresponding to these variables will be zero in an optimal solution.
For a MASTER Model with a relatively small number of constraints but a very large number of variables, the great majority of variables will never enter the BASIS.

We therefore go to a practical algorithm more widely used in mathematical programming. In this algorithm, columns are generated in course of optimization. A column (PROPOSAL) is added to the MASTER Model only when it seems worthwhile.

We therefore deal with only a subset of possible PROPOSALs. Such a truncated model is known as a RESTRICTED MASTER Model.
PROPOSALs are added to (and sometimes deleted from) the RESTRICTED MASTER Model in the course of optimization.

In general, only a very small number of the potential PROPOSALs will ever be generated and added to the RESTRICTED MASTER Model.

In order to describe this new Model, consider again the small multi-plant model mentioned above. Instead of using the MASTER Model (we were lucky enough to be able to obtain it from the graphical consideration in this very simple example), we will work with a RESTRICTED MASTER Model.

To start with, take only the PROPOSALs corresponding to σ_{11} and σ_{13} from the sub-model of Plant A and σ_{21} and σ_{24} from the sub-model of Plant B. This choice is largely arbitrary (<u>how it is made is not important to this discussion</u>).

In practice, a number of good PROPOSALs from each sub-model would be used to make up the first version of the RESTRICTED MASTER Model in order to have a reasonably realistic model with some substance.

The first RESTRICTED MASTER Model is therefore :

$$\text{Max.}\ \sigma = 250\,\sigma_{13} + 187.5\,\sigma_{24}$$

s.to:

Material:

$$90\,\sigma_{13} + 50\,\sigma_{24} \le 120$$

Convexity 1 :

$$\sigma_{11} + \sigma_{13} = 1$$

Convexity 2 :

$$\sigma_{21} + \sigma_{24} = 1$$

When this model is optimized, we can obtain a VALUATION for the Material. This VALUATION is the MARGINAL VALUE of the Material constraint in the optimal solution. Such MARGINAL VALUEs for constraints are sometimes known as SHADOW-PRICEs.*

---

* The SHADOW-PRICEs are discussed much fully together with their economic interpret ion in sec 6.2 , H. P. Williams , Model Building in Mathematical Programming.

Again, the MARGINAL VALUE associated with a constraint such as Material is the rate at which this optimal profit would increase for small (MARGINAL) increase in the right-hande-side.

[Note that : such VALUATIONs for constraints are possible]

Now, if this RESTRICTED MASTER Model is optimized, the SHADOW-PRICE on the Material constraint turns out to be 2.75 pounds. This can be taken as the value of p and used as an INTERNAL-PRICE by which Plants A and B are charged for each Kg of Material which they wish to use.

When Plant A is charged this INTERNAL-PRICE, it will re-form its Objective function taking into account the new charge.
The new Objective function comes from (1) for p=2.78, i.e.,

$$Z_1 = -1.12\, x_1 + 3.18\, x_2 \qquad (10)$$

If this objective function is used with the constraints of the sub-model of the Plant A, we obtain the optimal solution :

$$x_1 = 0 \quad \text{and} \quad x_2 = 12$$

This is clearly corresponds to the vertex (0 , 12) in the first previous figure. The PROPOSAL corresponding to this solution is the column-vector of σ_{14} . This is easily calculated to be :

$$\begin{bmatrix} 180 \\ 48 \\ 1 \\ 0 \end{bmatrix}$$

Then , a new variable (σ_{14} but with a different name) is therefore added to the RESTRICTED MASTER Model with this column of coefficients. This new PROPOSAL represent Plant A 's new provisional production plan given the new INTERNAL PRICE of Material.

For Plant B, when it charged at p = 2.78 per Kg of Material, the previous relation (2) gives its new Objective function as :

$$Z_2 = - 1.12 x_3 + 3.18 x_4 \qquad (11)$$

When this objective function with the constraints of the sub-model of Plant B, we obtain the solution :

$$x_3 = 0 \quad \text{and} \quad x_4 = 12.5$$

This corresponds the vertex (0 , 12.5) of the second previous figure. The PROPOSAL corresponding to this is the column vector of σ_{24} .

<u>This PROPOSAL has already been included in the first RESTRICTED MASTER Model. We therefore conclude that even if Plant B is charged at the suggested rate of 2.78 pounds per Kg of Material, it would not suggest a new PROPOSAL (Provisional production plan).</u>

Having added only the PROPOSAL corresponding to σ_{14} to the RESTRICTED MASTER Mode, it becomes :

$$\text{Max. } \sigma = 250\,\sigma_{13} + 180\,\sigma_{14} + 187.5\,\sigma_{24}$$

s.to:

Material:

$$90\,\sigma_{13} + 48\,\sigma_{14} + 50\,\sigma_{24} \leq 120$$

Convexity 1 :

$$\sigma_{11} + \sigma_{13} + \sigma_{14} = 1$$

Convexity 2 :

$$\sigma_{21} + \sigma_{24} = 1$$

Optimizing this model, the SHADOW PRICE on the Material turns out to be 1.76. We see that the previous VALUATION of 2.78 appears to be an OVERESTIMATE.

The CYCLE is now repeated and each Plant is INTERNALly charged for 1.67 per Kg of Material. This gives the following new Objective functions for Plants A and B as follows :

$$Z_1 = 3.32\,x_1 + 8.32\,x_2 \qquad (12)$$

$$Z_2 = \qquad 3.32\,x_3 + 8.32\,x_4 \qquad (13)$$

Using the Objective function (12) and the constraints of the sub-model for Plant A, the optimal solution becomes :

$$x_1 = 17.5 \quad \text{and} \quad x_2 = 5$$

This is the vertex (17.5 , 5) and gives the PROPOSAL corresponding σ_{13} . Since this PROPOSAL has already been incorporated in the RESTRICTED MASTER Model, Plant A has no new PROPOSAL to offer as a result of the revised INTERNAL PRICE of 1.67 per Kg of Material.

Also, Plant B optimizing its Objective function (13) subject to the constraints of its sub-model yields the solution :

$$x_3 = 0 \quad \text{and} \quad x_4 = 12.5$$

This is the vertex (0 , 12.5) which results in the PROPOSAL corresponding to σ_{24} . Since this PROPOSAL is already present in the RESTRICTED MASTER Model, Plant B also has no further useful PROPOSAL to add as a result of the revised charge of the Material.

We therefore conclude that Plants A and B have been submitted all the useful PROPOSALs that they can. The optimal solution to the latest version of the RESTR+ICTED MASTER Model gives the proportions in which these PROPOSALs should be.

For the present example, the optimal solution to the RESTRICTED MASTER Model is :

$$\sigma_{13} = 0.52 \quad , \quad \sigma_{14} = 0.48 \quad , \quad \sigma_{24} = 1$$

This enables us to calculate the optimal values of x_1, x_2, x_3, and x_4 by considering the vertex solutions of the sub-models correspondin to σ_{13}, σ_{14}, and σ_{24}.

So, we obtain :

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.52 \begin{bmatrix} 17.5 \\ 5 \end{bmatrix} + 0.48 \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

&

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 12.5 \end{bmatrix}$$

This gives us the optimal solution to the overall model as follows :

$$x_1 = 9.17 \quad , \quad x_2 = 8.33 \quad , \quad \text{and} \quad x_4 = 12.5$$

Notice that we have obtained the optimal solution to the overall model without solving it directly. Instead, we dealt with what would generally be much smaller models. The two types of models we have used are the sub-models and the RESTRICTED MASTER Model.

The Significance of Used Models :

THe Sub-Models :

THese models contain the details concern the individual sub-problems. For multi-plant model such as the one used here, the coefficients in the constraints only concern the particular Plant ; that is Process I and Process II, times, and capacities in each Plant.

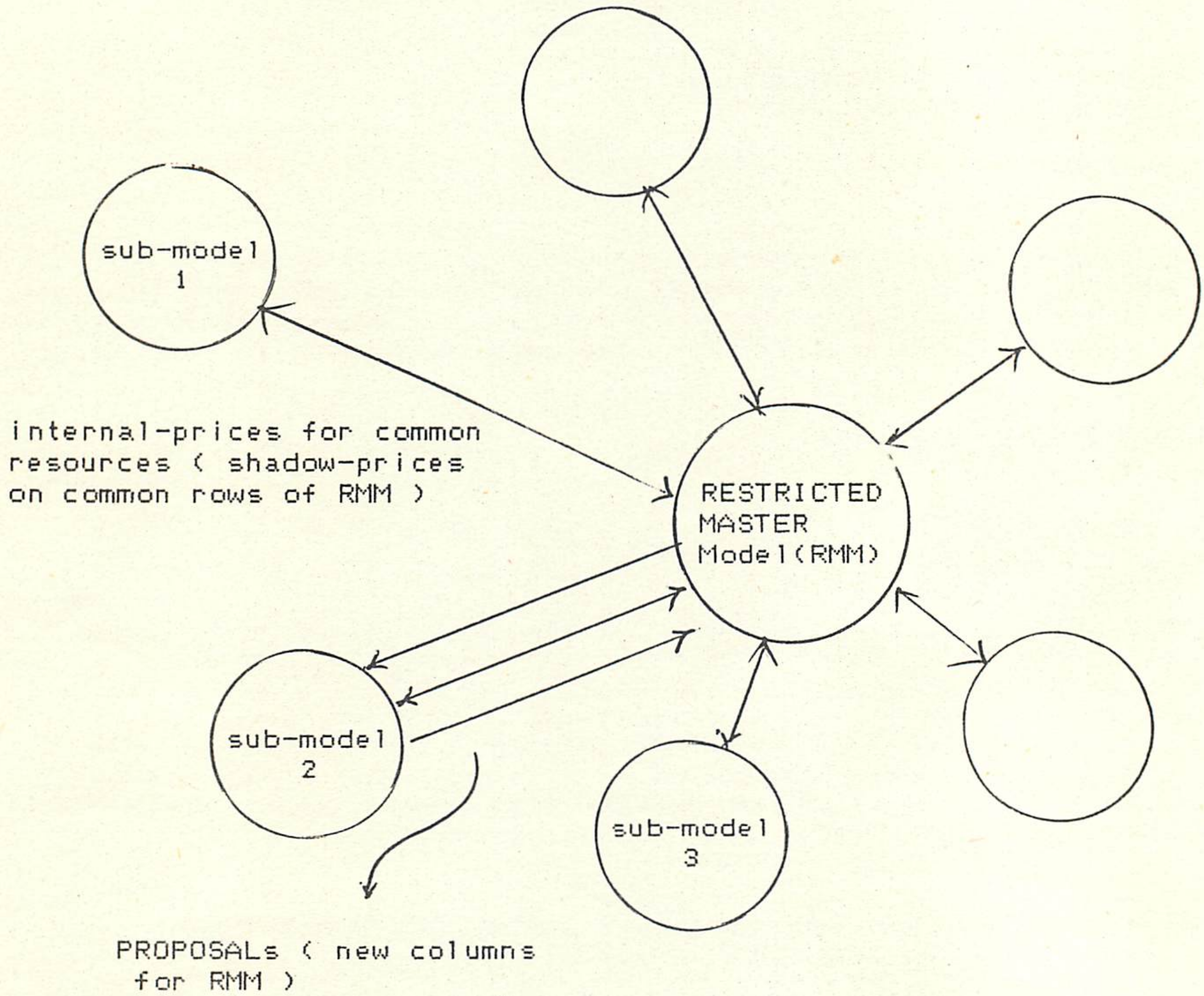
The RESTRICTED MASTER Model :

This RESTRICTED MASTER Model is an overall model for the Organization as a whole but unlike the overall model, it contains none of the technological detail relating to the individual sub-models. Such detail is left to the sub-models. Instead, the constraints for each sub-model are accounted for by a simple convexity constraint. In the present example, we had constraints for Plants A and B reduced to convexity constraints (convexity 1 and convexity 2 respectively page 24). On the other hand, the RESTRICTED MASTER Model does contain the common constraints in full details since its main purpose is to determine suitable VALUATIONs for the resources of limited supply, represented by these common constraints.

The Interaction Between Sub-models and RESTRICTED MASTER Model :

This means that it is possible to obtain the optimal solution to the overall model (usually very large) without building and solving it directly.

The process of interaction between the sub-models and the RESTRICTED MASTER Model can be represented diagrammatically as follows :



Conclusion :

In a multi-plant Organization, the individual Plants usually or probably be geographically separated. This would make the avoidance of including all their technical details in one Central-Model (RESTRICTED MASTER Model) desirable. In this case, each Plant might build and maintain its own model inside its own Plant, and solve it on its own computer. The Central Planning Board would maintain a RESTRICTED MASTER Model on another computer which would be linked to the computers of the individual Plants. Each model would then be executed independently but could supply the essential information of PROPOSALs and INTERNAL PRICEs to the other Models. It would then be possible to use such a system automatically to obtain an overall optimal solution for the Organization under consideration.

The process of DECOMPOSING a LP Model has a very good interest for Planners and Economists since it clearly represents a system of DECENTRALIZED-PLANNING. The existence of DECOMPOSITION Algorithm(s) such as the Dantzig-Wolfe Algorithm demonstrates that it is possible to plan for a Decentralized-PLANNING which achieves an optimal solution for the overall Organization. This is done by allowing the sub-Organization (Plant) to decide its own optimal polices, given limited resources from the Centre. In case of Dantzig-Wolfe Algorithm, these limited resources take the form of INTERNAL PRICEs. For other Algorithms, they may take the form of allocations.

- A discussion of large number of DECOMPOSITION Algorithms and their relation to Decentralized-Planning in real life is given in Atkins (1974).

- An account of computation; experience using Dantzig-Wolfe DECOMPOSITION is given by Beale, Hughes, and Small (1965).

- A very full description of the computational side of DECOMPOSITION is given by Lasdon (1970).

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```
2 REM ****************************************************************************
4 REM ... THIS PROGRAM FOR THE DECOMPOSITION PRINCIBLE OF
6 REM ... MULTIDIVISIONAL PROBLEMS (ANGULAR STRUCTRUE )
8 REM ...
10 REM ... VARIABLES DESCRIBTION
12 REM ... ====================
14 REM ...
16 REM ...  NN = THE TOTAL NO. OF SUBPROBLEMS
18 REM ...  ND =  "   SUBPROBLEM INDEX
20 REM ...   M =  "  NO. OF CONSTRAINTS ( ROWS ) FOR THE REFORMULATED
22 REM ...                  MASTER PROBLEM
24 REM ...   N =  "  NO. OF VARIABLES ( COLUMNS ) "    "  FORMULATED
26 REM ....                 MASTER PROBLEM
52 REM ...
54 REM ...  MO(I) = THE NO. OF ROW.S FOR THE SUBPROBLEM (I), I=1, NN
60 REM ...  NO(J) =  "   "   " COLM.S "   "    "         (J), J=1, NN
62 REM ...  MO(O) =  "   "   " ROW,S  "   "  COMMON CONSTR.'S
64 REM ...  ==================================================================
66 REM ...
68 REM ... THE FOLLOWING FOR THE ORIGINAL PROBLEM
70 REM ...*************************************
72 REM ...
74 REM ...  A(I, J, ND) = THE COIEFF.  MATRIX FOR THE COMMON CONSTRAINTS,
76 REM ...                I=1, MO(O),
78 REM ...                J=1, NO(ND)
80 REM ...               ND=NN+1, 2*NN
82 REM ...
84 REM ...  A(I1, J1, ND)= THE COEIFF. MATRIX FOR THE SUBPROBLEM NO. (ND)
86 REM ...                I1=1, MO(ND),
87 REM ...                J1=1, NO(ND),
88 REM ...                ND=1 , NN
90 REM ...
92 REM ...  ==================================================================
94 REM ... BJ(I,O)   = THE R.H.S CONSTR.S FOR THE COMMON CONSTR.S , I=1, MO(O)
98 REM ... BJ(I1,ND)=  "    "       "     "    "  SUBPROBLEM NO. (ND), ND=1, NN
100 REM .. CJ(I,ND) =  "  ROW VECTOR FOR THE COIEFF. OF THE OBJ. FUNCTION FOR
102 REM ...            SUBPROBLEM NO. ND , ND=1, NN
104 REM ...
106 REM ... THE FOLLOWING FOR THE REFORMULATED MASTER PROBLEM
108 REM ...==================================================
110 REM ... B(I,J)=THE BASIS MATRIX
112 REM ... CB(J) = "  COIEFF. OF THE OBJ. FUNCTION FOR THE B.V,S
114 REM ... IB(I) = "  INDEX     "  "   B.V,S
116 REM ...
118 REM ...    I1 = "  PIVOT ROW
120 REM ...    J1 = "   "   COLUMN
122 REM ... X(I1) = "  LEAVING BASIC VAR.  ( L.B.V )
124 REM ... X(J1) = "  ENTERING  "    "    ( E.B.V )
126 REM ...
132 REM ... *******************************************************************
134 REM ...
150 DIM A(20,30,5),B(20,20),MO(6),NO(6),E(20,20),IB(20),IN(20),CB(20),CN(20)
160 DIM RB(20),RN(20),X(20),P(20),IKK(6),B2(20,20),IA(20),BJ(20,6),CJ(20,5)
170 DIM IB1(20),IN1(20),CB1(20),CN1(20),X1(20,20),WJ(20),C(20),C1(20)
180 DIM BO(20),XX(20),B1(20,20)
190 REM **********************************************************************
200 REM ************** WE READ HERE THE DATA FORE THE ORIGINAL PROBLEM *******
208 REM **********************************************************************
209 READ NN
```

```
215 F$="######.###"
221 FOR I=1 TO NN :READ IKK(I):NEXT I
230 FOR I=0 TO NN
240 READ MO(I)
250 NEXT I
260 FOR J=1 TO NN
270 READ NO(J)
280 NEXT J
290 FOR K=1 TO NN
300 NK=NO(K)
310 FOR I=1 TO MO(0)
320 FOR J=1 TO NK
330 READ A(I,J,K)
342 NEXT J
345 NEXT I:NEXT K
360 FOR K=NN+1 TO 2*NN
370 L=K-NN
380 MM=MO(L)
390 NK=NO(L)+MM
400 FOR I=1 TO MM
410 FOR J=1 TO NK
420 READ A(I,J,K)
435 NEXT J
440 NEXT I:NEXT K
450 FOR K=0 TO NN
460 FOR I=1 TO MO(K)
470 READ BJ(I,K)
480 NEXT I:NEXT K
490 FOR J=1 TO NN
500 FOR I=1 TO NO(J)
510 READ CJ(I,J)
520 NEXT I:NEXT J
540 READ M
550 REM ********
560 REM ********
570 FOR I=1 TO M
580 READ IB(I)
590 NEXT I
600 REM
610 REM *********
620 FOR I=1 TO M
630 READ CB(I)
640 NEXT I
650 FOR J=1 TO M
660 READ BO(J)
670 NEXT J
680 [illegible] : KK=0 : IKK(0)=0
690 REM ****
700 REM *********
702 FOR I=1 TO M :FOR J=1 TO M
703 IF I=J THEN 705
704 B1(I,J)=0 :GOTO 706
705  B1(I,J)=1
706 NEXT J : NEXT I
707 ITER=0
709 ND=NN
719 FOR I=1 TO M
720 FOR J=1 TO M
730 IF I=J THEN 790
740 E(I,J)=0
```

```
750 B(I,J)=0
780 GOTO 820
790 E(I,J)=1
800 B(I,J)=1
820 NEXT J
830 NEXT I
840 REM ****************************************************
850 REM ********** WE SOLVE HERE THE L.P SUBPROBLEM NO. (ND)
860 REM ********** BY USING THE REVISED SIMPLEX METHOD
870 REM ****************************************************
880 REM
930 N21=NO(ND)+MO(ND)
940 NM1=N21-MO(ND)
945 M1=MO(ND)
946 NM3=NM1
960 REM ****************************************************************
965 REM *************                                   -1
970 REM *************      WE COMPUTE HERE   WJ=(CB)*(B)      *AJ -CJ
975 REM *************                                    1;MO
980 REM*****************************************************************
990 FOR I=1 TO MO(O)
1000 C(I)=0
1010 FOR J=1 TO M
1020 C(I)=C(I)+CB(J)*B1(J,1)
1030 NEXT J
1040 NEXT I
1050 FOR I=1 TO NO(ND)
1060 C1(I)=0
1070 FOR J=1 TO MO(O)
1080 C1(I)=C1(I)+C(J)*A(J,I,ND)
1090 NEXT J
1100 NEXT I
1110 FOR I=1 TO NO(ND)
1120 C1(I)=C1(I)-CJ(I,ND)
1130 NEXT I
1140 REM ***************************************************************
1150 REM *************** WE CREATE HERE CB1(J) ,CN1(J),IB1(J),IN(J)
1160 REM ***************************************************************
1180 FOR I=1 TO NM1
1190 IN1(I)=I
1200 NEXT I :LPRINT
1210 FOR J=NM1+1 TO N21
1360 IB1(J-NM1)=J
1370 NEXT J:LPRINT
1380 FOR J=1 TO NO(ND)
1390 CN1(J)=C1(J)
1400 NEXT J
1435 FOR I= NM1+1 TO N21
1445 CB1(I-NM1)=0
1455 NEXT I
1475 REM *****************
1480 GOSUB 4345
2465 GOSUB 4115
2475 GOSUB 3825
2477 KK=KK+1:LPRINT : IF RRR=10 THEN 2486
2478 FOR I=1 TO NO(ND)
2479 FOR J=1 TO MO(ND)
2480 IF IB1(J)=I THEN 2484
2481 NEXT J
2482 X1(I,KK)=0:PRINT X1(I,KK)
```

```
2483 GOTO 2485
2484 X1(I,KK)=X(I):PRINT X1(I,KK)
2485 NEXT I
2486 GOSUB 3825
2505 GOSUB 4115
2515 IF J1<>(-1) THEN 2665
2525 REM ...
2535 REM ...OPTMAL SOLUTION
2545 REM ...-----------------
2555 REM ...
2557 GOSUB 4356
2560 LPRINT
2565 LPRINT"     *** WHICH IS THE OPTIMAL SOLUTION ***"
2575 REM ...
2615 WJ(ND)=F
2625 IF ND >=NN THEN 2734
2645 GOTO 707
2665 FOR I=1 TO M1
2666 P(I)=0
2667 FOR J=1 TO M1
2668 P(I)=P(I)+B(I,J)*A(J,J1,ND+NN)
2669 NEXT J:NEXT I
2679  GOSUB 3485
2680 IF I1<>(-1) THEN 2710
2690 LPRINT"    ***UNBAUND SOLUTION ***"
2700 GOTO 707
2710 ITER=ITER+1
2720 GOSUB 3210
2730 GOTO 2465
2734 LPRINT
2735 LPRINT"         ..............................................................
.........  "
2740 LPRINT"         .     WE TEST HERE IF ALL WJ(J) >=0 ,I=1,2...,NN
         .     "
2750 LPRINT"         .     IF YES THEN THE CURRENT BASIC FEASIBLE SOLUTION IS OPTIMAL
 ,AND    ."
2760 LPRINT"         .     WE SHALL IDENTIFIY IT FOR THE ORIGINAL PROBLEM
         ."
2770 LPRINT"         .           ELSE
         ."
2780 LPRINT"         .      FIND  W=MIN [W1,W2,...,WNN ]
         ."
2790 LPRINT"         ..............................................................
.........  "
2793 LPRINT:LPRINT
2794 RRR=10
2795 JJJ=-1 : GGG=0
2800 FOR I=1 TO NN
2810 IF WJ(I) >= 0 THEN 2830
2811 WJ(I)=ABS(WJ(I))
2812 IF WJ(I) <  GGG THEN 2830
2814 JJJ=I
2816 GGG=WJ(I)
2830 NEXT I
2832 IF JJJ=-1 THEN 2859
2834 ND=JJJ:J1=IKK(JJJ):M1=M
2835 FOR I=1 TO M:X(I)=0:FOR J=1 TO M:X(I)=X(I)+B1(I,J)*BO(J):NEXT J:NEXT I
2837 PRINT J1,ND
2838 FOR I=1 TO MO(0)
2840  P(I)=0
```

```
.    IF YES THEN THE CURRENT BASIC FEASIBLE SOLUTION IS OPTIMAL ,AND   .
.    WE SHALL IDENTIFIY IT FOR THE ORIGINAL PROBLEM                    .
.          ELSE                                                        .
.     FIND  W=min {w1,w2,...,wnn }                                     .
........................................................................
```

```
                 *************************
                 *  SUBPROBLEM NO : 1    **
                 *************************

ITERATION NO. 0
****************

 X( 3 )=     20.000
 X( 4 )=     25.000
 Z( 1 )=    -24.000


ITERATION NO. 2
****************

 X( 1 )=      2.000
 X( 2 )=      3.000
 Z( 1 )=      0.000

*** WHICH IS THE OPTIMAL SOLUTION ***


                 *************************
                 *  SUBPROBLEM NO : 2    **
                 *************************

ITERATION NO. 0
****************

 X( 3 )=      2.000
 Z( 2 )=      0.000


ITERATION NO. 1
****************

 X( 1 )=      3.000
 Z( 2 )=    -24.000

*** WHICH IS THE OPTIMAL SOLUTION ***

........................................................................
.      WE TEST HERE IF ALL WJ(J) >=0 ,I=1,2...,NN                      .
```

```
.     IF YES THEN THE CURRENT BASIC FEASIBLE SOLUTION IS OPTIMAL ,AND   .
.     WE SHALL IDENTIFIY IT FOR THE ORIGINAL PROBLEM                    .
.           ELSE                                                        .
.      FIND  W=min {w1,w2,...,wnn }                                     .
.........................................................................
```

```
************************
*  SUBPROBLEM NO : 2  **
************************
```

```
ITERATION NO. 0
***************

 X( 3 )=      9.000
 Z( 2 )=    -24.000


ITERATION NO. 0
***************

 X( 3 )=     12.000
 Z( 2 )=      0.000

*** WHICH IS THE OPTIMAL SOLUTION ***
```

```
.........................................................................
.     WE TEST HERE IF ALL WJ(J) >=0 ,I=1,2...,NN                        .

.     IF YES THEN THE CURRENT BASIC FEASIBLE SOLUTION IS OPTIMAL ,AND   .
.     WE SHALL IDENTIFIY IT FOR THE ORIGINAL PROBLEM                    .
.           ELSE                                                        .
.      FIND  W=min {w1,w2,...,wnn }                                     .
.........................................................................
```

```
THE OPTIMAL SOLUTION TO THE ORIGINAL PROBLEM
********************************************
 X( 1 )       2.000
 X( 2 )       3.000

 X( 3 )       2.000
 X( 4 )       0.000

 OBJ.FUNCTION =     42.000
```