ARAB REPUBLIC OF EGYPT

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo (1322)

(1322)

EXPERIMENTAL OPTIMIZATION BY SIMULATION TECHNIQUE

BY

MOTAZ KHORSHID

AND

Alain CHECROUN

March 1982

CAIRO
SALAH SALEM St.-NASR CITY

The purpose of this paper is twofold , (i) to propose an integrated framework for studying management systems by simulation and (ii) to evaluate the possibility of using Simulation as an experimental optimization technique.

A job shop simulation model , which can be used to test both materials handling and dispatching rules , was developed in order to demonstrate the applicability of the proposed procedures* .

---

* This paper is published in the proceedings of the international congress on applied System research and cybernetics - pergamon press - New york 1981.

1 - INTRODUCTION

The attractive features that computer simulation offers, have encouraged operations researchers and statisticians to improve its practice through the use of various statistical techniques to analyse the obtained results [11,12,13,16,17,18,19,23].

The commun conclusion of these studies indicate the need to consider two main problems when investigating simulated systems. First, the particular circumstances of simulation experiments that may lead to misinterpretation of results[1]. Second, the difficulty to achieve the assumptions of the statistical theory, as independence, normality, and homogeneity of variances. So, either we hope that the selected techniques are not influenced by assumptions violation, or we manipulate simulation runs to match them.

The purpose of this research is two fold. First, to propose an integrated framework for studying complex systems by simulation. Second, to evaluate the possibility of using simulation to find the optimum solution. In order to demonstrate the applicability of the proposed procedures, we developed a model that can be used to test both materials handling and dispatching rules in a job shop production system.

As any statistical investigation, simulation user should consider the following steps when developing an experimental strategy:-

i) The choice of a sampling plan that specify how each test run is to be executed and how to determine simulation run length.

ii) The development of an experimental design that will yield the desired information.

iii) The selection of a data analysis technique in order to reach some conclusion about the simulated system.

(1) A list of these circumstances can be found in Fishman (11)P.262

In section 2, we propose an integrated mathematical base for studying management systems by simulation. The proposed optimization techniques, are presented in section 3. In section 4, we explain briefly the simulation model that will be the setting of the study. Finally, the design and analysis of two experiments are discussed in section 5.

2 - MATHEMATICAL FORMULATION

In many simulation models, the process of interest apears as a stochastic process (1).

$$\{Y(t); -\infty \leq t \leq \infty\} \qquad (1)$$

Since our research concerne discrete event digital simulations, we assume that during a small interval of time, the process shows little, if any change so that observing $Y(t)$ at periodic intervals result in no loss of information. In a more detailed manner, the sequence,

$$\{Y_t \ , \ t = 1,2...,\infty\} \qquad (2)$$

corresponds to the process $\{Y(t)\}$ at all integer values of the index t.

The index t may be the time; for example Y_t may define the number of jobs in a production system at the instant t of simulation. It may simply denote order, for example Y_t may represent the waiting time for the t^{th} customer to receive service in a queueing model.

In order to study several processes of interest, generated by different environmental conditions or input specifications, we should aquire a quantitative characterization of each of them.

---

(1) we will consider only the stochastic simulation models as most management systems inevitably appear random to some degree in nature.

The mean of the process;

$$u = E\{Y_t\} \quad (3)$$

serves generally as the mathematical descriptor, and by definition, the variance[1];

$$Var\,(Y_t) = E\,[(Y_t - u)^2] \quad (4)$$

and the autocovariance function,

$$R_s = E\,[(Y_t - u)(Y_{t+s} - u)];\; s = 1,2,\ldots, \quad (5)$$

Then regardless of the experimental objective and the type of simulated model, we should define a procedure for estimating "u" and for determining the accuracy of this estimator, i;e to select a sampling plan.

2.1 SAMPLING PLAN:

In simulation experiments sample size can be increased by prolonging simulation run, or by conducting separate runs. Consequently, there is two sampling plans, the autocorrelated observations and the replicated runs.

i) Autocorrelated observations:
Let $\{Y_t, t \in n\}$ a time series of length n observed during the simulation run, the mean u can be estimated by:

$$\bar{Y} = n^{-1} \sum_{t=1}^{n} Y_t \quad (6)$$

where $\bar{Y}$ is called"simular response" .

In order to determine the accuracy of $\bar{Y}$, we need to estimate its variance 12 :

$$Var\,(\bar{Y}) = n^{-1} \left[Var\,(Y_t) + 2 \sum_{s=1}^{n-1} (1-s/n)\, R_s \right] \quad (7)$$

(1) Assuming $\{Y_t\}$ a covariance stationary sequence [11,18]

Where Var(Y_t) and R_s are defined by (4),(5) and can be estimated by the formulas [18] :-

$$\hat{Var}(Y_t) = (n-1)^{-1} \sum_{t=1}^{n} (Y_t - \bar{Y})^2 \qquad (8)$$

$$\hat{R}_s = (n-s)^{-1} \sum_{t=1}^{n-s} (Y_t - \bar{Y})(Y_{t+s} - \bar{Y}); \qquad (9)$$

$$s = 1,2,..,(n-1)$$

ii) Replicated runs :-

We can generate independent observations in simulation experiment by repeating the run using different random numbers. Let

k = number of runs
n'= number of observations per run
n = total number of observations (=n'k)
$Y_{t,j}$= the observation generated at time t of run j.

The mean of each run is defined by:

$$Y_{n',j} = n'^{-1} \sum_{t=1}^{n'} Y_{t,j} \quad ; \quad j = 1,2,...,k \qquad (10)$$

and then simular response is calculated by:

$$\bar{Y} = k^{-1} \sum_{j=1}^{k} Y_{n',j} \qquad (11)$$

Since the sequence $\{ Y_{n',j} \; ; \; j = 1,2,...,k \}$ consist of k independent observations, Var $(\bar{Y})$ is given by:

$$Var(\bar{Y}) = Var(Y_{n',j})/k \qquad (12)$$

and can be estimated by:

$$\hat{Var}(\bar{Y}) = S^2/k \qquad (13)$$

where $S^2 = (k-1)^{-1} \sum_{j=1}^{k} (Y_{n',j} - \bar{Y})^2$

2.2 SIMULAR RESPONSE FUNCTION:

When experimenter select a sampling plan, he can proceed to the study of the response Y as a function of the environmental conditions or the experimental factors $(x_1, x_2, \ldots, x_p)$. Factors are categorized as qualitative and quantitative. Examples of qualitative factors are policy specifications, such that alternative dispatching rules in job shop, or discrete environmental conditions. Quantitative factors are examplified by input parameters that can usually be thought as continious variates.

Since the stochastic features are spowned in the simulation model by incorporating the random number seed as an integrated part of input specifications, the response $\bar{Y}$ becomes a random variable because it is a transformation not only of the experimental factors $(x_1, x_2, \ldots, x_p)$, but also of the randomly selected seed " r " [(1)]. This relation is defined by:

$$\bar{Y} = \phi\ (x_1,\ x_2, \ldots, x_p;\ r\ ) = \phi\ (\vec{x}\ r\ ) \qquad (14)$$

Although the random number seed " r " may be conceptually defined as a real number between o and 1, it could not be classified as quantitative factors, because $\bar{Y}$ will probably not be a continious function of it. Then it is unique among the other quantitative factors and expression (14) can be written:

$$\bar{Y} = \phi(x_1,\ x_2, \ldots, x_p) + \mathcal{E}\ (r\ ) \qquad (15)$$

where $\mathcal{E}\ (r\ )$ is a random effect dependent upon the random number seed :

Further, if we assume that $\mathcal{E}\ (r\ )$ is independent of experimental factors and has zero expectation, the expected simular response can be defined as:

---

(1) a detailed discussion of this point can be found in Mihram [18]

$$E(\bar{Y}) = \phi(x_1, x_2, \ldots, x_p) = \phi(\vec{x}) \qquad (16)$$

It is the nature of the unknown function $\phi(\vec{x})$, termed simular response function, that we try to investigate by the simulation experiment.

In practical situations, any attempt to develop the exact form of $\phi(\bar{x})$ could not be justified from economical point of view. In addition, for many experimental purposes, it is unnecessary to consider the form of the true function. A flexible graduating function, will often be satisfactory to express the relation ship between $E(\bar{Y})$ and the p factors. Further more, many experimental strategies, divide the whole operability region of the factors space, into a number of smaller regions of immediate interest. Withen these regions, the experimenter may feel it is reasonable to represent $\phi(\vec{x})$ by a known functional form, for example a polynomial, although he may know that such representation would be quite inadequate over the whole operability region.

As a result of the previous discussion, $\phi(\vec{x})$ may be approximated by:

$$E(\bar{Y}) \simeq f(x_1, x_2, \ldots, x_p \,;\, \theta_1, \theta_2, \ldots, \theta_1) = f(\vec{x}, \vec{\theta}) \quad (17)$$

Where f is a known functional form indexed by some unknown vector $\vec{\theta}$.

The way with which we investigate the function $f(\vec{x}, \vec{\theta})$, in order to yield information about simulated system, depends on the experimental objectives. Accordingly we distinguish two types of experiments, Exploratory and Optimization.

2.3 EXPLORATORY EXPERIMENTS :

If experimenter wishs to study the relative importance of the factors $\bar{x}$ as they affect the simular response $\bar{Y}$, he may use one of the following designs:

i) Screening designs

At the begining of investigation, specially with complicated simulation models, the experimenter may face the problem of so many factors. It may happen that not all the p factors are important but only a few, say p' factors. Therefore, he screen for them.

The statistical literature contains many designs, for example, fractional factorial designs [4,8,17], random disigns [24], group screening designs [21], and super saturated designs [3]. The investigator has to select the design which fit his particular experimental situation.

ii) Designs for estimating parameters

When experimenter has a prior knowledge about the simulated system, either due to theoritical background or from previous investigations. He may assume that a particular functional form $f(\vec{x},\vec{\theta})$ is a good approximation of the true response function $\phi(\bar{x})$, in such a way that bias due to inadequacy of $f(\vec{x},\vec{\theta})$ to represent $\phi(\vec{x})$ can be neglected. So his goal will be to select an experimental plan to estimate the unknown parameters $\vec{\theta}$ with high accuracy. Two basic approaches were proposed to develop an experimental design, either to use a simple factorial or fractional factorial design [4,8,17], or to select a design based on a variance criterion as D - optimal designs [15].

iii) Designs for exploring response surface

When knowledge about simulated system is limited, the object is to approximate, withen a given region of factors space, $\phi(\vec{x})$ by some graduating function $f(\vec{x},\vec{\theta})$ which most closely represent the true simular response function.

Accordingly the following design requirements have to be considered:

a) The design should allow sequentialization so that designs of higher order can be developed with minimum loss of information.

b) The design should consider not only sampling variation but also bias error.

c) The design should allow a check to be made on the representational accuracy of the postulated model.

2.4 OPTIMIZATION EXPERIMENTS:

The purpose of these experiments is to find the combination of factor levels at which simular response function $\phi(\vec{x})$ is optimized.

A particular attention will be devoted, in the next section, to the explanation and the applicability of this experiment.

To summarize, any attempt to develop an experimental method for investigating management systems by simulation, should select a sampling plan which define an efficient procedure for estimating the variance of simular response $\bar{Y}$. The estimated variance measures the accuracy of results and then can be used to determine the appropriate run length. Having accomplished this task, an experimental strategy may be defined for investigating the interdependence between the simular response $\bar{Y}$ and the experimental factors.

3 - EXPERIMENTAL OPTIMIZATION TECHNIQUES

The choice of an experimental strategy that will yield an optimal solution depends on the type of factors in the simulation model. When all factors are quantitative, an optimum seeking routine can be used in order to find the combination of factor levels that optimize the response $\bar{Y}$. But the existance of some qualitative factors, as policy specification or operating rules, limit the search procedure to the choice between a number of experimental alternatives.

3.1 The search for an optimum combination of factor levels;

When all factors are quantitatives, the investigator will wish to find in the smallest number of simulation runs, the Point $(x^{\circ}_{1}, x^{\circ}_{2}, \ldots, x^{\circ}_{p})$, withen the factors space, at which $\phi(\vec{x})$ is a minimum or a maximum.

Since simular response function is not known in advance and is subjected to random variation, we think that the most reasonable strategy will be to fit a sequential program of investigation consisting of the following steps :-

i) Divide the whole region of interest into a number of small subregions, so that we can explore adequately a small subregion with a moderate number of simulation runs.

ii) Use the results obtained in one subregion to move to a second in which simular response $\bar{Y}$ is better.

iii) Repeate the previous steps until the attainment of a near stationary region where no improvement in the simular response can be achieved.

iv) In this limited region, conduct a more detailed experiment in order to determine the local nature of the function $\phi(\vec{x})$.

In the following sections we discuss breifly the two main elements of this sequential program, seeking a near stationary region and exploring it.

3.1.1 Seeking a near stationary region

When the starting conditions of simulation are fairly remoted from the stationary point, an optimum seeking technique will be needed to move rapidly through the factors space to a near stationary region.

Brooks [6] compared four optimum seeking methods, steepest ascent, univariate, factorial, and random search. He concluded that, when sequential investigation is possible, steepest ascent seems to be superior to the others, except in case of large number of factors, where random search is more efficient(1).

Recently, Smith [25] showed that random search should not necessarily be the search technique selected in practical situations even in case of so many factors and he recommended the use of the steepest ascent.

Since the steepest ascent method is explained in detail in Box and Wilson [5] and Davies [8] , we just mention, here, some remarks that should be considered when applying the method to simulation experiment(2).

i) Since we use the error variance to test the adequacy of the fitted function and the significance of model parameters, an accurate estimate of the variance of $\bar{Y}$ is needed in order to avoid any wrong conclusion.

ii) As the steepest ascent method is affected by the size of the experimental error [5] , we may try to reduce it, by selecting a minimum variance design (see section 2.3), by increasing simulation run length, and if possible, by using a variance reduction techniques.

iii) If possible, provision should be made to estimate some of higher order coefficients that were not included in the postulated model. The study of these coefficients will provide some indication of whether the assumption that these terms can be ignored is a reasonable one or not.

3.1.2 Exploring the near stationary region

The experimenter may arrive at a near stationary region either as the result of successive application of steepest ascent method,

(1) This is explained by the fact that, in random search algorithm, the number of experimental trials is not a function of the number of factors.

(2) The method will be explained using an example model in section 5

or because he has already found it at the begining of his investigation. In either cases, only immediate neighbourhood need be explored to determine the local nature of response function $\phi(\bar{x})$ and this may be done without excessively large number of experimental points.

Although many authers have ignored the exploration of near stationary region, and are only satisfied by finding the approximated optimum point, we think that it is an important step in case of simulation for the following reasons:-

First, it should be remembered that because of random error and possible lack of fit between fitted equation and the true response $\phi(\bar{x})$, it must not be implied immediatly that the true surface has a maximum (or minimum) at the selected point. So in practice further exploration and confirmatory runs should be performed arround the stationary point of the fitted surface.

Second, the discovery of factors dependence of a particular type may give us an idea about the cost of departure from the optimum point, if it was impossible to reach it in practice. For example, finding the direction of a stationary ridge means that we can know the different combinations of factor levels that optimize the response $\bar{\bar{Y}}$. Then the choice between these alternatives can be decided according to the cost of each combination or according to an auxiliary response.

Two exploratory techniques are proposed in the statistical literature, Canonical analysis [5,8] and Ridge analysis [9]. The authors matched the two techniques in a single computer program in order to have more robust conclusions: This can be done by using canonical analysis to reveal the factor dependence withen the local stationary region, then using ridge analysis to evaluate the locus of the absolute maximum or minimum when augmenting the experimental region.

3.2 The choice between experimental alternatives

When simulation model contains qualitative factors, as managerial policies or operating rules, the search procedure will be reduced to the optimum choice between a number of experimental alternatives. More specifically, it is required to find the combination of factor levels corresponding to the best simular response $\bar{Y}$, such that the probability of correct selection (CS) is at least P^*, given that the difference Δ between the best and the next best simular response is at least Δ^*. This may be stated formally as

$$P_r\ (CS/\Delta \geqslant \Delta^*) \geqslant P^* \qquad (18)$$

The previous formulation of the problem permit the use of one of the multiple ranking procedures [2,17,22,23]. Most of these methods assume normality, independence and commun known or unknown variances.

In practical simulation models, the distribution of the response $\bar{Y}$ is not known, variances are not known and tend to differ, so either we manipulate simulation runs to meet these assumptions or we hope that the effect of their violation is negligible.

After consulting several multiple ranking procedures, the authers choosed three of them that seem to be attractive for simulation circumstances. The selected procedures are Bechhofer and Blumenthal [2], Paulson [22], and Sasser et al [23].

Bechhofer method is the only one extensivly tested for its sensitivity to assumptions violation, it is quite robust and relatively efficient [17]. Unfortunately, it cannot capitalize on favorable configurations of population means. Paulson's procedure give us the possibility to eliminate inferior populations, so it might be advantageous when comparing a large number of alternatives.

The authors conducted a comparative study using two simulation models, a job shop production system [16] and a dynamic model of the firm [19] . The results of this study indicate that Paulson method is the most efficient method, specialy when the number of alternatives is large. Bechhofer method seems to be the best procedure from robustess point of view, but unfortunately it require large sample sizes when deeling with a large number of alternatives. Sasser method, which is a heuristic version of Bechhofer, is less efficient than Paulson procedure.

To conclude, the results of this emperical study(1), show that no ideal procedure, that consider the particular circumstances of simulation experiment, does exist in present time. We recommend Paulson method when experimenter is wary about computer time and the number of alternatives is sufficiently large. Otherwize, Bechhofer procedure seems to be the most robust one and it is relatively efficient.

4. THE PROPOSED SIMULATION MODEL

Most researchs on job shop production systems using simulation technique assume that the major system constraint was machine availability [1, 7, 10] . Recently a particular attention has been devoted to the study of labor and machine limited production systems [14, 20] , where the performance of the shop was evaluated considering both dispatching and labor assignment rules.

Since, materials handling systems and transportation time between operations are ignored, till now, in most simulation models, the authors developed a model that can be used to test materials handling systems and rules, as well as the other operating polices. To our knowledge this is the first trial to incorporate materials handling system as an integrated part of a job shop

(1) See also Kleijnen [17]

simulation program.

In the following discussion the major characteristics of the model are briefly presented. For more detailed explanation, see Khorshid [16] , chapters 2 and 3 .

4.1 Job shop structure :

The processing operations are carried out in m work centers. Each work center is characterized by a number of machines (C_i, $i=1,2,\ldots,m$), a machining queue for jobs waiting for service, and a handling queue for jobs to be handled between centers. The shop contains a number of materials handling equipment which are organized and assigned to work centers according to some handling rules.

4.2 Job characteristics :

The time between successive arrivals of jobs in the shop is a random variable generated from certain probability density function a (.), with a mean interarrival time λ . Upon the arrival of a job, the sequence of work centers through which it must be routed are generated randomly using an $(m+1) \times (m+1)$ transition probability matrix P , with entries P_{ij} giving the probability of transition from work center i to work center j . The first row ($i=o$) represents entry into the system, while the first column ($j=o$) represents departure from the shop. The choice of the elements of the matrix P determine the flow pattern in the tested shop which lie between the extreme cases, the pure job shop and the flow shop.

The time to process a job, as well as its handling time are random variables generated from a selected probability density function. Mean handling time depends on the distance between work centers and the used materials handling equipment.

4.3 Materials handling systems

The model can be usedto test several handling systems, which vary according to the degree of centralized control exercised over the handling equipment. For example, in case of completely centralized systems, each handling equipment, upon completing a transportation operation at a work center, is given its next assignment by central control. On the contrary, in case of decentralized systems, handling equipment return to central control for reassignment, only when there is no work remaining in the work center at which it was previously assigned.

4.4 Operating policies

Materials handling and dispatching decisions are required to operate the job shop. An operating policy consists of a specific combination of the following rules:

i) The rules governing the choice of a job from machining queue
ii) " " " " " " " " " handling queue
iii) The rules governing the assignment of handling equipment to different work centers.
iv) The rules determining when a equipment is eligible for reassignment to another work center.

4.5 Performance criteria

The efficiency of each operating policy is evaluated used a general performance properties, that can be easily transformed to a cost criteria in practical situations. Examples of these measures are, mean flow time per job, mean queue length, etc.

5. Experimentation

The hypothetical shop used in the experimentation consists of 6 work centers, each of them contains 2 identical machines. Four materials handling equipment are used to transport jobs between work centers. The job interarrival time, service time.and handling time are random variables generated from a negative exponential distribution. The means of these variables are adjusted so that average machine utilisation attain 85% and handling equipement utilisation will be 95%. The job routing is of the pure job shop type, and is generated using the transition probability matrix. A specific layout configuration is proposed in order to find estimates for distances between work centers.

5.1 First experiment

Finding the best operating policy.

The purpose of this experiment is to choose the operating policy that lead to the best performance measure (simular response). Since out put of simulation experiment is stochastic in nature, a measure of the probability of correct selection is needed in order to justify the obtained results.

5.1.1 Experimental Factors

As it was indicated in section 4.4, an operating policy consist of a specific combination of dispatching and handling rules which we call, in statistical termonology, the experimental factors. The tested factors are(1):

i) The rules governing the choice of a job from machining queue (x_1)
   1) Shortest -imminent- processing time rule ($x_1 = 1$)
   2) Minimum slack per operation rule ($x_1 = 2$)

ii) Rules governing the choice of a job from handling queue (x_2)
   1) Shortest handling time rule ($x_2 = 1$)
   2) Minimum slack per operation rule ($x_2 = 2$)

iii) Rules governing the assignment of handling equipment (x_3)
   1) assign available equipment to the work center whose handling queue contains the maximum number of jobs ($x_3 = 1$)
   2) assign available equipment to the work center whose queue contains the job which has been in the system the longest periodof time ($x_3 = 2$)

(1) The choice of these rules and their definition are discussed in reference [16]

iv)Rules determining when an equipment is eligible for reassignment (x_4)

1) an equipment, upon completing a handling operation at a work center, is given its next assignment by central control (x_4=1)
2) an equipment, upon completing a handling operation at a work center, stays at work center as long as there is any job in its queue (x_4=2)

Table 1, defines the 16 operating policies representing all possible combinations of the decision rules.

Table 1. Definition of Policies

operating policies	Experimental factors (rules)				simular response
	x_1	x_2	x_3	x_4	
1	1	1	1	1	0.4278
2	2	2	1	1	0.1832
3	1	2	1	1	0.2393
4	2	1	1	1	0.2178
5	1	1	2	1	0.2523
6	2	2	2	1	0.1814
7	1	2	2	1	0.2338
8	2	1	2	1	0.2046
9	1	1	1	2	0.3281
10	2	2	1	2	0.1916
11	1	2	1	2	0.2386
12	2	1	1	2	0.1938
13	1	1	2	2	0.3253
14	2	2	2	2	0.1979
15	1	2	2	2	0.2406
16	2	1	2	2	0.1985

5.1.2. Simular response $\bar{Y}$:-

The selected measure of performance is the mean delay ration [10], which is defined for a particular job by the total service and handling time divided by the job flow time in the shop.

5.1.3. Sampling Plan :-

At the begining of investigation, a study of the equilibrium conditions is conducted using some pilot runs. The results showed that the number of observations to be deleted, in order to reduce the bias caused by the starting conditions, was 500. On the other hand, since multiple ranking procedures require the generation of independent observations, we think that the best sampling procedure is to make a single simulation run per operating policy, to exclude 500 observations in the begining, and then to divide the run into a set of subruns in a way that the averages of the subruns would be uncorrelated. The method of Fishman [13] was used to determine the minimum number of observations per subrun. An iterative algorithm was incorporated in the main simulation program in order to estimate sequentially the variance of $\bar{Y}$ and then to terminate simulation run when desired accuracy is achieved.

5.1.4. Obtained results :-

In section 3.2, we discussed multiple ranking procedures, and how they could be adopted to simulation circumstances. Table 2, shows the results of applying three of them, which were selected by the authors, to job shop simulation out put. All procedures choosed policy 1 as the best one, with probabilities of correct selection $P^* = 0.8, 0.95$ and different values of Δ^*. The results confirm also the superiority of Paulson method in case of large number of tested policies and favorable configurations of population means. This method requires a smaller sample sizes even with high probability of correct selection.

5.2 Second Experiment :

Finding optimum combination of factor levels:

In this experiment, we assume that simular response $\bar{Y}$ is related to a three quantitative factors by an unknown functional form. The purpose will be to find the values of the factors that optimize the simular response $\bar{Y}$.

Table 2. Results of multiple ranking procedures

P^*	Δ^*	Bechhofer			Paulson			Heuristic		
		Sample Size	Time in Sec.	Selected Policy	Sample Size	Time in Sec.	Selected Policy	Sample Size	Time in seconds	Selected Policy
0.80	0.01	11	1.1612	1	3	0.3855	1	7	0.4726	1
0.80	0.02	6	0.7800	1	2	0.3794	1	7	0.4818	1
0.80	0.03	4	0.6159	1	2	0.3784	1	7	0.4736	1
0.80	0.04	3	0.5571	1	2	0.3753	1	7	0.4731	1
0.95	0.01	17	1.769	1	6	0.4096	1	7	0.4690	1
0.95	0.02	8	0.9062	1	3	0.3820	1	7	0.4716	1
0.95	0.03	5	0.6840	1	2	0.3773	1	7	0.3912	1
0.95	0.04	4	0.6205	1	2	0.3773	1	7	0.4721	1

5.2.1 Experimental factors :

i) Due date control parameter (D1) :-

When a job arrives at the shop, an approximated date for its completion has to be estimated. This due date depends on the shop utilisation factor and the number of operations to be performed on the job. We selected a simple and popular calculation formula , which is given by :-

$$\text{Due Date} = r + D1.T$$

where r is the arrival date and T is the expected processing and handling time for the job. $D1 \geq 1$ is a control parameter.

ii) Dispatching Parameter (U):

Eilon et al [10] proposed a variant of the shortest Imminent processing time rule (SI), which they called SI^* rule. It consist of the assignment of a float value F to each job in the machining queue, where F is defined by :-

$$F = (D - r) - t_e - U$$

and r = current date

t_e = expected time for the uncompleted operations on the job.

D = Due Date

U = a given control parameter.

Jobs with $F \leq o$ are put in a priority queue, while those with $F > o$ are put in a normal queue, the latter being processed only when the periority queue vanishes. In each queue the SI rule applies.

The emperical study of Eilon et al [10] proved that this rule is superior to SI rule when selecting the optimum value of U.

iii) Handling parameter (UI) :-

The previous rule is modified to deal with the choice of a job from the handling queue. In this case we calculate the float value by :

$$F = (D - r) - t_e - UI$$

Where D, r, t_e are previously defined, and UI is a given control parameter specified by the experimenter.
The rest of the rule is the same except that we use the shortest handling time rule instead of the SI rule.

To summarize, the purpose of this experiment is to find the values of the parameters (D1, U, UI) that optimize the performance measure of a job shop characterized by a particular flow pattern and level of congestion. The selected measure is the mean flow time per job.

5.2.2 Seeking a near stationary point :

Since little is known about the orientation of the simular response function such that a good starting conditions could not be specified, it was thought that first order effects would be dominant, at least in the first stage of experimentation. Consé-quently, a linear model is fitted using 2^3 factorial experiment. The levels chosen for the factors being those given in table 3.

Table 3 : Factor levels for the 2^3 experiment

Factor levels in natural units	Factor levels in units of the design		Base Level	Unit
	-1	+1		
Due date parameter D1	1.5	3.5	2.5	1
Dispatching parameter U	20	60	40	20
Handling parameter UI	20	60	40	20

Thus, denoting the variables measured in unites of the design by X_1, X_2 and X_3 respectively, then

$X_1 = (D1 - 2.5)/1$; $X_2 = (U-40)/20$; $X_3 = (UI-40)/20$

and the linear model can be written.

$$E(\bar{Y}) = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3$$

The following table shows the used factorial design and out put results(1)

run	Factor Level			Mean flow time ($\bar{Y}$)
.	X_1	X_2	X_3	
1	-1	-1	-1	66.8984
2	+1	-1	-1	88.0471
3	-1	+1	-1	66.7253
4	-1	-1	+1	67.6155
5	+1	+1	-1	84.9666
6	+1	-1	+1	94.1092
7	-1	+1	+1	65.4243
8	+1	+1	+1	74.1050

Since complete factorial experiment provides measures of linear and interaction effects, the following estimates were obtained :-

$\hat{\theta}_o = 75.986$ $\qquad$ $\hat{\theta}_{12} = -2.589$

$\hat{\theta}_1 = 9.320$ $\qquad$ $\hat{\theta}_{13} = -0.526$

$\hat{\theta}_2 = 3.181$ $\qquad$ $\hat{\theta}_{13} = -2.367$

$\hat{\theta}_3 = 0.672$

and the approximated standard error of the $\hat{\theta}_i$'s, based on the estimated variance of $\bar{Y}$ (see section 5.1.3) was 1.9.

Three concluding remarks can be deduced from these results :-

(1) When number of factors is large, we can use a fractional factorial design which require a smaller number of experimental points [4]

i) Factor X_3 has small effect compared with the other factors. Then, either the system is independent of the factor levels, or the unit adopted for the factor is relatively small. To guard against the second possibility, larger unit must be used in the next set of trials. If the factor continue to give small effect, it can be excluded from further investigation .

ii) Since interaction effects are not small compared to some first order effects, the starting condition is probably not too remoted from the stationary region. This suggest that we reduce the steepest descent path by a small increment. The calculation of the optimum path is shown in table 4.

Table 4. Calculation of steepest descent path and subsequent trials.

	X_1	X_2	X_3	$\bar{Y}$
1)Base level	2.5	40	40	-
2)Unit	1	20	20	-
3)Estimated slop "$\hat{\theta}$"	9.320	-3.181	-0.672	-
4)Unit X $\hat{\theta}$	9.320	-63.622	-13.458	-
5)Changes in level per 5% change in X_1	0.466	- 3.181	-0.672	-
6)Path of Steepest descent 0%	2.5	40	40	69.789
5%	2.034	43.181	40.672	66.436
10%	1.567	46.362	41.345	65.424
15%	1.101	49.543	42.018	65.424

The results of table 4. show that no reduction of $\bar{Y}$ can be attained by the present path. So another cycle of steepest descent may be needed in order to improve the simular response.

Whan selecting factor levels for the next trials, we have to keep in mind that we are not too far from the optimum point. Thus if further progress is to be possible, without taking second order effects into account, the best chance of success lay in reducing the units of the factors; Table 5 indicate the new chosen levels.

Table 5. Factor levels for second group of trials

	-1	1	Base level	Unit
Due date Parameter D1	1	2	1.5	0.5
Dispatching parameter U	36	56	46	10
Handling Parameter UI	16	66	41	25

A second factorial experiment was conducted, and the obtained estimates were :

$\hat{\theta}_0 = 66.374$ $\hat{\theta}_{12} = -0.092$

$\hat{\theta}_1 = 0.950$ $\hat{\theta}_{13} = -1.222$

$\hat{\theta}_2 = -0.0496$ $\hat{\theta}_{23} = 0.106$

$\hat{\theta}_3 = -1.222$

We conclude from these estimates that a near stationary region is reached, since all first order effects are small compared with their standard error ($\sigma_{\hat{\theta}} \simeq 1.9$); Further more, the effect of factor X_3, which was not significant in the first group of trials, became better than the effect of X_2. Thus neglecting it in the exploratory phase of experimentation may lead to misinterpretation of results.

5.2.3 Exploring the near stationary region :-

In order to study the local nature of the response function, a central composite design [5] was used to fit the second order polynomial :-

$$E(\bar{Y}) = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3 + \theta_{11} X_1^2 + \theta_{22} X_2^2 + \theta_{33} X_3^2 + \theta_{12} X_1 X_2 + \theta_{13} X_1 X_3 + \theta_{23} X_2 X_3$$

This design consist of 2^3 factorial experiment, 6 axial points, and one center point. It requires a reasonable number of trials and can benefit from results obtained from the first order design. On the assumptionthat a second order equation provides an adequate model, unbiased estimates were obtained for quadratique and linear effects.

Taking partial derivatives with respect to each factor, and solving the obtained system of linear equations, the levels of X_1, X_2 and X_3 corresponding to minimum flow time were as follows:

$X_1^o = 0.781$; $X_2^o = 0.0445$; $X_3^o = 0.3153$

Consequently, the minimum simular response $\bar{Y} = 64.930$

Transforming these values from design units to natural units, we get :

D1 = 1.11 U = 46.445 $U_1 = 33.117$

When the fitted surface was reduced to the canonical form, we obtained the following results:

i) the canonical form was

$$\bar{Y} - 64.93 = -0.2808\, x_1^2 + 0.3845\, x_2^2 + 1.1362\, x_3^2$$

ii) The new coordinates (x_1, x_2, x_3) for any points are given in terms of the old coordinates (X_1, X_2, X_3) by the following table :-

	$(X_1+0.781)$	$(X_2-0.044)$	$(X_3+0.315)$
x_1	0.4972	-0.0502	0.8661
x_2	0.0806	0.9966	0.0115
x_3	-0.8638	0.0640	0.4996

Since the coefficients of the canonical form are different in sign and each of them is not small compared with the others; This suggested that a minimax surface exist in the near optimal region. This conclusion was confirmed by applying the ridge analysis program to the fitted second order equation.

Figure 2, shows the values of the absolute maximum and absolute minimum of simular response $\bar{Y}$ as the radius R of the sphere, representing the experimental region, varies.

6. CONCLUSION

This research demonstrate the importance of using statistical techniques to design and analyse computer simulation experiments.

The problems concerning the choice of a sampling procedure and, the development of an experimental plan, were formulated considering the particular circumstances of simulation. This formulation can guid the investigator to select the most suitable statistical technique as a function of the desired objective.

The two applications of the proposed experimental techniques to a job shop production model showed that simulation can be used efficiently to find the optimum operating conditions. The essential advantage of these techniques is that they provide information about the precision of the estimated optimum, for example, the probability of correct selection of the best alternative, or the cost of departure from the stationary point, if it was impossible to reach it in practice.

REFERENCES

1. Ashour, S; and S.D.Vaswani, "AGASP Simulation Study of Job Shop Scheduling", Simulation, Jan.1972 (PP.1-9)
2. Bechhofer, R.E; and S.Blumenthal; "A Sequential Multiple Decision Procedure for Selecting the Best One of Several Normal Populations with a Commun Unknown Variance, II: Monte Carlo Sampling Results, and New Computing Formulae", Biometrics, Vol.18, March 1962 (PP.52-67)
3. Booth, K.V.H; D.R.Cox "Some Systematic Supersaturated Designs" Technometrics; Vol.4, 1962 (P.489)

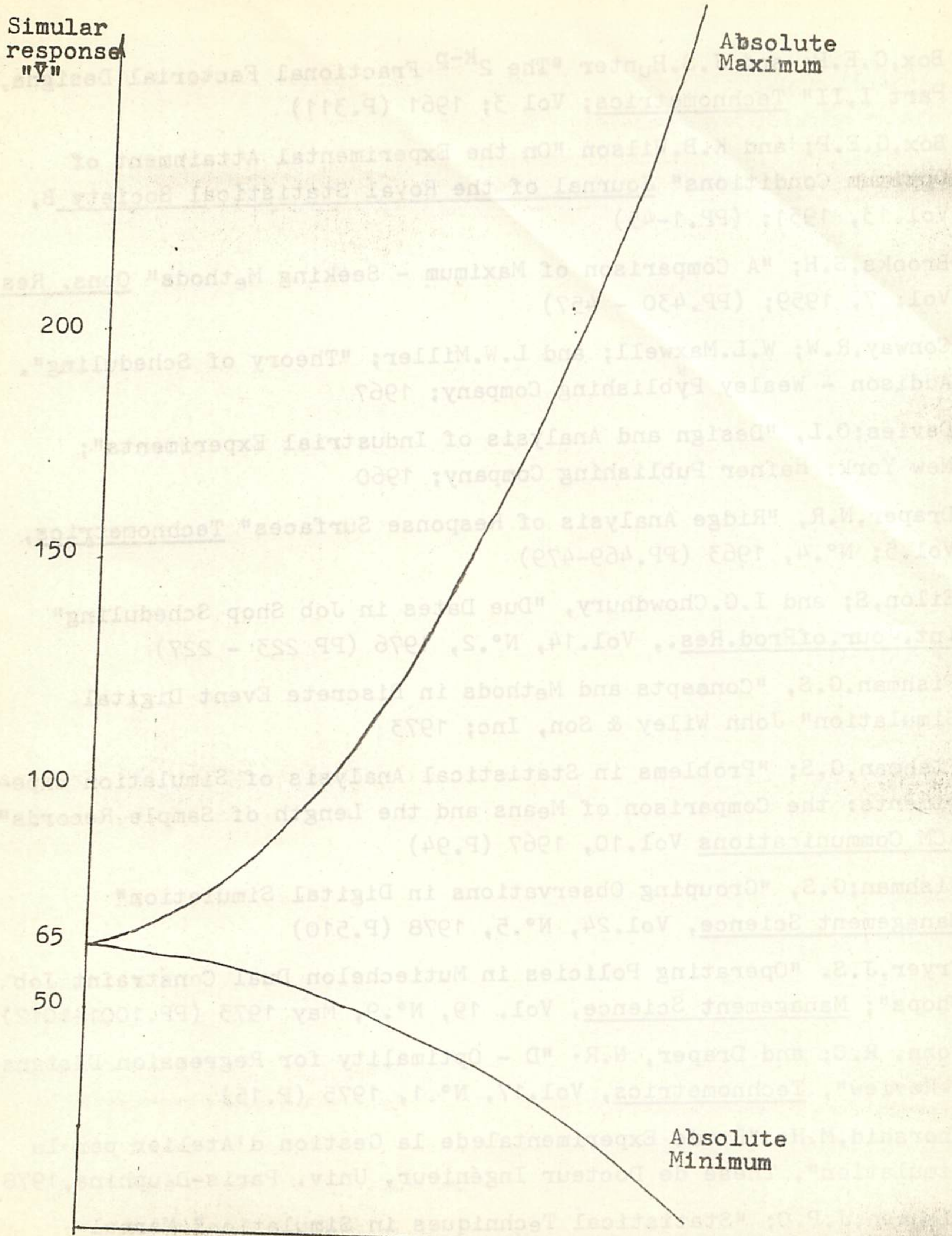

Figure 2 : Loci of stationary values as R varies

4. Box,G.E.P; and J.S.Hunter "The 2^{k-p} Fractional Factorial Designs, Part I,II" Technometrics; Vol 3; 1961 (P.311)

5. Box,G.E.P; and K.B.Wilson "On the Experimental Attainment of Optimum Conditions" Journal of the Royal Statistical Society B, Vol.13, 1951; (PP.1-45)

6. Brooks,S.H; "A Comparison of Maximum - Seeking Methods" Opns. Res. Vol. 7, 1959; (PP.430 - 457)

7. Conway,R.W; W.L.Maxwell; and L.W.Miller; "Theory of Scheduling", Addison - Wesley Publishing Company; 1967

8. Davies;O.L, "Design and Analysis of Industrial Experiments"; New York: Hafner Publishing Company; 1960

9. Draper,N.R, "Ridge Analysis of Response Surfaces" Technometrics, Vol.5; N°.4, 1963 (PP.469-479)

10. Eilon,S; and I.G.Chowdhury, "Due Dates in Job Shop Scheduling" Int.Jour.ofProd.Res., Vol.14, N°.2, 1976 (PP 223 - 227)

11. Fishman.G.S, "Consepts and Methods in Discrete Event Digital Simulation" John Wiley & Son, Inc; 1973

12. Fishman,G.S; "Problems in Statistical Analysis of Simulation Experiments: the Comparison of Means and the Length of Sample Records". ACM Communications Vol.10, 1967 (P.94)

13. Fishman;G.S, "Grouping Observations in Digital Simulation" Management Science, Vol.24, N°.5, 1978 (P.510)

14. Fryer,J.S. "Operating Policies in Mutiechelon Dual Constraint Job Shops"; Management Science, Vol. 19, N°.9, May 1973 (PP.1001-1012)

15. John, R.C; and Draper, N.R. "D - Optimality for Regression Designs A Review", Technometrics, Vol.17, N°.1, 1975 (P.15)

16. Khorshid,M.H; "Etude Experimentalede la Gestion d'Atelier par la Simulation", Thèse de Docteur Ingénieur, Univ. Paris-Dauphine,1978

17. Kleijnen,J.P.C; "Statistical Techniques in Simulation" Marcel Dekker, New York, Part II, 1975

18. Mihram, G.A; "Simulation : Statistical Foundation and Methodology" Academic Press; New York, 1972

19. Naylor,N.H; K,Wertz; and T.Wonnacott, "Methods for Analysing Data from Computer Simulation Experiments" <u>ACM.Comm</u>, Vol.10, N°.11, 1967 (PP 703 - 710)

20. Nelson,R.T; "Labor and Machine Limited Production Systems" <u>Management Science</u>, Vol.13, N°.9, May 1967, (P.648)

21. Patel, M.S; "Group Screening with More Than Two Stages", <u>Technometrics</u>, Vol. 4; 1962, (P.209).

22. Paulson, E; "A Sequential Procedure for Selecting the Population with the Largest Mean from K Normal Populations" <u>Ann. of Math. Stat</u>. Vol.35; 1964 (P.174)

23. Sasser, W.E; D.S.Burdic; D.A.Graham; and T.H.Naylor "The Application of Sequential Sampling to Simulation" <u>ACM.COMM</u>., Vol.13, N°.5; 1970 (P.287)

24. Satterthwaite, F.E; "Random Balance Experimentation"; <u>Technometrics</u> Vol. 1, 1959 (P.111)

25. Smith,D.E; "An Emperical Investigation of Optimum - Seeking in the Computer Simulation Situation", <u>opns. Res</u>; Vol 21; 1973 (P.475)