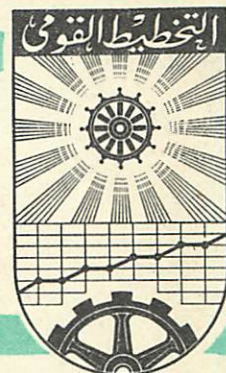


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PRACTICAL USE OF ECONOMIC-MATHE-
MATICAL MODELS IN PLANNING PRACTICE
(Applicable to input-output balance)

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The computers are used in the work of the USSR Gosplan in two main directions:

- implemetation of direct plan estimates;
- modelling of economic processes and plan estimates based on economic-mathematical models.

The direct plan estimates are built up on the basis of simple algorithms containing no logical operations. An example may be the estimates of requirements in material resources for production and operation needs that contain only operations of multiplying a vector by a matrix and aggregation. The direct plan estimates are connected with processing of big volumes of data (dozens and hundreds of thousands of numbers) and their implementation covers about 70% of machine time of computers available at the USSR Gosplan disposal. The said estimates are practically made for all sections of national economic plan.

It is known that the economic-mathematical models can not give an exhaustive description of actual economic processes. An attempt to take into account all details leads to bulky constructions and inevitably entails computing difficulties. Therefore, the structure of model takes into account only main essential features of described process and each of the models is designated for solution of certain problems. The system of interconnected models is required for multiform description of economic processes. In the meantime the USSR Gosplan has ~~zage-~~neral idea of such system of models, however applied practically

are only individual models experimentally tested, for solution of which there is a relevant mathematical apparatus and mainly necessary plan-economic information. It is natural that while using a complex of models that cover only a part of plan-economic problems a problem of organic coordination of planning economic-mathematical methods with so-called "traditional" methods of plan compilation arises. All economic-mathematical models to be practically applied in the USSR Gosplan are built up on the principle of maximum use, as initial information, of "traditional" plan estimates results with compulsory use of modelling results in further plan estimates.

All information-norm facilities of economic-mathematical models are worked out in accordance with general principles of forming the norm facilities of national economic plan and based on the estimates and substantiation of indicators to be contained in relevant sections of the state plan. The basic method of substantiation of the norms to be used in economic-mathematical models is a techno-economic designing with due account to the factors influencing the change of their value in the planned period, particularly the factors of technological progress. Considered hereby is an experience of work of advanced enterprises and production corporations, world achievements, possibilities that appear due to use of new design solutions, new structural materials and technological methods.

The major source of information for the plan input-output balance is the reported input-output balances that are made out by the Central Statistics Agency in monetary and physical terms as per nomenclature agreed upon with the USSR Gosplan. The said balances to the full extent are made out regularly once in 5-6 years.

Moreover, the reported balances with an enlarged sectoral nomenclature corresponding to the nomenclature of enlarged dynamic models to be used in the USSR Gosplan are annually made out. The enlarged balances are also supplemented with comprehensive production and distribution balances for the most important commodities in physical terms.

With respect to the fact that the USSR Gosplan is the directive body which decisions on plan problems are obligatory to everyone, it is in a position apart from its scientific-research institutes to involve all scientific-research organizations of the country in working out and realizing the economic-mathematical models. The said scientific-research organizations in conformity with the USSR Gosplan plan-economic assignment shall work out economic-mathematical models, form initial information and, sometimes carry out estimates on models and submit their results to the USSR Gosplan. As a rule, the proposed economic-mathematical models and initial information thereto undergo a compulsory expertise in the Gosplan departments. The estimates on models are usually made in the main Computing Centre of the Gosplan being responsible for introduction of economic-mathematical methods and computers in the planning, for implementation of operative plan estimates at the computers for the USSR Gosplan.

However, the major part of economic-mathematical models to be practically applied is worked out and prepared to introduction in the Gosplan.

In general, the process of realization of economic-mathematical models in the USSR Gosplan may be represented as follows:

Content of work	Plan-economic assignment	Economic-mathematical models, algorithms and methods of problems solution	Formation of initial information	Estimates on models	Analysis of results and correcting of initial data
Executives	USSR Gosplan Departments	USSR Gosplan Departments (Main Computing Centre scientific-research organizations (Scientific-Research Institute)	USSR Gosplan Departments	USSR Gosplan Main Computing Centre	USSR Gosplan Departments

The models to be practically used in the USSR Gosplan on contents of problems to be solved may be divided in the following main groups:

- national economic models (mainly input-output balance models);
- sectoral planning models (optimization of production plans, development and distribution of sectors, forecasts of demand for products of a sector, etc.);
- models of solving individual plan problems (forecast of population number, demand for consumer goods, etc.).

The input-output balances to be used in the USSR Gosplan differ from each other by a degree of nomenclature details (enlarged or comprehensive), applied measuring units (physical, monetary, physical-monetary), range of coverage of economic processes (national economic, regional, sectoral).

At the preliminary stages of national economic plan preparation used are the enlarged dynamic models of input-output balance that are built up for not more than 30 sectors in monetary terms. The simplified model is given in Appendix No. 1. On the basis of this model at the stage of working out the major trends of national economic development for the planned period the national economic plan central department in collaboration with other central departments makes alternative estimates with a view of working out a preliminary hypothesis on major national economic and input-output proportions of economic development in conformity with the draft plan social and economic conception. The estimates on model are made consecutively commencing from the first year of considered period. The estimates include determination of production and investment volumes in the sectors proceeding from the stipulated structure of final net production (estimated are various alternatives of final production that reflect different hypotheses of meeting the final demand).

The above model is the simplest model of national economic development. The USSR Gosplan uses also more complicated models, particularly, dynamic models that take into account in their structure the logs "capital investments-production increase" as well as dynamics of development of capital investments and production capacities. The basic ratios of these models are built up according to the following principle. The volume of capital investments each year of the planned period is determined as a sum of expense elements (it is shaded in the drawing) for construction of projects started in various years.

	t-3	t-2	t-1	t					
M_{t-3}	α_1	α_2	α_3	α_4					
M_{t-2}		α_1	α_2	α_3	α_4				
M_{t-1}			α_1	α_2	α_3	α_4			
M_t				α_1	α_2	α_3	α_4		

6.

$$Q_t = Q'_{t-3} \alpha_4 + Q'_{t-2} \alpha_3 + Q'_{t-1} \alpha_2 + Q'_t \alpha_1$$

or
$$Q^t = M_{t-3} q_{t-3} \alpha_4 + M_{t-2} q_{t-2} \alpha_3 + M_{t-1} q_{t-1} \alpha_2 + M_t q_t \alpha_1$$

where:

Q^t - capital investments in the sector in t year;

$Q'_{t-1}, Q'_{t-2}, \dots$ - capital investments in the projects that construction started in $t-1, t-2$ years;

$\alpha_1, \alpha_2, \dots$ - coefficients of development of construction estimated cost (i.e. share of construction expenses falling upon each year of construction period); $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$

M_{t-1}, M_{t-2}, \dots - production capacities of enterprises that construction started in years $t-1, t-2, \dots$

q - specific capital investments. (capital-output ratio)

Development of newly production capacities under construction is determined according to analogous principle. Let's consider an example when the enterprise in any sector is constructed within four years (at an average). Let's assume that a certain part of production capacities is being developed in the last

year of construction period (β_1 - coefficient of capacities development) and full development of capacities is completed within the next two years. Then the amount of developing new production capacities in year t is determined as a sum of elements (it is shaded in the drawing):

	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	t		
M_{t-5}	α_1	α_2	α_3	α_4				
				β_1	β_2	β_3		
M_{t-4}	α_1	α_2	α_3	α_4				
				β_1	β_2	β_3		
M_{t-3}	α_1	α_2	α_3	α_4				
				β_1	β_2	β_3		

$$\Delta M^t = M^{t-5}\beta_3 + M^{t-4}\beta_2 + M^{t-3}\beta_1$$

M^{t-5}, M^{t-4}, \dots - production capacities, construction started in years $t-5, t-4, \dots$

β_1, β_2, \dots - coefficients of developing production capacities (they are put in by an increasing result, i.e. $\beta_3 = 1.00$).

In general, major ratios of model:

- production and distribution balance

$$X_i^t - \sum_j a_{ij}^t X_j^t - \sum_j b_{ij}^t Q_j^t = y_i^t \quad (1)$$

- capital investments balance

$$Q_j^t = \sum_{k=1}^{\tau_j} M_j^{t-(\tau_j-k)} Q_j^{t-(\tau_j-k)} \alpha_j^k \quad (2)$$

- production capacities balance (provided the loading is full)

$$X_j^t = \bar{M}_j^t + \sum_{m=0}^{q_j-1} M_j^{t-\tau_j+m} \beta_j^{q_j-m} \quad (3)$$

where new symbols are as follows:

τ_j - average construction period of enterprises in sector j (in years);

\bar{M}_j^t - completely developed capacities in sector j
by year t with due account to removal;

\mathcal{Y}_j - by average continuity of production capacities
development period in sector j

The simplest alternative of model of input-output balance optimization may be received by introduction of some corrective^{objective} functions in formula (1) and (3) as well as purposeful function

$$\alpha^T \rightarrow \max \quad (T - \text{last year of planned period}) \quad \text{or} \quad \sum_{i,t} C_i^t \alpha^t \Delta y_i^t \rightarrow \max$$

where C_i^t - weight coefficient. Formula (1) introduces

$$\text{therein } y_i^t = y_{i \min}^t + \alpha^t \Delta y_i^t, \quad y_{i \min}^t -$$

- minimum permissible final net product for sector i in
year t ; Δy_i^t - elementary set of products that determines
the structure of final net product increase in year t .

Formula (3) introduces therein a symbol of inequality: $\chi_j^t \leq \bar{M}_j^t + \dots$

Unknown variable quantities in the said model are the production volumes χ^t , velocity of final net product increase α^t , capital investments in sector Q^t , size of production capacities and commencing dates of new enterprises construction M .

The above model may be used for solution of the following problem as well. There is a reported input-output balance as well as a list of projects that may be implemented in the plan period. It is required to determine the most expedient priority of projects implementation proceeding from available resources of capital investments, and, if the said resources are not sufficient, to reveal also the projects that shall be expeditiously implemented in the next plan period. To solve the above problem introduced is a conception of production methods

that mean, in this case, output of products in each sector at the existing enterprises and newly ones to be under construction in conformity with available projects. If the production methods are illustrated through S ($S = 1, 2, \dots, n$), then $X_j^{t,s=1}$ means, for example, a production volume in sector j in year t at existing enterprises; $X_j^{t,s=2}$ - production volume in sector j in year t at the enterprise which construction started in the previous plan period and that shall be commissioned in the considered period; $X_j^{t,s=3}$ - production volume in sector j in year t at enterprise No. 1 (Project No. 1) which construction started in the considered period; $X_j^{t,s=4}$ - ditto but for enterprise No. 2 (Project No. 2) and so on. Each method is characterized by own set of norms to be used in the model.

With due account to the above this model may be specified as follows:

$$(1) \sum_s X_i^{t,s} - \sum_{s,j} a_{ij}^{t,s} X_j^{t,s} - \sum_{s,j} b_{ij}^{t,s} Q_j^{t,s} = y_i^t$$

$$(2) Q_j^{t,s} = \sum_{k=1}^{T_j} M_j^{s,t-(T_j-k)} Q_j^{s,t-(T_j-k)} \alpha_j^{s,k}$$

$$(3) \sum_s X_j^{t,s} \leq \bar{M}_j^t + \sum_{s,m=0}^{y_j^s-1} M_j^{s,t-y_j^s+m} \beta_j^{s,y_j^s-m}$$

$$(4) \sum_{s,j} Q_j^{t,s} \leq \bar{Q}^t, \text{ or } \sum_{t,s,j} Q_j^{t,s} \leq \bar{Q}$$

\bar{Q}^t - amount of capital investments available in the national economy in year t for production development,

\bar{Q} - ditto for the period as a whole.

$$(5) \sum_{s,j} \ell_j^{t,s} X_j^{t,s} \leq L^t \quad \text{where } \ell_j^{t,s} \text{ - requirement in skilled manpower per unit of production in sector } j \text{ with production method } S \text{ in year } t;$$

L^t - availability of skilled workers in year t .

$$(6) \quad y_i^t = y_{i, \min}^t + \alpha^t \Delta y_i$$

$$(7) \quad \alpha^T \rightarrow \max \quad \text{or} \quad \sum_i \sum_t c_i^t \alpha^t \Delta y_i^t \rightarrow \max$$

It is natural that the content of purposeful formula (7) in the model may be changed depending on the targets set for the national economy in the considered period that will result in some correctives and other terms.

The above model facilitates to receive a sectoral structure of production and capital investments, evaluate requirements in manpower, select out of the total list of projects the most expedient priority of projects implementations. The following conditions are automatically observed, such as top-priority of completing the commenced construction sites, increase of loading level of existing production capacities.

As a result of problem solution obtained are mean weighed norms that take into account realization of planned projects of sectors development and, consequently, being planned norms. Therefore, application of such approach facilitates essentially a problem of building up the plan input-output balance providing to correct the reported data with the help of information that contains in the projects. It is natural hereby that the information contained in the projects shall reflect requirements of the model, i.e. apart from the size of production capacities each project shall stipulate requirements in raw and other materials (including fuel and electric power) to be consumed for current production (coefficients a_{ij}) and for capital investments, time-periods and coefficients of their development, coefficients of production capacities development.

The structure of input-output balance enlarged models and methods of their use differ subject to the problems to be set for a user of models. In particular, the complex of models may be used for coordination of long-term, medium-term and annual plans for the purpose of simplifying. For instance, the basic outlines of national economic development for 1990 are determined according to one of the models proceeding from which the basic indicators for ending years of five-year period (1980, 1985) are determined according to other model, the indicators for each year of the first five-year period (1976, 1977, 1978, 1979) are determined according to the third model. The said complex of models is worked out and used in the USSR Gosplan for operative estimates in the regime of plan preparation (See description of models in Appendix No. 2).

The enlarged monetary models are used at the preliminary stages of plan preparation. The results of alternative estimates according to such models after expertise and correcting in the Gosplan departments are a basis for working out comprehensive statistical input-output balances being of particular interest to the Gosplan, for they show monetary as well as commodity proportions in physical terms. (See Appendix No. 3). The model of the above balance includes 14 groups of equations: balances of products in physical and monetary terms with respect to Ministries, agencies and sectors, equations of commodity turnover, capital investments, manpower, etc. The commodity nomenclature of physical-monetary balances includes about 260 major commodities for perspective plans (medium-term and long-term) and about 700

commodities for annual plans. The balance contains also the data on products distribution by final product elements, national income, manpower required for provision of planned production level (See Balance Diagram). The initial data for estimating the balanced plan according to the model of physical-monetary input-output balance are the indicators for national income, export, import and overhaul repairs (See Diagram of Balance Estimate). The estimates of input-output balance are made at the preliminary stages of plan preparation for determining the balanced levels of products output in physical and monetary terms.

At the initial stages of plan preparation the estimates are made according to the formula: $(E-A)^{-1}Y = X$ where E- single matrix; at the completion stages- according to the formula $(E-A)X=Y$. To correct the plan in the process of its elaboration the mixed estimates are also made, when production volumes are ^edetermined for one products according to known values of final commodity and vice versa the final commodity is determined for other sectors.

The major result of working out the input-output balance being practically used in the USSR Gosplan is the estimates of coefficients of full expenses $B=(E-A)^{-1}$. Estimated are coefficients of material expenses (b_{ij}) as well as coefficients of spendings of capital investments, labour, fixed assets according to the following formulas:

$$Q_j = \sum_i b_{ij} q_i ; \quad F_j = \sum_i b_{ij} f_i$$

$$T_j = \sum_i b_{ij} t_i ;$$

where

Q_j, T_j, F_j - coefficient of full spendings of capital investments, manpower, fixed assets;

q_i, t_i, f_i - coefficient of specific capital investments, labour

(capital-output ratio)

spendings, fund intensity. The reference books of these norms are compiled for commodity nomenclature of input-output balance and contain the data on value and commodity structure of coefficients of direct and full spendings.

They are sent to the USSR Gosplan departments and used there for preliminary plan estimates, operative correcting of plan projections, economic analysis. The use of those reference books at solution of individual plan problems guarantees accounting for direct as well as indirect input-output (inter-commodity) relations. The latter is very important, for indirect spendings of resources surpass rather often in several times (sometimes in ten and more times) direct spendings, and a list of commodities in indirect spendings surpasses considerably a list of commodities to be directly consumed in the given production.

The analysis of economic processes with use of coefficients of full spendings facilitates to detect tendencies that may not be seen while accounting for only direct production relations. For instance, while analysing development of any sector a tendency to increase of production fund intensity may be detected that may be evaluated as negative phenomenon requiring certain arrangements. However, such a conclusion will be premature, for the fund intensity may considerably decrease in adjustment (directly or indirectly) sectors. The analysis of dynamics of full spendings coefficients may detect a cont-

rary tendency-reduction of full spendings of fixed assets for output of unit of products that is a progressive phenomenon.

The norms of full spendings estimated on the basis of input-output balance are used in individual sectoral and functioning models in which it is necessary to take into account national economic relations while deciding local questions.

The input-output balance being a major tool of plan balancing contains the systematized and intercoordinated information on the basis of which various economic-mathematical models may be built up.

One of the said models to be practically used in the Gosplan is called a model of adjusting by-annual targets of the five-year plan. The task to be achieved within the framework of this model may be described as follows. In the progress of five-year plan elaboration the input-output balances for each year of the period are made out, the said balances correspond to the plan approved by the Government. However, the conditions of plan realization may be changed eventually as against those for which the plan has been designated. For instance, resources of a number of products initially set in the plan vary (or due to a change of production capacity commissioning program, or due to clarifying the export-import program, or due to any other reasons connected with weather conditions). In this respect it is necessary to correct the plan for a certain year of five-year period so that the changes to be introduced therein would cause less damage to the economy (in case of revealing

any deficit for some resources) or with availability of resources increase as against the stipulated ones in the plan, to correct the plan so that to approach to a maximum extent to the desired higher level of economic development. The economic development level is expressed hereby through a certain fixed vector of final commodity. This task may be described as follows:

$$(1) \quad x_i - \sum_j a_{ij} x_j = y_i$$

$$(2) \quad x_i \geq \bar{x}_i$$

$$(3) \quad \sum_i c_i \frac{1}{y_i^0} (y_i - y_i^0)^2 \rightarrow \min$$

where:

y^0 - vector of final commodity that reflects the level of economic development stipulated by the five-year plan or desired for a considered year of five-year period,

c - weight coefficient.

The final plan alternative with use of this model is obtained after repeated solution of problem (5-6 alternatives). After each solution the result is sent to the expertise of specialists-planners whose opinion on plan quality (more often it is not expressed quantitatively but it reflects a meaning - "more" - "less") is taken into account by weight coefficients C in the further estimate.

While forming limitations (2) in case of any corrections of by-annual targets of the five-year plan) of some resources the following conditions may be used:

a) the limitation for top production level is set for deficient resources;

b) conditions for output at the amount to be not less than

in the five-year plan are created for some products to be of vital importance to the goals stipulated in the plan;

c) limitations for minimum output level, e.g. previous year production level, are set for some products which output decrease will lead to the smallest damage.

In our opinion the model of correcting the by-annual targets of five-year plan may be successfully used in the developing countries, particularly, in the ARE where any lack of initial data, changes in export-import programs, that greatly influence the economic development and do not often depend on domestic conditions, do not permit clearly enough to estimate availability of any resources for the five-year period.

The other direction of input-output balance use that is practically applied in the USSR Gosplan is to solve a problem of multistage optimization of national economic plan. (See Appendix No. 4). The said task is first and foremost connected with the problem of national economic coordination of individual sectors development plans for a long-term perspective. The complex of models shall be used for solution of this problem: of static input-output balance in physical terms, models of optimum plans of developing and distributing the sectors, input-output coordination model of sectoral plans. The key diagram of estimates may be represented as follows. - At the beginning estimated (forecast) is a plan of products output in physical terms with use of input-output balance model keeping in mind a preliminary character of estimates the certain interval is fixed, this

interval shall, in the opinion of specialists, comprise final values of products output volumes.

For instance, it is ascertained that the actual production volumes will be within the limits $x \pm 10\%$, where x is obtained from the preliminary estimates of input-output balance. The said extremes are selected as the initial data for estimates of sectoral plans.

The criterion function of sectoral plans model is an expression $C + EK$, where C - current expenditure, K - one-time expenditure (capital investments), E - coefficient of investments efficiency. The value of coefficient E depends, in principle, on specific economic situation to be created in any planned period. If there is any relative lack of capital investments, coefficient E shall be increased, if vice-versa, the society finds it necessary to increase capital investments for saving current expenditure, the said coefficient is decreased. There is a completely certain idea on possible maximum fluctuations of value E in the planning practice. Proceeding from the above the minimum and maximum possible values E are put in the estimates on sectoral models. Thus, four estimates are made for each sector:

- at maximum values X and E ;
- at minimum values X and E ;
- at minimum X and maximum E and vice versa.

The obtained four alternatives of sector development differ from each other by the requirement in capital investments, material resources (in commodity nomenclature of input-output balance)

labour resources. The plan alternatives are put in the input-output optimization model of sectoral plans coordination as production methods. As a result of solving the said model we receive a new production plan for the whole national economy, new values of coefficient E that serve as a basis for fixing new reliable intervals on which facilitates a new series of sectoral estimates is implemented that stipulates in its turn a new solution of input-output model. After four or five such iterations the final plan is obtained, the said plan is characterized by full use of labour and capital resources as well as maximum value of criterion function in the national economic model.

The similar diagram of input-output balance use is applied for intercoordination and other plan estimates (e.g. estimates of requirement in material resources).

As stated above, apart from national economic input-output balances the regional and sectoral balances are made out. The majority of Union Republics makes out an own input-output balance. The USSR Gosplan makes out internal sectoral intercommodity balances. The latter is made out in physical terms for the sectors having considerable internal sectoral products turnover. The said sectors include first and foremost chemical industry which internal sectoral turnover accounts for 70%. The nomenclature of such balances is considerably wider than that in national economic balances (e.g. the balance of chemical industry represents about 1400 commodities, the balance of aniline-paint represents about 2000 commodities) and includes, apart from products of considered sector, major types of raw

materials to be produced in other sectors as well.

The models of optimum production plans are built up on the basis of internal sectoral balances. For the sectors producing consumer goods such models are similar to a model of correcting the annual plan (see the above) and realize the task: to find the production plan that approaches at maximum the population demands at set limits of raw materials and production capacities.

Model of dynamic input-output
balance

$$X_i^t = \sum_j a_{ij}^t X_j^t + \sum_j b_{ij}^t Q_j^t + y_i^t$$

$$Q_j^t = \left(\frac{f_j^t X_j^t - \varphi_j^{t-1}}{g_j} + \omega_j^t \varphi_j^{t-1} \right) \frac{1 + \psi_j^t}{1 - \alpha_j^t}$$

where:

- X_i^t - volume of products output of sector i in year t ;
- a_{ij}^t - coefficient of direct material expenditure in year t ;
- b_{ij}^t - expenditure of products of sector i per unit of capital investments in sector j in year t ;
- Q_j^t - volume of capital investments in sector j in year t ;
- y_i^t - final products of sector i in year t with deduction of products directed to production capital investments (final net product);
- f_j^t - coefficient of fund intensity of sector j ;
- φ_j^{t-1} - fixed production assets in sector j at the end of a year $(t-1)$;
- ω_j^t - coefficients of funds to be removed in sector j in year t
- g_j - coefficients of ratio of funds average annual increase to annual increase;
- ψ_j^t - ratio of uncompleted construction increase to fund commissioning within a year;
- α_j^t - share of various capital investments in the total volume of investments.

Complex of models for estimates of long-term,
medium-term and annual plans

1. Model of plan estimate for the last year ($t=T$) of long-term period:

$$1) (1-h_{\psi})X_{\psi}^T - \sum_{j=1}^n a_{\psi j}^T x_j^T - \Delta_{\psi}^T S^T = \bar{Y}_{\psi}^T + W_{\psi}^T \quad (\psi=1, 2, \dots, \kappa)$$

$$2) y_f(1-\delta_f)X_f^T - \sum_j a_{fj}^T x_j^T + (1-\delta_f)u_f^0 \left(\sum_{t=1}^{T-1} \Delta_f^t \right) G_f - \Delta_f^T S^T = W_f V_f^T - F_f^T - \\ - (T-1)u_f^0(1-\delta_f) + \sum_{t=1}^{T-1} V_f^t - \bar{Y}_f^T \quad (f=\kappa+1, \dots, n)$$

$$3) X_f^T - u_f^0 \Delta_f^T G_f = u_f^0$$

$$4) \sum_{j=1}^n a_{pj}^T x_j^T = P_p^T$$

Indices:

i, j - indices of sectors; ($i, j = 1, 2, \dots, n$)

ψ - indices of sectors producing elements of circulating funds;
 ($\psi \in i, j$; $\psi = 1, 2, \dots, \kappa$)

f - indices of sectors creating elements of fixed assets; ($f \in i, j$;
 $f = \kappa+1, \dots, n$)

p - indices of limitation for a number of people engaged in material production;

t - index of year ($t=0, 1, 2, \dots, T$), where 0-index of base year;

T -index of last year of plan period.

Parameters and exogenous variables:

$a_{\psi j}^t$ - rates of direct material expenditure of products ψ per unit output of products j in year t ;

a_{fj}^t - rate of fund intensity of products j by f type of fixed assets in year t ;

a_{pj}^t - rate of labour intensity of products j in year t ;

h_{ψ} - rate of increase of circulating funds, stocks and reserves of products ψ ;

φ_f - coefficient of evenness of fixed assets commissioning, type f ;

δ_f - rate of increase of utilized fixed assets, type f ;

ω_f - coefficient of evenness of fixed assets removal, type f ;

$$\Delta y_{\psi}^t = \bar{y}_{\psi}^t - \bar{y}_{\psi}^t$$

$\bar{y}_{\psi}^t, \bar{y}_{\psi}^t$ - top and bottom limits of volume of

private and public consumption of products ψ in year t ;

$\Delta y_f^t = \bar{y}_f^t - \bar{y}_f^t$; \bar{y}_f^t, \bar{y}_f^t - top and bottom limits of average annual volume of fixed non-production funds of type f in year t ;

W_{ψ}^t - export-import balance and other elements of final consumption of products ψ

Δf^t - coefficient that fixes for each year, by what times the increase of investments volume in fixed assets of type f to their base level this year is more than in the first year of period;

V_f^t - volume of removing fixed assets of type f in year t ;

u_f^0 - volume of investments in fixed assets of type f in the base year;

F_f^1 - volume of fixed assets of type f at the beginning of the first year ($t=1$) of plan period;

p_p^t - average annual number of people engaged in material production in year t .

Unknown variables:

x_{ψ}^t - volume of products output of sector ψ in year t ;

x_f^t - volume of investments in fixed assets of type f in year t ;

σ_f - relative increase of annual investments in fixed assets of type f to the base level in the first year of period;

S^t - value determining a degree of move from \bar{y}_j^t to $\bar{\bar{y}}_j^t$.

2. The plan estimates for the end years of intermediate five-year periods are made out according to the above model, in this respect determined preliminary is the volume of fixed assets at the beginning of tenth and fifth year of plan period (at continuity of long-term period of 15 years) according to the formula:

$$\text{for tenth year } F_{f10} = F_{f15}^* - (1 - \delta_f) \left[5 - \left(\sum_{t=10}^{14} \Delta'_{ft} \right) G'_f \right] X_{f15}^* + \sum_{t=10}^{14} W''_{ft};$$

$$\text{for fifth year } F_{f5} = F_{f15}^* - (1 - \delta_f) \left[10 - \left(\sum_{t=5}^{14} \Delta'_{ft} \right) G'_f \right] X_{f15}^* + \sum_{t=10}^{14} W''_{ft}$$

3. Model of plan estimate for each year of first five-year period.

$$X_{\psi}^t = \sum_{j=1}^n a_{\psi j}^t X_j^t + Z_{\psi}^t; \quad (\psi = 1, 2, \dots, \kappa; t = 1, 2, 3, 4);$$

$$X_f^t = \sum_{j=1}^n a_{fj}^t X_j^t + \sum_{j=1}^n k_{fj}^t + K_{fH}^t + Z_f^t; \quad (f = \kappa + 1, \dots, n; t = 1, 2, 3, 4);$$

$$\sum_{j=1}^n a_{pj}^t X_j^t = P_p^t \quad (t = 1, 2, 3, 4);$$

$$X_j^t \leq X_j^t \leq \bar{X}_j^t \quad (j = 1, 2, \dots, n; t = 1, 2, 3, 4);$$

$$K_f^t \leq \sum_{j=1}^n k_{fj}^t \leq \bar{K}_f^t \quad (f = \kappa + 1, \dots, n; t = 1, 2, 3, 4);$$

$$K_{fH}^t \leq K_{fH}^t \leq \bar{K}_{fH}^t \quad (f = \kappa + 1, \dots, n; t = 1, 2, 3, 4);$$

$$\sum_{t=1}^4 K_{fj}^t = K_{fj}^* \quad (f = \kappa + 1, \dots, n; j = 1, 2, 3, 4);$$

$$\bar{y}_j^t \leq \frac{1}{a_{fj}^t} \left[F_j^1 + (1 - \delta_f) \sum_{\tau=1}^{t-1} K_{fj}^{\tau} + G_f (1 - \delta_f) K_{fj}^t - \sum_{\tau=1}^{t-1} V_{fj}^{\tau} - \omega_f V_{fj}^t \right]; \quad \begin{matrix} (j=1, \dots, n) \\ (f=\kappa+1, \dots, n) \\ (t=1, 2, 3, 4) \end{matrix}$$

The sign \star shows that the values are taken and solution of previous problem

$$\min Q = \sum_t \sum_j \frac{1}{C_j^t} \left(\frac{\tilde{x}_j^t - \tilde{x}_j^{*t}}{\tilde{x}_j^{*t}} \right)^2 + \sum_t \sum_j \sum_f \frac{1}{P_j^t} \left(\frac{K_{fj}^t - K_{fj}^{*t}}{K_{fj}^{*t}} \right)^2 + \\ + \sum_t \sum_j \frac{1}{q_j^t} \left(\frac{x_j^t - x_j^{*t}}{x_j^{*t}} \right)^2,$$

where:

\tilde{x}_ψ^t - unknown volume of final net products of sector ψ in year t ;

\tilde{x}_f^t - unknown volume of final use of investments in fixed assets of type f in year t ;

K_{fj}^t - unknown volume of investments in fixed assets of type f of sector j in year t ;

K_{fn}^t - unknown volume of investments in fixed assets of type f of non-productive sphere in year t ;

$\underline{x}_j^t, \bar{x}_j^t$ - top and bottom limits for production volumes of sector j in year t to be determined by production dynamics to be estimated according to the model within the whole long-term perspective;

$\bar{K}_f^t, \underline{K}_f^t$ - analogous determined limits for investments;

K_{fj}^{*t} - obtained-from-estimates (item 2) total investments volume of type f in sector j that provides outcome for the value of fixed assets of type f in sector j at the beginning of fifth year of first five-year period;

C_j^t, P_j^t, q_j^t - autonomously determined parameters of purposeful function that facilitate to control the task decision.

Statistical physical-monetary input-
output balance

I. Model of balance

1. Balances of products

$$\sum_{j=1}^{l-1} a_{ij} x_j + \sum_{j=l}^{k-1} a_{ij}^l x_j^l + \sum_{j=k}^{m-1} a_{ij}^k x_j^k + \sum_{j=m}^n a_{ij}^m x_j^m + y_i + \alpha_i = x_i \quad (i = 1, 2, \dots, l-1)$$

where:

$$y_i = a_{iT} T + a_{iU} U + a_{iK_p} K_p + a_{iK_n} K_n + a_{iR} R + a_{iZ} Z + a_{iQ} Q + a_{iE} E + a_{iI} I$$

2. Balances of products in physical terms for the Ministries

$$x_i^l = b_{ij} x_j \quad (i = l, l+1, \dots, k+1)$$

3. Balances of gross products of Ministries

$$(1 - \gamma_i) x_i^k = \sum_{j=l}^{k-1} v_{ij} p_j x_j^l + \delta_i \quad (i = k, k+1, \dots, m-1)$$

4. Balances of gross products of enlarged sectors

$$(1 - \beta_i) x_i^m = \sum_{j=k}^{m-1} \sigma_{ij} x_j^k + S_i \quad (i = m, m+1, \dots, n)$$

5. Equation of commodity turnover $T = g_T C$ 6. Equation of various resources of consumption fund $U = g_U C$ 7. Equation of production investments $K_p = h_{Kp} \varphi_p + \tilde{K}_p$ 8. Equation of non-production investments $K_n = h_{Kn} \varphi_n + \tilde{K}_n$ 9. Equation of increase of circulating assets, stocks and reserves. $Z = q_z G$ 10. Equation of other expenditure $Q = q_Q G$ 11. Equation of consumption fund $C = q_C G$

12. Equation of accumulation of fixed production assets

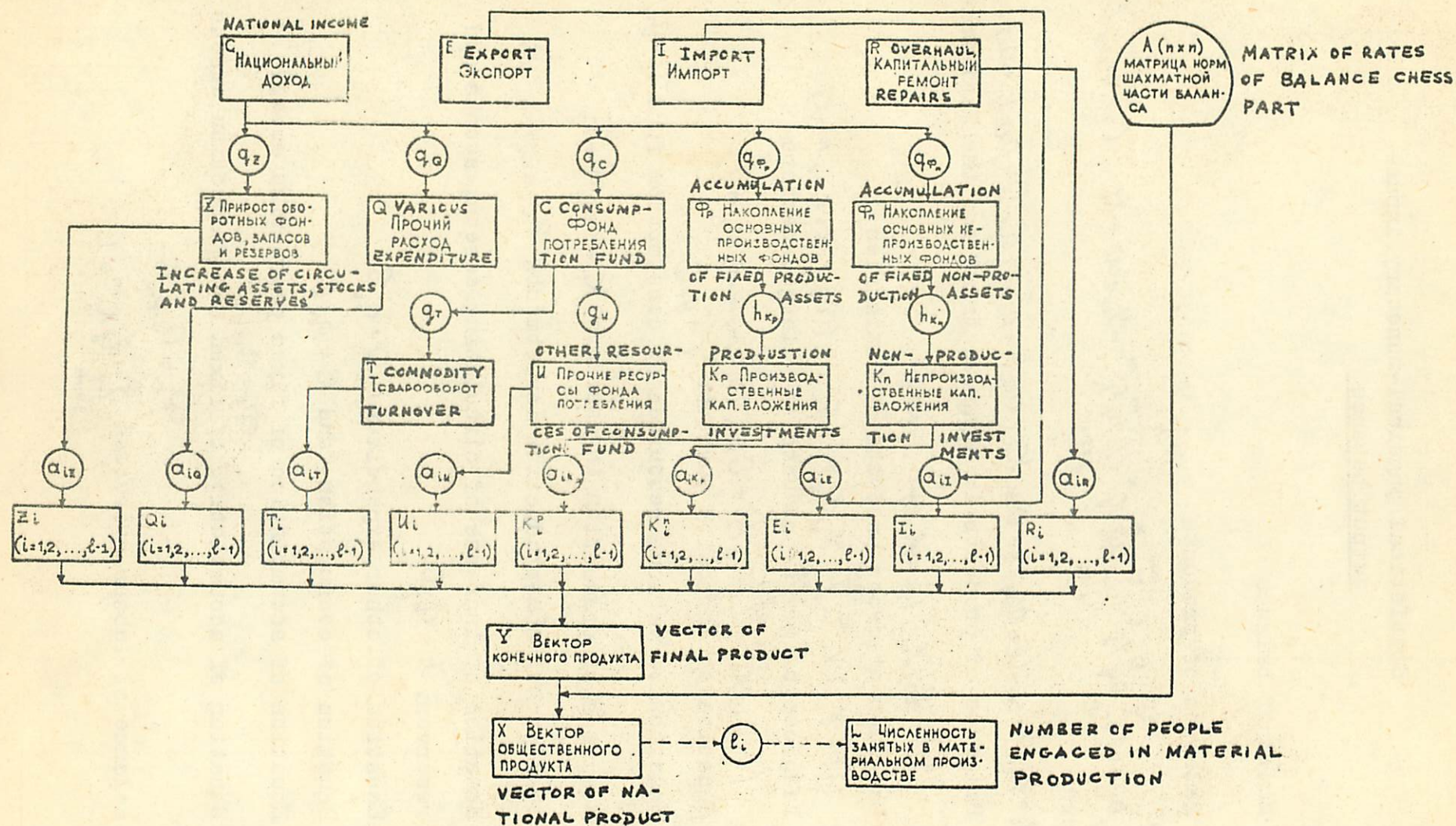
$$\varphi_p = q_{\varphi p} G$$

13. Equation of accumulation of fixed non-production assets

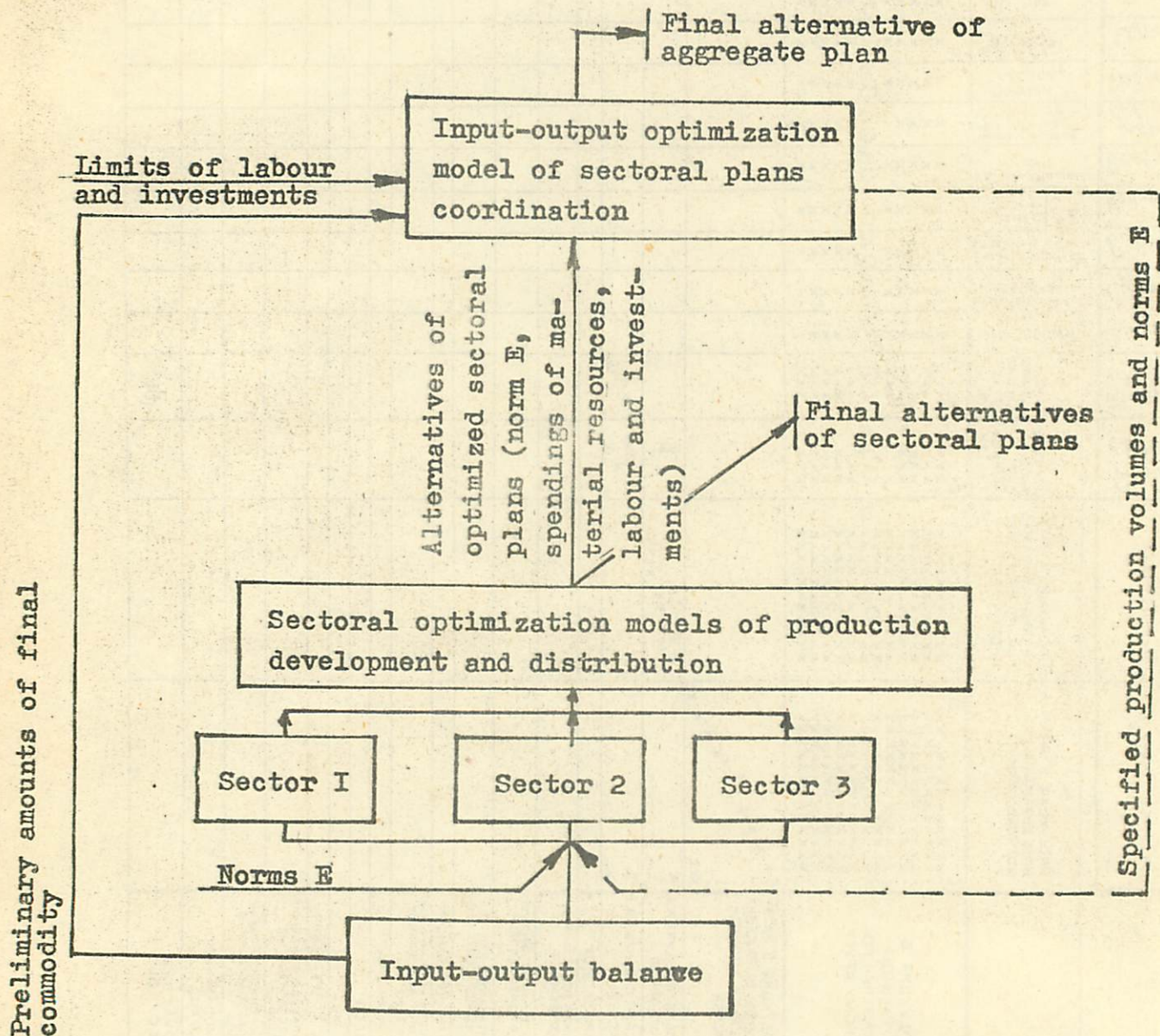
$$\varphi_n = q_{\varphi n} G$$

14. Balance of labour resources $\sum_{j=m}^n l_j x_j^m = L$

DIAGRAM OF ESTIMATES FOR MODEL OF PHYSICAL-MONETARY INPUT-OUTPUT BALANCE
 3. СХЕМА РАСЧЕТОВ ПО МОДЕЛИ НАТУРАЛЬНО-СТОИМОСТНОГО МЕЖОТРАСЛЕВОГО БАЛАНСА



Key diagram of multistage optimization
of national economic plan



Imitation Model - "Berthage -Transport Fleet" (Main clauses)

The berthage functioning is going on in complicated conditions of probabilities. Due to the fact that modern mathematical apparatus cannot discover the main dependencies appearing in the process of functioning and development of the "berthage-transport fleet" system and consequently does not provide to a necessary extent the determination of the system's optimum parameters an imitation modelling method on fast-operating computers could be used for solving this task. The imitation model programme contains the system formalized analogue reflecting its structure, functional dependences, connections and dynamics of processes flowing therein. In the process of imitation with the help of computer the system functioning is reproduced. Such a reproduction allows to obtain a quantitative evaluation of the system functioning quality conforming to those data on the system, which were fed to the computer's memory. Information of the system is contained first in the model structure and second in the initial data (regulated variables). Logic of estimates is given by the programme in conformity with logical block diagram of the model. Values of determinative functions are estimated according to given formulas or equations of probable values-according to Monte-Carlo.

Model Designation. The imitation "berthage-transport fleet" model is designed for the following:

- evaluation of the installation efficiency, development and operation of the sea port berthage;

- analysis of operation quality of existing or designed sea ports and fixing of optimum regime in their operation, determination of optimum number of moorages and optimum intensity of handling transportations vessels;

- evaluation of alternatives arising while elaboration of perspective plans of developing sea ports and providing the acception of optimum solution in the sphere of developing both ports and transport fleet.

In particular the use of model allows to answer the following questions:

- how will the system behave if changes take place with the port freight turnover, its structure, direction and reasons of transportations?

- how will the system behave if changes take (or might take) place with a number of operating berths, their inter-group specialization, depth near their cordon, intensity of handling the vessels or simultaneous vessel handling rates?

- how the system will behave if changes take place in the structure of transport fleet (e.g. cargo carrying capacities or vessel specialization) or in distributing the fleet according to directions of transportations?

- how will all these changes influence the total value of expenditures for the "port-fleet" system?

- what solutions shall be taken so as to provide the most rational development of the system and achieve therewith the optimum regime of its functioning?

Model Contents. The main components of the imitation model are as follows:

- regulated variables;
- logic-mathematical presentation of the system;
- imitation programme designed for realizing the model at the computer.

Referred to regulated variables are as follows:

1. Variables, characterizing the port turnover:
 - 1.1. Freight turnover;
 - 1.2. Structural composition;
 - 1.3. Distribution per the basic directions of transportations;
 - 1.4. Seasons of freight turnover.
2. Variables, characterizing the port berthage:
 - 2.1. Number of berths and their specialization per basic types of cargoes;
 - 2.2. Inter-group specialization of berths, i.e. their distribution per basic directions of transportations;
 - 2.3. Depth near berth cordon;
 - 2.4. Diagram of equipping berths with transshipping equipment and normative values of vessel handling intensity relevant to those diagrams;
 - 2.5. Rates of vessels simultaneous handling;
 - 2.6. Value of given construction and operational expenditures for each berth considered.

3. Variables characterizing vessels:

- 3.1 Parameters-medium and medium-square deviation of vessel distribution function, fed for each individual direction of transportations;
- 3.2. Upper borders of vessel freight capacity groups fed according to the rates of their handling;
- 3.3. Maximum admissible value of vessel draught for each individual direction of transportations;
- 3.4 Value of given construction and operational expenditures in average per one day for one vessel of individual direction of transportations while its mooring in the port.

Besides, as constants the estimated period of the port operation are fed in days every month, factor of influence of meteorological conditions for the duration of vessel delay in the port as well as a number (iterations) necessary for obtaining stable solution.

The imitation model structure contains the reflection of the port freight turnover, berthage, process of vessels in-coming for handling, process of distributing vessels per capacity, loading with cargoes considered, draught and individual directions of transportations, process of their handling, as well as vessels delays because of the moorages occupied, their inter-group specialization and various depths near the berth cordon.

Equation of freight carrying capacity:

$$(1) \quad Q_n = \sum_i \sum_j q_{ij}$$

where:

q_{ij} - individual flow of the port goods traffic (thou. tons);

i - index of goods traffic (cargo) ($i=1,2 \dots \kappa$)

~~R~~

J - index of transportation direction.

Each flow is characterized by its structural composition per goods groups fixed for relevant berths, seasons of transportations, and parameters of vessels to be handled and their value characteristic.

Berthage is fed in the shape of the whole complex of individual groups of berths specialized for trans-shipment not allowing joint handling of goods traffics:

$$(2) \quad N_n = \sum_i \sum_r n_{ir} \quad \text{where:}$$

n_{ir} - an individual berth designed for trans-shipment of "i"-load.

r - index of berth ($1,2 \dots N_i^n$)

Applied for each berth beside per types of loads is also the group specialization (i.e. fixing j -transportation directions of i -loads), depth near cordon, normative intensity of vessel handling, value of given construction and operational expenditures.

Budget of operation time for berths or total time of berths operational time for handling vessels is estimated while modelling by the formula:

$$(3) \quad T_{os}^n = \sum_i \sum_r t_{ir}^n \quad , \text{where:}$$

t_{ir}^n - time of operation of (r) berth of (i) group for handling vessels with (i) type of loads, berth-days.

And hereby:

$$(4) \quad t_{ir}^n = \sum_s t_{irs}^{\varphi.0\delta.}$$

where,

$t_{irs}^{\varphi.0\delta.}$ - duration of handling of (S) vessel with (i) load near (r) berth, vessel-days.

S - index of vessel in-coming to (r) berth for handling.

$$(S=1, 2, \dots, \sqrt{ir})$$

Total time of idling berths is estimated by formula:

$$(6) \quad T_{np}^n = \sum_i \sum_r \tau_{ir}^n, \text{ where:}$$

τ_{ir}^n - duration of idling time of (r) berth of (i) group, berth-days.

Hereby:

$$(6) \quad \tau_{ir}^n = T - t_{ir}^n, \text{ where;}$$

T - duration of the port operation period considered, days.

Factor of using berths per time:

$$(7) \quad \rho_i = \frac{\sum_r t_{ir}^n}{T N_i^n}$$

Distribution of time periods between moments of vessels in-coming for handling in conditions of operation of the Soviet ports is sufficiently well approximated (probability of concordance 0.8-0.9) for exponential distribution of the type

$$(8) \quad f(t) = \lambda e^{-\lambda t}$$

where:

$$\lambda = \frac{1}{\lambda^*}$$

λ - average

number of vessels, in-coming per day*

* Distributions of Gauss and Johnson could be used in the model in such cases, when on the basis of special survey a possibility and an expediency of their use in local conditions will be shown

While estimating the moments of vessels in-coming for handling the (λ) parameter is defined for each (j) direction of transporting $(i-x)$ loads, i.e.

$$(9) \quad \lambda_{ij} = \frac{q_{ij}}{\tau \bar{G}_{ij}}, \text{ where:}$$

\bar{G}_{ij} - average freight capacity of the vessel of (j) direction of transporting $(i-x)$ cargoes, thou. tons.

Empiric function of distributing the freight capacity of vessels in-coming to the port for handling may be approximated (probability of concordance - not less than 0.8) by gamma-distribution, i.e.

$$(10) \quad f(G) = \frac{1}{\Gamma(\alpha)} \beta^\alpha G^{\alpha-1} e^{-\beta G}, \quad \alpha = \frac{\bar{G}^2}{\sigma^2}, \quad \beta = \frac{\bar{G}}{\sigma^2}$$

Values of \bar{G} and σ are fed in accordance with the evaluation of transport fleet structure.

To define quantities of loads to be shipped or unloaded the factors of loading vessels are used. Those factors are defined by parameters of vessels to be handled and the structure of goods traffics considered.

For each moment (obtained while imitating) of the next ship in-coming for handling defined is not only its affiliation to (j) direction of $(i-x)$ loads, value of its cargo carrying capacity and loading, but also importance of its draught and duration of its handling. Draught value is estimated by the formula below:

$$(11) \quad H_{ijs} = a_0 + a_1 \sqrt{x_{ijs}} \quad \text{where:}$$

D_{ijs} - dead weight of "S" vessel of "j" direction of shipments of "i" cargo.

$$a_0 = 5, a_1 = 0.95, D_s = 1.1 G_s$$

The total anchorage time of vessels is estimated as per the following formula:

$$(12) \quad T_{00}^{\varphi} = \sum_i \sum_j \sum_s t_{ijs}^{\varphi} \quad \text{where:}$$

t_{ijs}^{φ} - time under handling of vessels S ,

"j" direction of shipments of "i" cargo, vessel day.

Hereby,

$$(13) \quad t_{ijs}^{\varphi} = \frac{G_{ijs}}{M_{ijrs}} \psi + t_{ijs}^{\varphi_0} \quad \text{where: } G_{ijs} - \text{tonnage of "S"}$$

vessel; M_{ijrs} - rate of handling intensity of "S" vessel, "j" direction of shipments, of "i" cargo at "r" berth, thou. tons/vessel-day.

ψ - coefficient that considers any increase in time anchorage of transport vessels at the port due to unfavourable meteorological conditions;

$t_{ijs}^{\varphi_0}$ - duration of auxiliary operations at handling of vessel; vessel-day.

Duration of vessel delay due to occupied berths is formed in accordance with the formula:

$$(14) \quad T_{3n}^{\varphi} = \sum_i \sum_j \sum_s \tau_{ijs}^{\varphi_{3n}} \quad \text{where:}$$

$\tau_{ijs}^{\varphi_{3n}}$ - duration of "S" vessel delay, "j" direction of shipments of cargo "i" due to occupied berths of "i" group, vessel-day.

As aforestated, the port berthage is divided into berth groups that handle individual (joint handling is not permitted) types of cargo and it is understood hereby that individual berths

of this group can have intergroup specialization to be provided for by their attachment for handling of vessels "j" - directions of cargo "i" to be considered. Delay of the first type is a delay due to occupied berths, for the berths of other groups do not handle vessels with "i" type of cargo. Duration in delay of the second type is estimated as per the formula:

$$(15) \quad T_{cn}^{\varphi} = \sum_i \sum_j \sum_s \tau_{ijs}^{\varphi cn} \quad \text{where:}$$

$\tau_{ijs}^{\varphi cn}$ - duration in delay of "S" vessel, "j" direction of shipments of type of cargo due to intergroup specialization of berths of "i" group, vessel-day.

Any delay of vessels due to insufficient depth at berth is a delay of third type. Duration of this delay is estimated at imitation according to the formula:

$$(16) \quad T_{rn}^{\varphi} = \sum_i \sum_j \sum_s \tau_{ijs}^{\varphi rn} \quad \text{where:}$$

$\tau_{ijs}^{\varphi rn}$ - duration in delay of "S" vessel, "j" direction of cargo shipments, due to various depths at berths of "i" group, vessel-day.

The total amount of given construction-operation expresses as per the system "berthage-transport fleet" is determined according to the formula:

$$(17) \quad R = \sum_i \sum_z R_{iz} + \left\{ \left(\sum_i \sum_j \sum_s \tau_{ijs}^{\varphi} \bar{R}_{ijs}^{\varphi} \right) + \left(\sum_i \sum_j \sum_s \tau_{ijs}^{\varphi 3n} \bar{R}_{ij}^{\varphi} \right) + \right. \\ \left. + \sum_i \sum_j \sum_s \tau_{ijs}^{\varphi cn} \bar{R}_{ij}^{\varphi} \right\} + \left(\sum_i \sum_j \sum_s \tau_{ijs}^{\varphi rn} \bar{R}_{ij}^{\varphi} \right) \quad \text{where:}$$

R_{iz} - amount of given expenses on erection and operation of "z" berth of "i" group, thou.roubles;

\bar{R}_{ij}^{φ} - amount of given construction-operation expenses at an average per day of anchorage at the port per one ves-

sel „S“, direction „j“ of shipments of „i“ type of cargo, thou.roubles.

The logic (block diagram) flow chart of computer imitation of system functioning process is provisionally divided in the following blocks:

Block I. Input of initial data.

Block II Determination of moments of vessel arrived for handling.(Formula 8)

Block III Estimate of tonnage (10),loading(as per input coefficients), draughts (11) of vessels arriving for handling

Block IV The first check is carried on: whether there is any free berth by the time of successive vessel calling or not. If all the berths of considered specialized group are occupied, then the control of imitation process is given to Block V,if though one berth is free-to BlockIV

Block V The delay time of vessels pending due to occupied berth (14) is estimated and accumulated.

Block VI The second check is carried on: whether it is possible to place the arrived vessel under handing to any free berth in accordance with its specialization or not,i.e. according to the given diagram of jets „j“ to be attached to the berths of group „i“ . If not-control is shifted to Block VII,if yes- to Block VIII.

Block VII The delay time of vessels due to the stipulated diagram of specialization of individual berths (15) is estimated and accumulated.

Block VIII The third check is carried on: whether the depth at any free berth is sufficient or not, the berth specialization corresponds to conditions of arrived vessel acceptance, to place this vessel for handling to a given berth. If not - process-control is shifted to Block IX (i.e. the arrived vessel is delayed until such a berth is freed, specialization and depth of the berth permit to receive such a vessel for handling), if yes - to Block XI (i.e. the arrived vessel is taken for handling to the free berth).

Block IX. The delay time of vessels due to insufficient depth at berths (16) is estimated and accumulated.

Block X. The total delay time of vessels pending the handling is estimated. The said amount is determined according to the accumulated data of blocks V, VII, IX.

Block XI The anchorage time of vessels under handling is estimated and accumulated: duration of anchorage - as per formula (15), totalling - as per formula (12).

Block XII The occupied time (4) and the idle time (6) of each separate berth are estimated and accumulated.

Block XIII The occupied time (3) and the idle time (5) of all considered berths as well as coefficient of their use by time (7) are estimated.

Block XIV The monetary indicators for berths and vessels (17) are estimated, i.e. economic efficiency of considered alternative of erection, development or operation of sea port berthage.

Block XV. Provision of estimate results for printing.