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معهد التخطيط القومي

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INPUT OUTPUT ANALYSIS
A PILOT EXAMPLE

by

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القاهرة
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An Introduction:

Dr. Nazih Deif and his group, of the ministry of the national planning, has constructed recently, an Input/Output table for the year 1962.*

The Operations Research group has the opportunity of working out in collaboration with Dr. Deif, the different applications of that I/O table.

The purpose of this paper is to present these results, to be taken as an example of what an I/O table can provide us.

It is hoped that this pilot application will be considered when more elaborate I/O table is constructed.**

This paper is devided into three sections. The first section is a brief description of the I/O table used. The second section is a theoritical formulation of the applications of the I/O tables, while the third section contains the numerical results.

Section I. The I/O table

In the I/O table provided by Dr. Deif, the sectors considered had been aggregated into 13 sectors. The components of these sectors are given in table 1.

The I/O technical coefficients are given in table 2. The coefficients in each cell are devided into two components: the first component corresponds to inputs from local production while the second component coresponds to imported inputs.

The primary factors had been devided into three parts:

1- Wages.

2- Customs and duties.

3- Residual.

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* A separate paper will be given by Dr. Deif explaining in detail the structure of that table.

** There is now, in the Operations Research Center; under the supervision of Dr. Aglan; a project for constructing an elaborate I/O table.

Table 3 is a reformulation of table 2 where the import components in each column had been aggregated to form the import row no 14.

Table -1-

The sectors used in the I/O table and their components .

Sector No	Name of the sectors and its components.
1	Agriculture, including, foresting, animal breeding husbandry and animal products.
2	Minning and Quarying, including petroleum extraction.
3	Food, beverages and tobacco products.
4	Ginning and Pressing
5	Textiles and Clothing industries, including foot wear.
6	Chemical industries; basic chemicals, pharmthetical products and printing and publishing.
7	Non-metalic industries; cement, ceramics, bricks.
8	Basic metal industries.
9	Engineering industries; electric and non-electric machinary, transport equipment and rolling stock, and other metal products.
10	Other industries; wood products, furnitures, paper products, automobile tyres and other rubber products.
11	Energy and Fuel; electricity and Petroleum refinary products.
12	Construction; rail, road, houses and industrial plant construction.
13	Services; transportation, communication, Suez Canal, housing, water sewage, health, education (private sector), trade and finance, personal services(including hotels), recreation and professional services.

The I/O technical coefficients

(Local Production and Import I/O coefficients)

Table (2) 1.

k \ h		1	2	3	4	5	6	7
1	Imports	0.000 474	----	0.069 934	----	0.010 209	----	----
	Local	0.193 073	----	0.438 509	0.909 037	0.008 932	0.001 289	0.025 735
	Total	0.193 547	----	0.508 443	0.909 037	0.019 141	0.001 289	0.025 735
2		----	----	----	----	----	0.016 753	----
		----	----	0.000 734	----	----	0.014 175	0.055 147
		----	----	0.000 734	----	----	0.030 928	0.055 147
3		----	----	0.034 875	----	----	0.050 258	----
		0.011 690	----	0.107 930	----	0.000 851	0.036 082	----
		0.011 690	----	0.142 805	----	0.000 851	0.086 340	----
4		----	----	----	----	----	----	----
		0.001 738	----	0.012 298	----	0.203 318	----	----
		0.001 738	----	0.012 298	----	0.203 318	----	----
5		----	----	----	----	0.022 544	----	----
		0.001 580	----	0.007 526	0.008 918	0.318 162	0.006 443	----
		0.001 580	----	0.007 526	0.008 918	0.340 706	0.006 443	----
6		0.020 063	0.010 169	0.001 101	----	0.018 715	0.125 000	----
		0.036 493	0.040 678	0.010 095	0.000 595	----	0.081 186	0.055 147
		0.056 556	0.050 847	0.011 196	0.000 595	0.018 715	0.206 186	0.055 147
7		----	0.003 390	----	----	----	----	----
		----	----	----	----	----	----	0.062 500
		----	0.003 390	----	----	----	----	0.062 500
8		----	----	----	----	----	----	----
		----	----	----	----	----	0.006 443	0.011 029
		----	----	----	----	----	0.006 443	0.011 029

Table (2) 2.

9	0.000 948 0.002 844 0.003 792	0.006 780 0.027 118 0.033 898	0.001 285 0.000 551 0.001 836	0.001 189 0.000 595 0.001 784	0.004 679 0.023 828 0.028 507	0.014 175 0.001 289 0.015 464	0.022 059 0.003 676 0.025 735
10	— 0.001 264 0.001 264	0.006 780 0.010 169 0.016 949	— 0.001 836 0.001 836	— — —	— 0.021 268 0.021 268	— 0.015 464 0.015 464	— 0.128 676 0.128 676
11	0.002 212 0.014 218 0.016 430	0.003 390 0.010 169 0.013 559	0.002 937 0.010 830 0.013 767	0.000 595 0.005 351 0.005 946	0.001 701 0.022 118 0.023 819	0.010 309 0.047 680 0.057 989	0.014 706 0.073 529 0.088 235
12	— 0.000 474 0.000 474	— 0.010 169 0.010 169	— 0.001 101 0.001 101	— — —	— 0.000 851 0.000 851	— 0.003 866 0.003 866	— 0.003 676 0.003 676
13	— 0.019 273 0.019 273	— 0.037 288 0.037 288	0.030 470 0.030 470	— 0.047 562 0.047 562	0.030 625 0.030 625	— 0.063 144 0.063 144	— 0.091 912 0.091 912
$\sum_k X'_{kh}$	0.282 647	0.135 591	0.621 880	0.972 058	0.629 953	0.277 061	0.511 027
B'_h	0.023 697	0.030 509	0.110 132	0.001 784	0.057 848	0.216 495	0.036 765
$B'_h + \sum_k X'_{kh}$	0.306 344	0.166 100	0.732 012	0.973 842	0.687 801	0.493 557	0.547 972
W'_h	0.219 589	0.172 881	0.043 686	0.008 323	0.135 262	0.139 175	0.180 149
r'_h	0.005 213	0.003 391	0.113 069	0.000 595	0.027 222	0.054 124	0.011 030
ϵ'_h	0.468 854	0.657 628	0.111 233	0.017 240	0.149 715	0.313 144	0.261 029
G.T	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

Table (2) 3.

k \ h		8	9	10	11	12	13	G.T.
	Imports	—	—	—	—	—	—	0.015 775
1	Local	—	—	0.032 609	—	—	0.000 321	0.198 693
	Total	—	—	0.032 609	—	—	0.000 321	0.214 468
		0.133 779	—	—	0.051 980	—	—	0.003 673
2		0.086 957	—	—	0.160 891	0.010 050	0.000 801	0.008 158
		0.220 736	—	—	0.212 871	0.010 050	0.000 801	0.011 831
		—	—	0.008 696	—	—	—	0.009 009
3		—	—	0.069 565	—	—	—	0.027 992
		—	—	0.078 261	—	—	—	0.037 001
		—	—	—	—	—	—	—
4		—	—	—	—	—	—	0.021 497
		—	—	—	—	—	—	0.021 497
		—	—	—	—	—	—	0.002 049
5		—	0.003 322	0.008 696	—	0.006 533	0.004 532	0.034 256
		—	0.003 322	0.008 696	—	0.006 533	0.004 532	0.036 305
		—	0.001 107	0.050 000	—	—	—	0.011 638
6		0.003 344	0.047 619	0.145 652	0.018 564	0.015 075	0.022 737	0.026 214
		0.003 344	0.048 726	0.195 652	0.018 564	0.015 075	0.022 737	0.037 852
		—	—	—	—	—	—	0.000 039
7		—	—	0.004 348	—	0.127 538	0.001 122	0.009 279
		—	—	0.004 348	—	0.127 538	0.001 122	0.009 318
		0.043 478	0.025 471	—	—	—	—	0.001 392
8		0.033 445	0.129 568	0.013 043	—	0.083 794	0.004 763	0.010 401
		0.076 923	0.155 039	0.013 043	—	0.083 794	0.004 763	0.011 793

Table (2) 4.

9	0.026 756 0.013 378 0.040 134	0.102 015 0.297 719 0.399 734	0.019 565 0.050 000 0.069 565	0.008 663 0.009 901 0.018 564	---- 0.038 693 0.038 693	---- 0.056 667 0.056 667	0.004 833 0.010 555 0.015 388
10	---- ---- ----	---- 0.012 182 0.012 182	0.160 870 0.113 043 0.273 913	---- ---- ----	---- 0.044 724 0.044 724	---- 0.002 981 0.002 981	0.002 938 0.013 571 0.016 509
11	0.073 579 0.043 478 0.117 057	0.005 537 0.028 793 0.034 330	0.006 522 0.030 435 0.036 957	0.030 941 0.084 158 0.115 099	---- 0.016 080 0.016 080	---- 0.013 558 0.013 558	0.003 982 0.022 038 0.026 020
12	---- 0.003 344 0.003 344	---- 0.005 537 0.005 537	---- 0.002 174 0.002 174	---- 0.002 475 0.002 475	---- ---- ----	---- 0.003 442 0.003 442	---- 0.001 392 0.001 392
13	---- 0.080 268 0.080 268	---- 0.046 346 0.046 346	---- 0.043 478 0.043 478	---- 0.074 257 0.074 257	---- 0.193 467 0.193 467	---- 0.273 301 0.273 301	---- 0.124 188 0.124 188
$\sum_k X'_{kh}$	0.264 214	0.571 086	0.513 043	0.350 246	0.535 954	0.384 225	0.508 234
B'_h	0.277 592	0.134 130	0.245 653	0.091 584	----	----	0.055 328
$B'_h + \sum_k X'_{kh}$	0.541 806	0.705 216	0.758 696	0.441 830	0.535 954	0.384 225	0.563 562
W'_h	0.183 946	0.172 591	0.141 304	0.133 663	0.311 558	0.204 808	0.167 028
r'_h	0.073 579	0.015 504	0.054 348	0.025 990	----	----	0.032 555
ϵ'_h	0.200 669	0.106 689	0.045 652	0.398 517	0.152 488	0.410 967	0.236 855
G.T	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

The I/O technical coefficients
(Local Production + Import row).

Table (3). 1

k \ h	1	2	3	4	5	6	7
1	0.193 073	0.000 000	0.438 509	0.909 037	0.008 932	0.001 289	0.025 735
2	0.000 000	0.000 000	0.000 734	0.000 000	0.000 000	0.014 175	0.055 147
3	0.011 690	0.000 000	0.107 930	0.000 000	0.000 851	0.036 082	0.000 000
4	0.001 738	0.000 000	0.012 298	0.000 000	0.203 318	0.000 000	0.000 000
5	0.001 580	0.000 000	0.007 526	0.008 918	0.318 162	0.006 443	0.000 000
6	0.036 493	0.040 678	0.010 095	0.000 595	0.000 000	0.081 186	0.055 147
7	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.062 500
8	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.006 443	0.011 029
9	0.002 844	0.027 118	0.000 551	0.000 595	0.023 828	0.001 289	0.003 676
10	0.001 264	0.010 169	0.001 836	0.000 000	0.021 268	0.015 464	0.128 676
11	0.014 218	0.010 169	0.010 830	0.005 351	0.022 118	0.047 680	0.073 529
12	0.000 474	0.010 169	0.001 101	0.000 000	0.000 851	0.003 866	0.003 676
13	0.019 273	0.037 288	0.030 470	0.047 562	0.030 625	0.063 144	0.091 912
B'_h	0.023 697	0.030 509	0.110 132	0.001 784	0.057 847	0.216 495	0.036 765
M'_h	0.219 589	0.172 881	0.043 686	0.008 323	0.135 262	0.139 157	0.180 149
Duties	0.005 213	0.003 391	0.113 069	0.000 595	0.027 222	0.054 124	0.011 030
Resid.	0.468 854	0.657 628	0.111 232	0.017 241	0.149 715	0.313 144	0.261 029
G.T.	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

Cont. Table (3) . 2

k \ h	8	9	10	11	12	13
1	0.000 000	0.000 000	0.032 609	0.000 000	0.000 000	0.000 321
2	0.086 957	0.000 000	0.000 000	0.160 891	0.010 050	0.000 801
3	0.000 000	0.000 000	0.069 565	0.000 000	0.000 000	0.000 000
4	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000
5	0.000 000	0.003 322	0.008 696	0.000 000	0.006 533	0.004 532
6	0.003 344	0.047 619	0.145 652	0.018 564	0.015 075	0.022 737
7	0.000 000	0.000 000	0.004 348	0.000 000	0.127 538	0.001 122
8	0.033 445	0.129 568	0.013 043	0.000 000	0.083 794	0.004 763
9	0.013 378	0.297 719	0.050 000	0.009 901	0.038 693	0.056 667
10	0.000 000	0.012 182	0.113 043	0.000 000	0.044 724	0.002 981
11	0.043 478	0.028 793	0.030 435	0.084 158	0.016 080	0.013 558
12	0.003 344	0.005 537	0.002 174	0.002 475	0.000 000	0.003 442
13	0.080 268	0.046 346	0.043 478	0.074 257	0.193 467	0.273 301
B'_h	0.277 592	0.134 130	0.245 653	0.091 584	0.000 000	0.000 000
W'_h	0.183 946	0.172 591	0.141 304	0.133 663	0.311 558	0.204 808
Duties	0.073 579	0.015 504	0.054 348	0.025 990	0.000 000	0.000 000
Resid.	0.200 669	0.106 681	0.045 652	0.398 571	0.152 488	0.410 967
G.T.	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

Section II. Theoritical Formulation.

I/O analysis have been discussed elaboratory in the literature.

We shall follow, however Ragner Frisch's representation which is given in many of his writings*.

We feel that it will be useful if we explain here in some details and in a systematic way the different formulae which has been given by R. Frisch.

Let k denotes the delivering sector and h denotes the receiving sector. The symbols k and h take the values from 1,...,n.

Let X_{kh} be the deliveries (in volume figure) from sector k to sector h. $(k,h = 1,\ldots,n)$.

Let B_h, W_h, r_h and ϵ_h represent respectively the deliveries to sector h from the imports, wages, customs & duties and residual.

If X_h is the total production of sector h then we have the following relations.

$$\sum_{k=1}^{n} X_{kh} + B_h + W_h + r_h + \epsilon_h = X_h \qquad (h=1,\ldots,n) \ \ldots \ (1)$$

Let D_k be the final demands in sector k.

By definition we shall have

$$\sum_{h} X_{kh} + D_k = X_k \qquad (k=1,\ldots,n) \ \ldots \ (2)$$

Assuming constant technical coefficients, then we shall have

$$\frac{X_{kh}}{X_h} = X'_{kh} = \text{const.}$$

$$\frac{B_h}{X_h} = X'_h = \text{const.} \qquad \ldots \ (3)$$

$$\frac{W_h}{X_h} = W'_h = \text{const.}$$

* See for example : Memo No 3 To the National Planning Committe Cairo by Ragnar Frisch, Dec. 1957, " Advance list of Some Formulae for Decomposition In Interflow Tables.

$$\frac{r_h}{X_h} = r'_h = \text{const.}$$

$$(h=1,\ldots,n)$$

$$\frac{\epsilon_h}{X_h} = \epsilon'_h = \text{const.}$$

It is obvious that :

$$\sum_k X'_{kh} + B'_h + W'_h + r'_h + \epsilon'_h = 1$$

From 2 & 3 we see that

$$\sum_{h=1}^{n} (\delta_{kh} - X'_{kh}) X_h = D_k \quad (k = 1,\ldots,n) \quad \ldots (4)$$

where $\delta_{kh} = 1$ for $k = h$

$= o$ for $k \neq h$

Putting

$$(\delta - X')_{kh} = \delta_{kh} - X'_{kh} \quad \ldots (5)$$

then equations 4 take the form $\sum_h (\delta - X')_{kh} \cdot X_h = D_k \quad (k=1,\ldots,n)$

(6)

Notice that $(\delta - X')_{kh}$ is an n by n matrix. It is simply the n by n input/output technical coefficients matrix substracted from the n by n unit matrix.

This resulting matrix is shown in table 4.

From equation 6 it is seen that .

$$X_h = \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \; D_k \quad (h=1,\ldots,n) \quad \ldots (7)$$

where $(\delta - X')^{-1}_{hk}$ is the inverse of the matrix $(\delta - X')_{kh}$.

The inverse is given in table 5.

Denote

ΔD_k = Change in the final demand from sector k.

ΔX_h = The corresponding change in the total production of sector h. From equations 7 we see that

$$\Delta X_h = \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \; \Delta D_k \quad \ldots (8)$$

Equations 8 enable us to determine the change in the total production of sector h corresponding to given changes in the final demands of sector k. $(k=1,\dots,n)$.

These changes in the final production ΔX_h will entail corresponding changes in the imports, wages, customs & duties and residual. To get these corresponding changes multiply the changes ΔX_h by the corresponding technical coefficients.

Thus the change ΔB_h in the imports delivered to sector h due to changes ΔD_k in the final demands of sectors k $(k=1,\dots,n)$ is given by:

$$\Delta B_h = B_h' \Delta X_h = B_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\dots,n) \quad \dots (9)$$

Similarly the changes Δw_h, Δr_h, ΔE_h due to changes ΔD_k is given by

$$\Delta W_h = W_h' . \Delta X_h = W_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\dots,n) \quad \dots (10)$$

$$\Delta r_h = r_h' . \Delta X_h = r_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\dots,n) \quad \dots (11)$$

$$\Delta \epsilon_h = \epsilon_h' . \Delta X_h = \epsilon_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\dots,n) \quad \dots (12)$$

Defining the value added as

$$V_h = W_h + \epsilon_h \qquad \dots (13)$$

We see that the corresponding change in ΔV_h will be given by:

$$\Delta V_h = (W_h' + \epsilon_h') \Delta X_h = (W_h' + \epsilon_h') \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\dots,n) \quad \dots (14)$$

From equations 9,...,12 & 14 we see that the matrices $B_h' (\delta - X')^{-1}_{hk}$, $W_h' (\delta - X')^{-1}_{hk}$, $r_h' (\delta - X')^{-1}_{hk}$, $\epsilon_h' (\delta - X')^{-1}_{hk}$ and $V_h' (\delta - X')^{-1}_{hk}$, can be computed once for all. Each h,k element of any of these matrices will simply be the change in the corresponding primary factor's component of the sector h due to a unit change in the final demand of sector k.

$(\delta - X')_{kh}$

Table (4) 1.

k \ h	1	2	3	4	5	6	7
1	+0.806 997	0.000 000	-0.438 509	-0.909 037	-0.008 932	-0.001 289	-0.025 735
2	0.000 000	+1.000 000	-0.000 734	0.000 000	0.000 000	-0.014 175	-0.055 147
3	-0.011 690	0.000 000	+0.892 070	0.000 000	-0.000 851	-0.036 092	0.000 000
4	-0.001 738	0.000 000	-0.012 298	+1.000 000	-0.203 318	0.000 000	0.000 000
5	-0.001 580	0.000 000	-0.007 526	-0.008 918	+0.681 838	-0.006 443	0.000 000
6	-0.036 493	-0.040 678	-0.010 095	-0.000 595	0.000 000	+0.918 814	-0.055 147
7	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	+0.937 500
8	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	-0.006 443	-0.011 099
9	-0.002 844	-0.027 118	-0.000 551	-0.000 595	-0.023 828	-0.001 289	-0.003 676
10	-0.001 264	-0.010 169	-0.001 836	0.000 000	-0.021 268	-0.015 464	-0.128 676
11	-0.014 218	-0.010 169	-0.010 830	-0.005 351	-0.022 118	-0.047 680	-0.073 529
12	-0.000 474	-0.010 169	-0.001 101	0.000 000	-0.000 851	-0.003 866	-0.003 676
13	-0.019 273	-0.037 288	-0.030 470	-0.047 562	-0.030 625	-0.063 144	-0.091 912

Cont. Table (4) 2.

k \ h	8	9	10	11	12	13
1	0.000 000	0.000 000	-0.032 609	0.000 000	0.000 000	-0.000 321
2	-0.086 957	0.000 000	0.000 000	-0.160 891	-0.010 050	-0.000 801
3	0.000 000	-0.000 000	-0.069 565	0.000 000	0.000 000	0.000 000
4	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000
5	0.000 000	-0.003 322	-0.008 696	0.000 000	-0.006 533	-0.004 532
6	-0.003 344	-0.047 619	-0.145 652	-0.018 564	-0.015 075	-0.022 737
7	0.000 000	0.000 000	-0.004 348	0.000 000	-0.127 538	-0.001 122
8	+0.966 555	-0.129 568	-0.013 043	0.000 000	-0.083 794	-0.004 763
9	-0.013 378	+0.702 281	-0.050 000	-0.009 901	-0.038 693	-0.056 667
10	0.000 000	-0.012 182	+0.886 957	0.000 000	-0.044 724	-0.002 981
11	-0.043 478	-0.028 793	-0.030 435	+0.915 842	-0.016 080	-0.013 558
12	-0.003 344	-0.005 537	-0.002 174	-0.002 475	+1.000 000	-0.003 442
13	-0.080 268	-0.046 346	-0.043 478	-0.074 257	-0.193 467	-0.273 301

Cont. Table (5) 2.

h \ k	8	9	10	11	12	13	$\sum_k$
1	0.000995	0.006385	0.105642	0.001657	0.015490	0.004924	3.615100
2	0.099707	0.028668	0.015245	0.178111	0.035321	0.008571	1.502443
3	0.000838	0.005354	0.097772	0.001788	0.008161	0.002455	1.361273
4	0.000274	0.001952	0.005237	0.000328	0.002932	0.002169	1.344985
5	0.001287	0.009221	0.018945	0.001493	0.013794	0.010478	1.583532
6	0.015023	0.085600	0.195118	0.035926	0.052765	0.043317	1.856796
7	0.000897	0.001622	0.006036	0.000899	0.137220	0.002510	1.221936
8	1.040630	0.195886	0.030171	0.005913	0.103768	0.023262	1.457569
9	0.035592	1.443183	0.093510	0.033637	0.091691	0.115217	2.013165
10	0.002769	0.023073	1.134606	0.004006	0.074525	0.008052	1.491386
11	0.054666	0.062900	0.057897	1.099377	0.047467	0.028692	1.665477
12	0.005347	0.009958	0.004716	0.005338	1.003328	0.005888	1.064906
13	0.131058	0.134160	0.110893	0.129376	0.319011	1.396324	2.909092
$\sum_h$	1.389083	2.007962	1.875788	1.497849	1.905473	1.651859	23.087660

$$(\delta - X')^{-1}_{hk}$$

Table (5) 1.

h \ k	1	2	3	4	5	6	7
1	1.253422	0.002890	0.635734	1.142899	0.361827	0.031589	0.051646
2	0.004822	1.004512	0.006208	0.005861	0.009503	0.026791	0.079123
3	0.018820	0.003225	1.131414	0.017357	0.010248	0.046533	0.017306
4	0.003231	0.000346	0.018116	1.005723	0.300306	0.003099	0.001272
5	0.004039	0.001483	0.015231	0.017331	1.473312	0.012154	0.004764
6	0.052808	0.051554	0.041824	0.051215	0.028271	1.099643	0.103732
7	0.000223	0.001643	0.000411	0.000334	0.000661	0.000954	1.068526
8	0.002004	0.008037	0.002420	0.003173	0.010128	0.010715	0.021462
9	0.009657	0.046290	0.011265	0.015883	0.064611	0.015523	0.037106
10	0.003265	0.014112	0.004989	0.003737	0.037945	0.020907	0.159399
11	0.023784	0.017565	0.027783	0.029389	0.050125	0.062414	0.103418
12	0.001146	0.011011	0.002193	0.001383	0.002616	0.005311	0.006671
13	0.043044	0.065937	0.074839	0.107345	0.108126	0.111763	0.177215
$\sum_h$	1.420265	1.228605	1.954311	2.401630	2.457679	1.447396	1.881640

It is obvious that these matrices can be computed once for all and they are characteristic of our input-output structure.

Tables 6,7,8,9&10 in section III give the elements of these matrices.

Now consider table 6. The vertical sum for given column k i.e, for given sector k, gives the total import required to meet (on the current account) unit change in the final demand of sector k. That means for an increase in the final demand of sector k by one unit, it is necessary to import an amount equal to

$$\sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk} \qquad \ldots (15)$$

This means that the real increase in production, or output will be equal to

$$1 - \sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk} \qquad \ldots (16)$$

and this will correspond to a change equal to $\sum_{h=1}^{n} B'_h . \, (\delta - X')^{-1}_{hk}$ in the import.

Thus

$$\frac{1 - \sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk}}{\sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk}} \qquad \ldots (17)$$

will give the net* increase in the output of sector k per unit of the required import.

The quantity $\dfrac{1 - \sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk}}{\sum_{h=1}^{n} B'_h \, (\delta - X')^{-1}_{hk}}$ is very important as $\ldots (18)$

giving an index measuring the interaction of production with imports.

That quantity is given at row no 15 in table 6.

This however will not put into consideration the constrains or limits to the production or other factors.

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* By net we mean the output substracted from it the imports corresponding to that output.

In all the previous analysis, we did not specify the nature of the change in the demand of sector k.

Remembering the composition of the final demand, that change may be to meet either one or more of the following:

1- Change in the consumption whether on the household or the government levels.
2- Change in investment requirments.
3- Change in the inventory level.
4- Change in the exports requirments whether net or gross.

It is obvious that ΔD_k can correspond to any one or more of the above factors, with the results, that the different formulae given in equations 9,...,12 & 14, will correspond to the category under which ΔD_k enter.

In the previous discussion so far, we had been concerned with the effect of the change in the final demand on the various economic parameter as wages, production .. & so on. This concept of "change in final demand" had been obtained through equation. 8.

However we can carry on the above discussion, but consider the final demand itself instead of the change in it.

This is obvious if we consider equation 7 instead of equation 8.

Similarly, we can individualy consider the components of the final demand instead of the final demand in the aggregate form.

To be specific, we can take the net export component of the final demand. Denote that pattern by A_k, we have the following expressions corresponding to formulea 9,...,12, & 14.

$$B'_h . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} A_k . \qquad h=1,\ldots,n \qquad \ldots (19)$$

$$W'_h . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} A_k . \qquad h=1,\ldots,n \qquad \ldots (20)$$

$$r'_h . \sum_{h=1}^{n} (\delta - X')^{-1}_{hk} A_k . \qquad h=1,\ldots,n \qquad \ldots (21)$$

$$\epsilon'_h . \sum_{h=1}^{n} (\delta - X')^{-1}_{hk} A_k . \qquad h=1,\ldots,n \qquad \ldots (22)$$

$$V'_h . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} A_k . \qquad h=1,\ldots,n \qquad \ldots (23)$$

The above expressions give the effect of the net export pattern A_k $(k=1,\dots,n)$ on the imports, wages, customs & duties and value added to sector h.

Similarly we can get corresponding expression for the corresponding pattern of consumption, investment and so on.

For example assuming given patterns* of A_k, C_k, I_k, as shown in table 11, tables 12, 13, & 14 give the effects of these patterns on the imports delivered to sector h, value added of sector h and total production of sector h.

In the above discussion we assume that these final demands or the change in them are arbitrary. This however may not be the case. In fact we may have, from another considerations, restrictions on the total production X_h or even on the net creditor position $\sum_k (A_k - B_k)$ and so on.

This will put some restriction on the uses of the inverse of the I/O table.**

As an example, let us assume that one of the sectors, has fixed given capacity. Take for example, the sector of agriculture i.e, sector no 1, and assume that X_1 is constant. We would like to see how this will affect our analysis.

Assuming X_1 to be at given level, implies that the final demands D_k $(k=1,\dots,n)$ will be constrained through the following relation:

$$X_1 = \sum_{k=1}^{n} (\delta - X')^{-1}_{1k} \; D_k \qquad \dots \quad (24)$$

This relation will enable us to get D_1, the final demand in sector 1, in terms of X_1, D_2, D_3, ..., D_n. With that value of D_1, we can substitute in the remaining formulae relating X_2, ..., X_n with the final demands, i.e, in the formulae

$$X_h = \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \; D_k \qquad (h=2,\dots,n) \; . \; (25)$$

To get the following formula

$$X_h = \alpha_{h1} \; X_1 + \sum_{k=2}^{n} \alpha_{hk} \, D_k \qquad (h=2,\dots,n)$$

---

* The patterns C_k, I_k, have been provided by Dr. Deif, while the pattern A_k has been obtained from another study of allocation problem, this another study will be given later a separate paper.

** To allow for these restictions, we have to use the technique of interflow analysis & mathematical programming. This will be discussed in another report.

Notic that D_1 obtained from equation (24) will decrease as final demands from other sectors increase. This is an obvious result of the assumption that total production from sector 1 is to be maintained at constant level, which means that the direct and indirect requirment on that production due to increase in the production of the other sector, will be on the expense of decreasing the final demand of that sector of assumed constant capacity. This may be achieved by decreasing consumption or investment or net export, etc. *

Again it can be seen by substituting the values of D_1 in equation (25) to get $X_2,\ldots,X_n$ in terms of $D_2,\ldots,D_n$, that the change in the production due to change in final demand is less than what it would be, in case the capacity of sector 1 is not constant. This is due to the fact that direct and indirect effects due to changes in sector 1 is now not considered in this case.

Table 15 & 16 give D_1 & $X_2,\ldots,X_n$ as functions of X_1 & $D_2,\ldots,D_n$ for an economy characterized by the input/output coefficients in table 2, and where we assume the production of sector 1 is set at constant level. The different applications of the Input - Output table as given through equations 9,...,12 & 14 will still hold when one of the sectors is assumed to be of constant capacity except for the following changes: (for illustration again consider sector 1 to be that one of constant capacity)

$$\sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \, \Delta D_k \quad \ldots \quad (h = 1, \ldots, n)$$

will change to

$$\sum_{k=2}^{n} \alpha_{hk} \, D_k \qquad (h = 2,\ldots,n)$$

and $\Delta X_1 = 0$.

Again equations 15,...,18 will hold except

$$\sum_{h=1}^{n} B'_h (\delta - X')^{-1}_{hk}$$

will change to

$$\sum_{h=2}^{n} B'_h \, \alpha_{hk}.$$

$k = 1,\ldots,n$ will change to $k = 2,\ldots,n$.

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* In another words any of the possible components of the final demand of that sector.

Tables 17,...,21 give these corresponding changes when sector 1 is assumed to be of constant capacity.
However equations 19,...,23 will still hold without any alteration because these equations correspond to certain patterns of export, consumption, ... etc. which necessitate given levels of capacities consistent with these patterns. So the concept of preassumed level of capacity for given sector can't be applied in general in that case.

Section III.

The Tabular Results

$$B'_h\ (\delta - X')^{-1}_{hk} \quad \& \quad \frac{1 - \sum_h B'_h\ (\delta - X')^{-1}_{hk}}{\sum_h B'_h\ (\delta - X')^{-1}_{hk}} = \gamma_k$$

Table (6) 1.

h \ k	1	2	3	4	5	6	7
1	0.029 702	0.000 068	0.015 065	0.027 083	0.008 574	0.000 749	0.001 224
2	0.000 147	0.030 647	0.000 189	0.000 179	0.000 290	0.000 817	0.002 414
3	0.002 073	0.000 355	0.124 605	0.001 912	0.001 129	0.005 125	0.001 906
4	0.000 006	0.000 001	0.000 032	0.001 794	0.000 536	0.000 006	0.000 002
5	0.000 234	0.000 086	0.000 881	0.001 003	0.085 227	0.000 703	0.000 276
6	0.011 433	0.011 161	0.009 055	0.011 088	0.006 121	0.238 067	0.022 457
7	0.000 008	0.000 060	0.000 015	0.000 012	0.000 024	0.000 035	0.039 284
8	0.000 556	0.002 231	0.000 672	0.000 881	0.002 811	0.002 974	0.005 958
9	0.001 295	0.006 209	0.001 511	0.002 130	0.008 666	0.002 082	0.004 977
10	0.000 802	0.003 467	0.001 226	0.000 918	0.009 321	0.005 136	0.039 157
11	0.002 178	0.001 609	0.002 544	0.002 692	0.004 591	0.005 716	0.009 471
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.048 434	0.055 894	0.155 795	0.049 692	0.127 290	0.261 410	0.127 126
γ_k	19.646 653	16.890 990	5.418 691	19.123 964	6.856 077	2.825 408	6.866 211

Cont.

Table (6) 2.

h \ k	8	9	10	11	12	13	$\sum_k$
1	0.000 024	0.000 151	0.002 503	0.000 039	0.000 367	0.000 117	0.085 667
2	0.003 042	0.000 875	0.000 465	0.005 434	0.001 077	0.000 261	0.045 838
3	0.000 092	0.000 590	0.010 768	0.000 197	0.000 899	0.000 270	0.149 920
4	0.000 001	0.000 003	0.000 009	0.000 001	0.000 005	0.000 004	0.002 399
5	0.000 074	0.000 533	0.001 096	0.000 086	0.000 798	0.000 606	0.091 603
6	0.003 252	0.018 532	0.042 242	0.007 778	0.011 423	0.009 378	0.401 987
7	0.000 033	0.000 060	0.000 222	0.000 033	0.005 045	0.000 092	0.044 924
8	0.288 871	0.054 376	0.008 375	0.001 641	0.028 805	0.006 457	0.404 609
9	0.004 774	0.193 574	0.012 542	0.004 512	0.012 299	0.015 454	0.270 026
10	0.000 680	0.005 668	0.278 719	0.000 984	0.018 307	0.001 978	0.366 363
11	0.005 006	0.005 761	0.005 302	0.100 685	0.004 347	0.002 627	0.152 531
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.305 849	0.280 123	0.362 243	0.121 390	0.083 372	0.037 244	2.015 867
γ_k	2.269 587	2.569 860	1.760 578	7.237 911	10.994 435	25.849 962	

$W'_h \cdot (\delta - X')^{-1}_{hk}$

Table (7) 1.

h \ k	1	2	3	4	5	6	7
1	0.275 238	0.000 635	0.139 600	0.250 968	0.079 453	0.006 937	0.011 341
2	0.000 834	0.173 661	0.001 073	0.001 013	0.001 643	0.004 632	0.013 679
3	0.000 822	0.000 141	0.049 427	0.000 758	0.000 448	0.002 033	0.000 756
4	0.000 027	0.000 003	0.000 151	0.008 371	0.002 499	0.000 026	0.000 011
5	0.000 546	0.000 201	0.002 060	0.002 344	0.199 283	0.001 644	0.000 644
6	0.007 349	0.007 174	0.005 820	0.007 127	0.003 934	0.153 023	0.014 435
7	0.000 040	0.000 296	0.000 074	0.000 060	0.000 119	0.000 172	0.192 494
8	0.000 369	0.001 478	0.000 445	0.000 584	0.001 863	0.001 971	0.003 948
9	0.001 667	0.007 989	0.001 944	0.002 741	0.011 151	0.002 679	0.006 404
10	0.000 461	0.001 994	0.000 705	0.000 528	0.005 362	0.002 954	0.022 524
11	0.003 179	0.002 348	0.003 714	0.003 928	0.006 700	0.008 342	0.013 823
12	0.000 357	0.003 431	0.000 683	0.000 431	0.000 815	0.001 655	0.002 078
13	0.008 816	0.013 504	0.015 328	0.021 985	0.022 145	0.022 890	0.036 295
$\sum_h$	0.299 705	0.212 855	0.221 024	0.300 838	0.335 415	0.208 958	0.318 432

Table (7) 2.

Cont.

h \ k	8	9	10	11	12	13	$\sum_k$
1	0.000 218	0.001 402	0.023 198	0.000 364	0.003 401	0.001 081	0.793 836
2	0.017 237	0.004 956	0.002 636	0.030 792	0.006 106	0.001 482	0.259 744
3	0.000 037	0.000 234	0.004 271	0.000 078	0.000 357	0.000 107	0.059 469
4	0.000 002	0.000 016	0.000 044	0.000 003	0.000 024	0.000 018	0.011 194
5	0.000 174	0.001 247	0.002 563	0.000 202	0.001 866	0.001 417	0.214 192
6	0.002 091	0.011 912	0.027 152	0.004 999	0.007 343	0.006 028	0.258 386
7	0.000 162	0.000 292	0.001 087	0.000 162	0.024 720	0.000 452	0.220 131
8	0.191 420	0.036 032	0.005 550	0.001 088	0.019 088	0.004 279	0.268 114
9	0.006 143	0.249 080	0.016 139	0.005 805	0.015 825	0.019 885	0.347 454
10	0.000 391	0.003 260	0.160 324	0.000 566	0.010 531	0.001 138	0.210 739
11	0.007 307	0.008 407	0.007 739	0.146 946	0.006 345	0.003 835	0.222 612
12	0.001 666	0.003 102	0.001 469	0.001 663	0.312 595	0.001 834	0.331 780
13	0.026 842	0.027 477	0.022 712	0.026 497	0.065 336	0.285 978	0.595 805
$\sum_h$	0.253 690	0.347 417	0.274 884	0.219 165	0.473 537	0.327 534	3.793 456

$$R'_h \cdot (\delta - X')^{-1}_{hk}$$

Table (8) I.

h \ k	1	2	3	4	5	6	7
1	0.006 534	0.000 015	0.003 314	0.005 958	0.001 886	0.000 165	0.000 269
2	0.000 016	0.003 406	0.000 021	0.000 020	0.000 032	0.000 091	0.000 268
3	0.002 128	0.000 365	0.127 929	0.001 963	0.001 159	0.005 261	0.001 957
4	0.000 002	0.000 000	0.000 011	0.000 598	0.000 179	0.000 002	0.000 001
5	0.000 011	0.000 040	0.000 415	0.000 472	0.040 106	0.000 331	0.000 130
6	0.002 858	0.002 790	0.002 264	0.002 772	0.001 530	0.059 517	0.005 614
7	0.000 002	0.000 018	0.000 005	0.000 004	0.000 007	0.000 011	0.011 786
8	0.000 147	0.000 591	0.000 178	0.000 233	0.000 745	0.000 788	0.001 579
9	0.000 150	0.000 718	0.000 175	0.000 246	0.001 002	0.000 241	0.000 575
10	0.000 177	0.000 767	0.000 271	0.000 203	0.002 062	0.001 136	0.008 663
11	0.000 618	0.000 457	0.000 722	0.000 764	0.001 303	0.001 622	0.002 688
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.012 742	0.009 167	0.135 305	0.013 233	0.050 011	0.069 165	0.033 530

Cont.

Table (8) 2.

h \ k	8	9	10	11	12	13	$\sum_k$
1	0.000 005	0.000 033	0.000 551	0.000 009	0.000 081	0.000 026	0.018 846
2	0.000 338	0.000 097	0.000 052	0.000 604	0.000 120	0.000 029	0.005 094
3	0.000 095	0.000 605	0.011 055	0.000 202	0.000 923	0.000 278	0.153 920
4	0.000 000	0.000 001	0.000 003	0.000 000	0.000 002	0.000 001	0.000 800
5	0.000 035	0.000 251	0.000 516	0.000 041	0.000 376	0.000 285	0.043 108
6	0.000 813	0.004 633	0.010 561	0.001 944	0.002 856	0.002 344	0.100 496
7	0.000 010	0.000 018	0.000 067	0.000 010	0.001 514	0.000 028	0.013 480
8	0.076 569	0.014 413	0.002 220	0.000 435	0.007 635	0.001 712	0.107 245
9	0.000 552	0.022 375	0.001 450	0.000 522	0.001 422	0.001 786	0.031 214
10	0.000 150	0.001 254	0.061 664	0.000 218	0.004 050	0.000 438	0.081 054
11	0.001 421	0.001 635	0.001 505	0.028 573	0.001 234	0.000 746	0.043 286
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.079 988	0.045 315	0.089 644	0.032 558	0.020 213	0.007 673	0.598 543

$$\epsilon'_h \cdot (\gamma - X')^{-1}_{hk}.$$

Table (9) 1.

h \ k	1	2	3	4	5	6	7
1	+0.587 672	+0.001 355	+0.298 066	+0.535 853	+0.169 644	+0.014 811	+0.024 214
2	+0.003 171	+0.660 595	+0.004 083	+0.003 854	+0.006 249	+0.017 619	+0.052 934
3	+0.002 093	+0.000 359	+0.125 849	+0.001 931	+0.001 140	+0.005 176	+0.001 925
4	+0.000 056	+0.000 006	+0.000 312	+0.017 340	+0.005 178	+0.000 053	+0.000 022
5	+0.000 604	+0.000 222	+0.002 280	+0.002 595	+0.220 577	+0.001 820	+0.000 713
6	+0.016 537	+0.016 144	+0.013 097	+0.016 038	+0.008 853	+0.344 347	+0.032 483
7	+0.000 058	+0.000 429	+0.000 107	+0.000 087	+0.000 173	+0.000 249	+0.278 916
8	+0.000 402	+0.001 613	+0.000 486	+0.000 637	+0.002 032	+0.002 150	+0.004 307
9	+0.001 030	+0.004 938	+0.001 202	+0.001 694	+0.006 893	+0.001 656	+0.003 959
10	+0.000 149	+0.000 644	+0.000 228	+0.000 171	+0.001 732	+0.000 954	+0.007 277
11	+0.009 480	+0.007 001	+0.011 073	+0.011 714	+0.019 978	+0.024 876	+0.041 219
12	+0.000 175	+0.001 679	+0.000 334	+0.000 211	+0.000 399	+0.000 810	+0.001 017
13	+0.017 690	+0.027 098	+0.030 756	+0.044 115	+0.044 436	+0.045 931	+0.072 830
$\sum_h$	+0.639 117	+0.722 083	+0.487 873	+0.636 240	+0.487 284	+0.460 452	+0.520 916

Cont. Table (9) 2.

h \ k	8	9	10	11	12	13	$\sum_k$
1	+0.000 467	+0.002 994	+0.049 531	+0.000 777	+0.007 263	+0.002 309	+1.694 954
2	+0.065 570	+0.018 853	+0.010 026	+0.117 131	+0.023 228	+0.005 637	+0.988 049
3	+0.000 093	+0.000 596	+0.010 875	+0.000 199	+0.000 908	+0.000 273	+0.151 417
4	+0.000 005	+0.000 034	+0.000 090	+0.000 006	+0.000 051	+0.000 037	+0.023 189
5	+0.000 193	+0.001 381	+0.002 836	+0.000 224	+0.002 065	+0.001 569	+0.237 078
6	+0.004 704	+0.026 805	+0.061 100	+0.011 250	+0.016 523	+0.013 564	+0.581 445
7	+0.000 234	+0.000 423	+0.001 576	+0.000 235	+0.035 818	+0.000 655	+0.318 961
8	+0.208 822	+0.039 308	+0.006 054	+0.001 187	+0.020 823	+0.004 668	+0.292 489
9	+0.003 797	+0.153 960	+0.009 976	+0.003 588	+0.009 782	+0.012 291	+0.214 766
10	+0.000 126	+0.001 053	+0.051 797	+0.000 183	+0.003 402	+0.000 368	+0.068 085
11	+0.021 788	+0.025 070	+0.023 076	+0.438 180	+0.018 919	+0.011 436	+0.663 811
12	+0.000 815	+0.001 518	+0.000 719	+0.000 814	+0.152 995	+0.000 898	+0.162 385
13	+0.053 861	+0.055 135	+0.045 573	+0.053 169	+0.131 103	+0.573 843	+1.195 541
$\sum_h$	+0.360 475	+0.327 130	+0.273 229	+0.626 943	+0.422 880	+0.627 548	+6.592 170

$V_h' \cdot (\delta - X')^{-1}_{hk} \cdot$

Table (10) 1.

h \ k	1	2	3	4	5	6	7
1	0.862 910	0.001 990	0.437 667	0.786 821	0.249 097	0.021 747	0.035 555
2	0.004 005	0.834 256	0.005 156	0.004 868	0.007 892	0.022 250	0.065 712
3	0.002 916	0.000 500	0.175 276	0.002 689	0.001 588	0.007 209	0.002 681
4	0.000 083	0.000 009	0.000 463	0.025 710	0.007 677	0.000 079	0.000 033
5	0.001 151	0.000 423	0.004 340	0.004 939	0.419 860	0.003 464	0.001 358
6	0.023 885	0.023 318	0.018 917	0.023 165	0.012 787	0.497 370	0.046 918
7	0.000 098	0.000 725	0.000 181	0.000 147	0.000 292	0.000 421	0.471 410
8	0.000 771	0.003 091	0.000 931	0.001 220	0.003 895	0.004 121	0.008 255
9	0.002 697	0.012 928	0.003 146	0.004 436	0.018 044	0.004 335	0.010 363
10	0.000 610	0.002 638	0.000 933	0.000 699	0.007 094	0.003 909	0.029 801
11	0.012 659	0.009 349	0.014 787	0.015 642	0.026 678	0.033 219	0.055 043
12	0.000 532	0.005 110	0.001 018	0.000 642	0.001 214	0.002 465	0.003 096
13	0.026 505	0.040 602	0.046 084	0.066 100	0.066 581	0.068 821	0.109 125
$\sum_h$	0.938 822	0.934 939	0.708 899	0.937 078	0.822 699	0.669 410	0.839 350

Cont. - Table (10) 2.

k / h	8	9	10	11	12	13	$\sum_k$
1	0.000 685	0.004 396	0.072 728	0.001 141	0.010 664	0.003 390	2.488 790
2	0.082 808	0.023 809	0.012 661	0.147 923	0.029 334	0.007 118	1.247 792
3	0.000 130	0.000 829	0.015 147	0.000 277	0.001 264	0.000 380	0.210 886
4	0.000 007	0.000 050	0.000 134	0.000 008	0.000 075	0.000 055	0.034 383
5	0.000 367	0.002 628	0.005 399	0.000 425	0.003 931	0.002 986	0.451 270
6	0.006 795	0.038 717	0.088 252	0.016 249	0.023 866	0.019 592	0.839 831
7	0.000 396	0.000 716	0.002 663	0.000 397	0.060 538	0.001 107	0.539 091
8	0.400 242	0.075 341	0.011 604	0.002 274	0.039 911	0.008 947	0.560 603
9	0.009 940	0.403 041	0.026 115	0.009 394	0.025 607	0.032 177	0.562 221
10	0.000 518	0.004 314	0.212 121	0.000 749	0.013 933	0.001 505	0.278 824
11	0.029 095	0.033 478	0.030 815	0.585 126	0.025 264	0.015 271	0.886 423
12	0.002 481	0.004 621	0.002 188	0.002 477	0.465 590	0.002 732	0.494 165
13	0.080 702	0.082 612	0.068 285	0.079 667	0.196 439	0.859 821	1.791 346
$\sum_h$	0.614 166	0.674 552	0.548 112	0.846 107	0.896 416	0.955 081	10.385 625

Patterns of Components of
Final Demand.
Table (11).

C_k	I_k	A_k
124.30	0.00	- 6.32
0.10	0.00	18.30
508.50	0.00	20.15
0.40	0.00	100.86
160.00	0.00	41.10
26.00	0.00	- 7.97
2.30	0.00	4.00
0.00	0.00	1.00
54.40	64.40	-119.78
10.50	0.00	9.13
51.80	0.00	0.01
0.00	202.20	0.00
356.20	0.00	111.99

Table (12)

	$\sum_{k=1}^{13} (\delta - X')^{-1}_{hk} \cdot A_k$	$B'_h \sum_{k=1}^{13} (\delta - X')^{-1}_{hk} A_k$	$V'_h \cdot \sum_{k=1}^{13} (\delta - X')^{-1}_{hk} A_k$
1	+ 135.816 854	+ 3.218 452	+ 93.502 162
2	+ 18.753 502	+ 0.572 151	+ 15.574 952
3	+ 25.149 868	+ 2.769 805	+ 3.896 167
4	+ 114.181 005	+ 0.203 699	+ 2.918 923
5	+ 62.787 160	+ 3.632 049	+ 17.892 896
6	− 3.886 666	− 0.841 444	− 1.757 943
7	+ 4.514 316	+ 0.165 969	+ 1.991 617
8	− 18.574 714	− 5.156 192	− 7.144 114
9	− 153.507 137	− 20.589 912	− 42.870 245
10	+ 11.277 470	+ 2.770 344	+ 2.108 391
11	+ 10.740 121	+ 0.983 623	+ 5.716 258
12	+ 0.027 613	0.0	+ 0.012 814
13	+ 160.017 045	0.0	+ 98.534 496

Table (13)

	$\sum_{k=1}^{13} (\delta-X)^{-1}_{hk} \; C_k$	$B'_h \; \sum_{k=1}^{13} (\delta-X')^{-1}_{hk} \; C_k$	$V'_h \; \sum_{k=1}^{13} (\delta-X')^{-1}_{hk} \; C_k$
1	541.657 308	12.835 653	372.900 182
2	20.096 719	.613 131	16.690 506
3	581.867 185	64.082 197	90.141 701
4	59.099 148	.105 433	1.510 811
5	248.892 359	14.397 676	70.928 598
6	85.205 703	18.446 609	38.538 625
7	3.917 429	.144 024	1.728 283
8	22.995 284	6.383 307	8.844 331
9	140.040 004	18.783 566	39.109 252
10	30.810 265	7.568 634	5.760 164
11	98.175 642	8.991 318	52.252 415
12	4.377 719	0.0	2.031 463
13	576.604 091	0.0	355.058 384

Table (14)

	$\sum_{k=1}^{13} (\delta-X')^{-1}_{hk} \; I_k$	$B'_h \sum_{k=1}^{13} (\delta-X')^{-1}_{hk} \; I_k$	$V'_h \sum_{k=1}^{13} (\delta-X')^{-1}_{hk} \; I_k$
1	3.543 272	.083 965	2.439 341
2	8.988 125	.274 219	7.464 719
3	1.994 952	.219 708	.309 054
4	.718 559	.001 282	.018 369
5	3.382 979	.195 695	.964 071
6	16.181 723	3.503 262	7.319 009
7	27.850 341	1.023 918	12.286 958
8	33.596 948	9.326 244	12.921 890
9	111.480 905	14.952 934	31.133 495
10	16.554 856	4.066 750	3.095 030
11	13.648 587	1.249 992	7.264 242
12	203.514 217	0.0	94.439 958
13	73.143 928	0.0	45.040 202

Table (15) 1.

h \ k	D_1	D_2	D_3	D_4	D_5	D_6	D_8
X_2	+0.004822	+1.004512	+0.006208	+0.005861	+0.009503	+0.026791	+0.079123
X_3	+0.018820	+0.003225	+1.131414	+0.017327	+0.010248	+0.046533	+0.017306
X_4	+0.003231	+0.000346	+0.018116	+1.005723	+0.000306	+0.003099	+0.001272
X_5	+0.004039	+0.001483	+0.016231	+0.017331	+1.473312	+0.012154	+0.004764
X_6	+0.052808	+0.051554	+0.041824	+0.051215	+0.028271	+1.099643	+0.103732
X_7	+0.000223	+0.001693	+0.000411	+0.000334	+0.000661	+0.000954	+1.069526
X_8	+0.002004	+0.008037	+0.002420	+0.003173	+0.010128	+0.010715	+0.021462
X_9	+0.009657	+0.046290	+0.011265	+0.015883	+0.064611	+0.015523	+0.037106
X_{10}	+0.003265	+0.014112	+0.004989	+0.003237	+0.037945	+0.020907	+0.159399
X_{11}	+0.023784	+0.017565	+0.027783	+0.029389	+0.050125	+0.082414	+0.103418
X_{12}	+0.001163	+0.011011	+0.002193	+0.001383	+0.002616	+0.005311	+0.006671
X_{13}	+0.043044	+0.065937	+0.074039	+0.107345	+0.108126	+0.111763	+0.077215
$\sum$	+0.166843	+1.225715	+1.336693	+1.258731	+2.095852	+1.415807	+1.779994

$$D_1 = -0.002306\,D_2 - 0.507199\,D_3 - 0.091823\,D_4 - 0.288571\,D_5 - 0.025202\,D_6 - 0.041204\,D_7 - 0.000794\,D_8$$

	+1.225330	+1.252070	+1.106600	+2.047689	+1.411602	+1.773119

Cont. Table (15) 2.

h \ k	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}
X_2	+0.099707	+0.028668	+0.015245	+0.178111	+0.035321	+0.008571
X_3	+0.000839	+0.005354	+0.097772	+0.001788	+0.008161	+0.002455
X_4	+0.000274	+0.001952	+0.005237	+0.000328	+0.002932	+0.002169
X_5	+0.001287	+0.009221	+0.018945	+0.001493	+0.013794	+0.010478
X_6	+0.015023	+0.085600	+0.195118	+0.035926	+0.052765	+0.043317
X_7	+0.000897	+0.001622	+0.006036	+0.000899	+0.137220	+0.002710
X_8	+1.040630	+0.195886	+0.030171	+0.005913	+0.103768	+0.023262
X_9	+0.035592	+1.443188	+0.093510	+0.033637	+0.091691	+0.115217
X_{10}	+0.002769	+0.023073	+1.134606	+0.004006	+0.074525	+0.008052
X_{11}	+0.024666	+0.062900	+0.057897	+1.099377	+0.047467	+0.028692
X_{12}	+0.005347	+0.009958	+0.004716	+0.005338	+1.003328	+0.005888
X_{13}	+0.131058	+0.124160	+0.110893	+0.129376	+0.319011	+1.396324
Σ	+1.388088	+2.001577	+1.770146	+1.496192	+1.889983	+1.646935
	$-0.005094\ D_9$	$+0.084283 D_{10}$	$-0.001322 D_{11}$	$-0.012358 D_{12}$	$-0.003928 D_{13}$	$+0.797816\ X_1$
	+1.387956	+2.000727	+1.756084	+1.495971	+1.887921	+1.646280

Table (16) 1.

h \ k	X_1	D_2	D_3	D_4	D_5	D_6	D_7
X_2	+0.003847	+1.004501	+0.003762	+0.001464	+0.008111	+0.026669	+0.078924
X_3	+0.015015	+0.003182	+1.121869	+0.000196	+0.004815	+0.046059	+0.016531
X_4	+0.002578	+0.000339	+0.016477	+1.002777	+0.299373	+0.003018	+0.001139
X_5	+0.003222	+0.001474	+0.013162	+0.013648	+1.472146	+0.012052	+0.004598
X_6	+0.042131	+0.051432	+0.015040	+0.003063	+0.013027	+1.098312	+0.101556
X_7	+0.000178	+0.001642	+0.000218	+0.000131	+0.000597	+0.000948	+1.068517
X_8	+0.001599	+0.008032	+0.001504	+0.001346	+0.009550	+0.010664	+0.021379
X_9	+0.007704	+0.046268	+0.006367	+0.007078	+0.061823	+0.015280	+0.036708
X_{10}	+0.002605	+0.014104	+0.003333	+0.000760	+0.037002	+0.020825	+0.159264
X_{11}	+0.018975	+0.017510	+0.015720	+0.007702	+0.043259	+0.061815	+0.102438
X_{12}	+0.000914	+0.011008	+0.001612	+0.000338	+0.002285	+0.005282	+0.006624
X_{13}	+0.034341	+0.065838	+0.053007	+0.068096	+0.095700	+0.110678	+0.175441
$\sum_h$	+0.133109	+1.225330	+1.252071	+1.106599	+2.047688	+1.411602	+1.773119

Cont. Table (16) 2.

k / h	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	$\sum_{2\to13}$
X_2	+0.099 703	+0.028 643	+0.014 839	+0.178 105	+0.035 261	+0.008 552	1.488 534
X_3	+0.000 823	+0.005 258	+0.096 186	+0.001 763	+0.007 928	+0.002 381	1.306 991
X_4	+0.000 271	+0.001 936	+0.004 965	+0.000 324	+0.002 892	+0.002 156	1.335 667
X_5	+0.001 284	+0.009 200	+0.018 605	+0.001 488	+0.013 744	+0.010 462	1.571 883
X_6	+0.014 981	+0.085 331	+0.190 667	+0.035 856	+0.052 112	+0.043 110	1.704 487
X_7	+0.000 897	+0.001 621	+0.006 017	+0.000 899	+0.137 217	+0.002 509	1.221 293
X_8	+1.040 628	+0.195 876	+0.030 002	+0.005 910	+0.103 743	+0.023 254	1.451 788
X_9	+0.035 584	+1.443 134	+0.092 696	+0.033 624	+0.091 572	+0.115 179	1.985 313
X_{10}	+0.002 766	+0.023 056	+1.134 331	+0.004 002	+0.074 485	+0.008 039	1.481 967
X_{11}	+0.054 647	+0.062 779	+0.055 892	+1.099 346	+0.047 173	+0.028 599	1.596 880
X_{12}	+0.005 346	+0.009 952	+0.004 619	+0.005 336	+1.003 314	+0.005 883	1.061 599
X_{13}	+0.131 024	+0.133 941	+0.107 265	+0.129 319	+0.318 479	+1.396 155	2.784 943
$\sum_h$	+1.387 954	+2.000 727	+1.756 084	+1.495 972	+1.887 920	+1.646 279	18.991 345

$B_h' \cdot \alpha_{hk}$

Table (17) 1.

h \ k	2	3	4	5	6	7	8
2	0.030 646	0.000 115	0.000 045	0.000 247	0.000 814	0.002 408	0.003 042
3	0.000 351	0.123 554	0.000 022	0.000 530	0.005 073	0.001 821	0.000 091
4	0.000 001	0.000 029	0.001 789	0.000 534	0.000 005	0.000 002	0.000 000
5	0.000 085	0.000 763	0.000 790	0.085 161	0.000 697	0.000 266	0.000 074
6	0.011 135	0.003 256	0.000 663	0.002 820	0.237 779	0.021 986	0.003 243
7	0.000 060	0.000 011	0.000 005	0.000 022	0.000 035	0.039 284	0.000 033
8	0.002 230	0.000 390	0.000 374	0.002 651	0.002 960	0.005 935	0.288 870
9	0.006 206	0.000 854	0.000 949	0.008 292	0.002 050	0.004 924	0.004 773
10	0.003 465	0.000 819	0.000 187	0.009 090	0.005 116	0.039 124	0.000 680
11	0.001 604	0.001 440	0.000 705	0.003 962	0.005 661	0.009 382	0.005 005
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Σ_h	0.055 783	0.131 231	0.005 529	0.113 309	0.260 190	0.125 132	0.305 810

Cont. Table (17) 2.

h \ k	9	10	11	12	13	$\sum_h$
2	0.000 874	0.000 453	0.005 434	0.001 076	0.000 261	0.045 414
3	0.000 579	0.010 593	0.000 194	0.000 873	0.000 262	0.143 942
4	0.000 003	0.000 009	0.000 001	0.000 005	0.000 004	0.002 383
5	0.000 532	0.001 076	0.000 086	0.000 795	0.000 605	0.090 930
6	0.018 474	0.041 278	0.007 763	0.011 282	0.009 333	0.369 013
7	0.000 060	0.000 221	0.000 033	0.005 045	0.000 092	0.044 901
8	0.054 374	0.008 328	0.001 641	0.028 798	0.006 455	0.403 005
9	0.193 568	0.012 433	0.004 510	0.012 283	0.015 449	0.266 290
10	0.005 664	0.278 652	0.000 983	0.018 297	0.001 975	0.364 050
11	0.005 750	0.005 119	0.100 683	0.004 320	0.002 619	0.146 249
12	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.279 875	0.358 162	0.121 328	0.082 774	0.037 055	1.876 177

$W'_h \cdot \alpha_{hk}$

Table (18) I.

k / h	2	3	4	5	6	7	8
2	0.173 659	0.000 650	0.000 253	0.001 402	0.004 611	0.013 644	0.017 237
3	0.000 139	0.049 010	0.000 009	0.000 210	0.002 012	0.000 722	0.000 036
4	0.000 003	0.000 137	0.008 346	0.002 492	0.000 025	0.000 009	0.000 002
5	0.000 199	0.001 783	0.001 846	0.199 125	0.001 630	0.000 622	0.000 174
6	0.007 157	0.002 093	0.000 426	0.001 813	0.152 837	0.014 132	0.002 085
7	0.000 296	0.000 054	0.000 024	0.000 108	0.000 171	0.192 492	0.000 162
8	0.001 477	0.000 258	0.000 248	0.001 757	0.001 962	0.003 933	0.191 419
9	0.007 985	0.001 099	0.001 222	0.010 670	0.002 637	0.006 335	0.006 141
10	0.001 993	0.000 471	0.000 107	0.005 229	0.002 943	0.022 505	0.000 391
11	0.002 340	0.002 101	0.001 029	0.005 782	0.008 262	0.013 692	0.007 304
12	0.003 430	0.000 502	0.000 105	0.000 712	0.001 646	0.002 064	0.001 666
13	0.013 484	0.010 856	0.013 947	0.019 600	0.022 668	0.035 932	0.026 835
$\sum_h$	0.212 162	0.069 014	0.027 562	0.248 900	0.201 404	0.306 082	0.253 452

Cont. Table (18) 2.

h \ k	9	10	11	12	13	$\sum_k$
2	0.004 952	0.002 565	0.030 791	0.006 096	0.001 478	0.257 338
3	0.000 230	0.004 202	0.000 077	0.000 346	0.000 104	0.057 097
4	0.000 016	0.000 041	0.000 003	0.000 024	0.000 018	0.011 116
5	0.001 244	0.002 517	0.000 201	0.001 859	0.001 415	0.212 615
6	0.011 874	0.026 533	0.004 990	0.007 252	0.005 999	0.237 191
7	0.000 292	0.001 084	0.000 162	0.024 720	0.000 452	0.220 017
8	0.036 031	0.005 519	0.001 087	0.019 083	0.004 277	0.267 051
9	0.249 072	0.015 998	0.005 803	0.015 805	0.019 879	0.342 646
10	0.003 258	0.160 286	0.000 565	0.010 525	0.001 136	0.209 409
11	0.008 391	0.007 471	0.146 942	0.006 305	0.003 823	0.213 442
12	0.003 101	0.001 439	0.001 662	0.312 591	0.001 833	0.330 751
13	0.027 432	0.021 969	0.026 486	0.065 227	0.285 944	0.570 380
$\sum_h$	0.345 893	0.249 624	0.218 769	0.469 833	0.326 358	2.929 053

$r'_h \cdot \alpha_{hk}$

Table (19) 1.

k / h	2	3	4	5	6	7	8
2	0.003 409	0.000 013	0.000 005	0.000 028	0.000 090	0.000 268	0.000 338
3	0.000 360	0.126 849	0.000 022	0.000 544	0.005 208	0.001 869	0.000 093
4	0.000 000	0.000 010	0.000 597	0.000 178	0.000 002	0.000 001	0.000 000
5	0.000 040	0.000 359	0.000 372	0.040 075	0.000 328	0.000 125	0.000 035
6	0.002 784	0.000 814	0.000 166	0.000 705	0.059 445	0.005 497	0.000 811
7	0.000 018	0.000 002	0.000 001	0.000 007	0.000 010	0.011 786	0.000 010
8	0.000 591	0.000 103	0.000 099	0.000 703	0.000 785	0.001 573	0.076 568
9	0.000 717	0.000 099	0.000 110	0.000 959	0.000 237	0.000 569	0.000 552
10	0.000 767	0.000 181	0.000 041	0.002 011	0.001 132	0.008 656	0.000 150
11	0.000 455	0.000 409	0.000 200	0.001 124	0.001 607	0.002 662	0.001 420
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.09 141	0.128 839	0.001 613	0.046 334	0.068 844	0.033 006	0.079 977

Cont. Table (19) 2.

k / h	9	10	11	12	13	$\sum_k$
2	0.000 097	0.000 050	0.000 604	0.000 120	0.000 029	0.005 051
3	0.000 595	0.010 876	0.000 199	0.000 896	0.000 269	0.147 780
4	0.000 001	0.000 003	0.000 000	0.000 002	0.000 001	0.000 795
5	0.000 250	0.000 506	0.000 041	0.000 374	0.000 285	0.042 790
6	0.004 618	0.010 320	0.001 941	0.002 821	0.002 333	0.092 254
7	0.000 018	0.000 066	0.000 010	0.001 514	0.000 028	0.013 471
8	0.014 412	0.002 208	0.000 435	0.007 633	0.001 711	0.106 821
9	0.022 374	0.001 437	0.000 521	0.001 420	0.001 786	0.030 780
10	0.001 253	0.061 649	0.000 218	0.004 048	0.000 437	0.080 542
11	0.001 632	0.001 453	0.028 572	0.001 226	0.000 743	0.041 503
12	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0
$\sum_h$	0.045 250	0.088 568	0.032 541	0.020 054	0.007 622	0.561 787

$\epsilon'_h \cdot \alpha_{hk}$

Table (20) 1.

h \ k	2	3	4	5	6	7	8
2	0.660 588	0.002 474	0.000 963	0.005 334	0.017 538	0.051 903	0.065 567
3	0.000 354	0.124 788	0.000 022	0.000 536	0.005 123	0.001 839	0.000 092
4	0.000 006	0.000 284	0.017 289	0.005 161	0.000 052	0.000 020	0.000 005
5	0.000 221	0.001 974	0.002 043	0.220 402	0.001 804	0.000 688	0.000 192
6	0.016 106	0.004 710	0.000 959	0.004 079	0.343 930	0.031 802	0.004 691
7	0.000 429	0.000 078	0.000 034	0.000 156	0.000 247	0.278 914	0.000 234
8	0.001 612	0.000 282	0.000 270	0.001 916	0.002 140	0.004 290	0.208 822
9	0.004 936	0.006 79	0.000 755	0.006 595	0.001 630	0.003 916	0.003 796
10	0.000 644	0.000 152	0.000 035	0.001 689	0.000 951	0.007 271	0.000 126
11	0.006 979	0.006 266	0.003 070	0.017 242	0.024 638	0.040 829	0.021 781
12	0.001 679	0.000 246	0.000 052	0.000 348	0.000 805	0.001 010	0.000 815
13	0.027 057	0.021 784	0.027 985	0.039 330	0.045 485	0.072 100	0.053 847
$\sum_h$	0.720 611	0.163 717	0.053 477	0.302 788	0.444 343	0.494 582	0.359 968

Cont. Table (20) 2.

h \ k	9	10	11	12	13	$\sum_k$
2	0.018 836	0.009 759	0.117 127	0.023 189	0.005 624	0.978 902
3	0.000 585	0.010 699	0.000 196	0.000 882	0.000 265	0.145 379
4	0.000 033	0.000 086	0.000 006	0.000 050	0.000 037	0.023 028
5	0.001 377	0.002 785	0.000 223	0.002 058	0.001 566	0.235 334
6	0.026 721	0.059 706	0.011 228	0.016 319	0.013 500	0.533 750
7	0.000 423	0.001 571	0.000 235	0.035 818	0.000 655	0.318 793
8	0.039 306	0.006 020	0.001 186	0.020 818	0.004 666	0.291 329
9	0.153 955	0.009 889	0.003 587	0.009 769	0.012 287	0.211 795
10	0.001 053	0.051 784	0.000 183	0.003 400	0.000 367	0.067 655
11	0.025 022	0.022 277	0.438 167	0.018 802	0.011 399	0.636 470
12	0.001 518	0.000 704	0.000 814	0.152 993	0.000 897	0.161 881
13	0.055 045	0.044 082	0.053 146	0.130 884	0.573 774	1.144 520
$\sum_h$	0.323 874	0.219 362	0.626 098	0.414 982	0.625 037	4.748 839

$$(W + \epsilon)_h' \cdot \alpha_{hk}$$

Table (21) 1.

h \ k	2	3	4	5	6	7
2	0.834247	0.003124	0.001216	0.006736	0.022149	0.065547
3	0.000493	0.173798	0.000030	0.000746	0.007135	0.002561
4	0.000009	0.000421	0.025635	0.007653	0.000077	0.000029
5	0.000420	0.003757	0.003889	0.419528	0.003435	0.001310
6	0.023263	0.006803	0.001385	0.005892	0.496768	0.045934
7	0.000724	0.000131	0.000058	0.000263	0.000418	0.471406
8	0.003089	0.000540	0.000518	0.003673	0.004102	0.008223
9	0.012921	0.001778	0.001977	0.017265	0.004267	0.010252
10	0.002637	0.000623	0.000142	0.006918	0.003893	0.029775
11	0.009319	0.008367	0.004099	0.023024	0.032900	0.05452
12	0.005108	0.000748	0.000157	0.001060	0.002451	0.003074
13	0.040541	0.032640	0.041932	0.058930	0.068153	0.108032
$\sum_h$	0.932771	0.232730	0.081038	0.551688	0.645748	0.800664

Cont.

Table (21) 2.

h \ k	8	9	10	11	12	13	$\sum_k$
2	0.082804	0.023788	0.012324	0.147918	0.029285	0.007103	1.236241
3	0.000127	0.000815	0.014901	0.000273	0.001228	0.000369	0.202476
4	0.000007	0.000049	0.000127	0.000008	0.000074	0.000055	0.034144
5	0.000366	0.002622	0.005302	0.000424	0.003917	0.002981	0.447951
6	0.006776	0.058595	0.086239	0.016218	0.023570	0.019499	0.770941
7	0.000396	0.000715	0.002655	0.000397	0.060537	0.001107	0.538807
8	0.400241	0.075337	0.011539	0.002273	0.039901	0.008944	0.558380
9	0.009938	0.403027	0.025887	0.009390	0.025573	0.032166	0.554441
10	0.000517	0.004310	0.212070	0.000748	0.013925	0.001503	0.277061
11	0.029085	0.033413	0.029748	0.585109	0.025107	0.015221	0.849913
12	0.002481	0.004618	0.002143	0.002476	0.465584	0.002730	0.492630
13	0.080681	0.082478	0.066051	0.079631	0.196111	0.859717	1,714898
$\sum_h$	0.613419	0.669767	0.468986	0.844865	0.884812	0.951395	7.677883

N.A.