The Seventh Study

The effectiveness of worked examples - collaborative reflection - problem solving instruction on students' mathematics achievement

By

Nawaf Awadh Kallaf Alreshidi

Assistant Professor of Mathematics Education at the University of Ha'il

Abstract

The present study aims to assess the effectiveness of worked examples - collaborative reflection - problem solving instruction on students' mathematics achievement, in comparison to worked examples - problem solving instruction. This study conducted a quasi-experimental investigation on(84) second grade intermediate school students, who were divided into two groups, comprising (43) students in the experimental group, and (41) students in the control group. The results show that the average scores in both the knowing and applying domains of the students who were taught using worked examples - collaborative reflection - problem solving instruction (the experimental group) were higher than the average scores of the students who were taught using worked examples - problem solving instruction (the control group) in both the 'knowing' and 'applying' domains.

Keywords: problem solving, mathematics achievement, worked example, collaborative reflection.

فعالية استراتيجية الأمثلة المحلولة – تأمل تعاوني- حل المشكلات على تحصيل الطلاب في الرياضيات

د. نواف عوض خلاف الرشيدي

أستاذ المناهج وطرق تدريس الرياضيات المساعد – جامعة حائل-كلية التربية

المستخلص:

هدفت الدراسة الحالية إلي تقييم فعالية استراتيجية "الأمثلة المحلولة – تأمل تعاوني- حل المشكلات" على تحصيل الطلاب في الرياضيات بالمقارنة مع استراتيجية " الأمثلة المحلولة- حل المشكلات". وقد تكونت عينة الدراسة من(٨٤) طالباً من الصف الثاني متوسط، حيث تم تقسيم الطلاب إلى مجموعتين: المجموعة التجريبية ضمت(٤٣) طالبًا ، وضمت المجموعة الضابطة (٤١) طالبًا. حيث تم استخدام المنهج شبه التجريبي من خلال تدريس طلاب المجموعة التجريبية باستخدام استراتيجية "الأمثلة المحلولة – تأمل تعاوني- حل المشكلات" بينما درست المجموعة الضابطة باستخدام استراتيجية "الأمثلة المحلولة- حل المشكلات". وقد أظهر نتائج الدراسة أن متوسط درجات الطلاب في المجموعة التجريبية أعلى من متوسط درجات الطلاب في المجموعة الضابطة وذلك في مجالي المعرفة والتطبيق.

الكلمات المفتاحية: حل المشكلات، التحصيل الرياضي، الأمثلة المحلولة، التأمل التعاوني.

Introduction:

Previous research has revealed that learning that relies heavily on worked examples is more effective for transfer and learning than learning that relies on problem solving alone (Nievelstein, Van Gog, Van Dijck & Boshuizen, 2013; Renkl, 2017; Rourke & Sweller, 2009; Schwonke et al. 2009; Sweller & Cooper, 1985). The agreed reason for this is that learning relying on problem solving alone increases the cognitive load in students' working memory, while learning relying heavily on worked examples does not. In learning relying on problem solving only, vast amounts of working memory resources are dedicated to solving the problem rather than obtaining new knowledge about the domain (Kirschner, Sweller & Clark, 2006; Sweller, Ayres & Kalyug, 2011). Conversely, however, with learning that relies heavily on worked examples, a sufficient amount of working memory capacity can be devoted to formatting a schema for later problem solving (Paas & van Gog, 2006; Simamora, Sidabutar & Surya, 2017).

In order to construct the schema, the research asserts that constriction and abstraction for schema are promoted when learners are provided with various examples, particularly when they are simultaneously taught to make comparisons between these examples (Gentner, Loewenstein, & Thompson, 2003; Gentner, Loewenstein, Thompson & Forbus, 2009; Renkl, 2014; Rittle-Johnson & Star 2009; Silver, Ghousseini, Gosen, Charalambous & Strawhun, 2005; Ziegler & Stern, 2016). However, the effectiveness of learning relying heavily on worked examples relies on the learners' self-explanation activities (Atkinson, Renkl & Merrill, 2003; Hausmann & VanLehn, 2008; Renkl, 2002; Rittle-Johnson, Loehr & Durkin, 2017). There are two strategies that can help students to reflect on their learning: self-explanation [that is, individual] or reflection with peers [that is, collaborative reflection] (Mason, & Singh, 2010). This can occur when their learning is compared and contrasted to the learning of other learners (Collins, 1991; Liu, 2017).

Self-explanation is when a learner engages in thinking to generate explanations for themselves, in an attempt to make sense of new learning (Chi, de Leeuw, Chiu & LaVancher, 1994; Rittle-Johnson, 2006). Self-explanation supports integration and generalisation of knowledge which, in turn, improves performance later (Rittle-Johnson & Loehr (2017). Collaborative reflection can provide opportunities for students to exchange different perspectives when they have discussions with others, particularly when others see things in a different way, ask questions, or challenge their beliefs (Krutka, Bergman, Flores, Mason & Jack, 2014; Larsen, London & Emke, 2016).

Schwarz, Dreyfus and Hershkowits (2004) state that in collaborative reflection, students '...integrate and generalise accepted arguments. They recapitulate actions and draw lessons from their experiences' (p. 170). Therefore, reflective dialogue can facilitate learners to generalise practical examples into explicit knowledge (Liu, 2017). A learner needs multiple perspectives and feedback on their own understanding (Carless & Boud, 2018).

Reflection can work as a tool to transform knowledge within concrete examples into abstract principles. Kolb (1984) stressed that abstract concepts can be transformed from concrete experience by reflection by eliciting meaningful learning from examples. Several scholars assert that reflection on the process of tasks is thinking in their broader meanings (Agouridas & Race, 2007; Heyler, 2015; Winitzky, 1992). Moreover, the findings of several studies indicate that, in mathematics, reflective thinking supports meaningful learning (Inoue & Buczynski, 2011; McNaught, 2010). When students conduct reflective processes, they perceive the experience, frame it and gain meaning from it (Pagano & Roselle, 2009).

Agouridas and Race (2007) argued that reflection is a process of understanding and personalizing the contents, process, and the logics for what individuals have already learnt. Therefore, reflection in learning is essential for learners to revisit what they have already learnt in order to achieve in-depth and improved learning (Chang, 2019). Numerous studies show that reflection plays an important role in improving learning and constructing knowledge (Barnett, 2006; Tok, 2008; Chen, Wei, Wu, & Uden, 2009; Kızılkaya, 2009; Phan, 2009; White & Frederiksen, 1998). There are two terms associated with reflective thinking which are: reflection-on-action (reflection after actions), and reflection-in-action (reflection during actions) (Schön, 1983).

There are several types of forms used in practice reflection, including portfolios, journals, reports, and summaries (Helyer & Kay, 2015; Helyer, 2015). Journal writing, for example, is an effective tool for learners to explore, express, organise, and reflect ideas about mathematical processes and content (Freeman, Higgins & Horney, 2016; Inoue & Buczynski, 2011; Suhaimi, Shahrill, Tengah & Abbas, 2016). Reflation by writing demands conscious awareness which represents thought (Vygotsky, 1986). However, writing environments must be monitored by teachers to bring about improvement (Freeman, et al., 2016). In mathematics, when students share their logic with others through their writing, it has to be written with clarity and precision in order that its meaning is understandable to the reader. Therefore, the mathematical reasoning must be logical, and the communication must also be coherent and sound (Freeman, et al., 2016).

Writing naturally takes many forms, comprising language (recordings of verbal communication and written text), formulas, numbers, and mathematical symbols, and visual representations (diagrams, graphs, charts, flow charts, maps, pictures, and models) (Schleppegrell, 2010).

Multimodal writing is recommended, as it provides ways for students to understand abstract concepts through drawing, and other models. It also allows learners to communicate their ideas to others, containing their teachers and peers. Hence, writing can provide instructors with a window into their students' minds which makes their thinking more visible (Bagley & Gallenberger, 1992; Ball, 1994).

Reflective activities can be transformed using a group discussion strategy or reflective dialogue strategy into a social activity, as discussed above (Aldahmash, Alshmrani & Almufti, 2017; Kohen & Kramarski, 2012; Wille, 2017). The students can reflect on their thoughts by utilising the 'thinking aloud' strategy (Taggart & Wilson, 2005), which can be done by using a set of questions to guide their reflections. Furthermore, reflection can increase students' awareness of their thoughts and actions, as well as increase their recall of prior knowledge (Larsen, London & Emke, 2016). Chang, (2019) believed that when students carried out reflections, retention was increased, as a result of repeatedly retrieving the information from memory. According to Gibbs (1988, p9), 'It is not sufficient to have an experience in order to learn. Without reflecting on this experience it may quickly be forgotten, or its learning potential lost'.

Studies concerning the use of collaborative reflection to learn from worked examples are minimal; hence, this is the focus of the present study. Students were asked to study multiple worked examples provided and explained by their teacher. They were then given opportunities to reflect collaboratively on their learning, and on how the examples compare to each other. Lastly, students will be provided with problems to solve individually. The aim of this study is to assess the effectiveness of worked examples - collaborative reflection - problem solving instruction on students' mathematics achievements, in comparison to using worked examples - problem solving instruction. Students' mathematics achievements were assessed over the two different domains - knowing and applying. In mathematics, the knowing domain includes the concepts and facts of mathematics, while the applying domain includes the application of mathematical tools in various contexts, see (Mullis, Martin, Foy & Hooper 2016).

This study will attempt to answer the following questions:

(1) When compared with worked examples - problem solving instruction, how does worked examples - collaborative reflection - problem solving instruction affect mathematics students' learning in the 'knowing' domain?

(2) When compared with worked examples - problem solving instruction, how does worked examples - collaborative reflection - problem solving instruction affect mathematics students' learning in the 'applying' domain?

Method:

Participants and design:

The participants were 84 eighth-grade students from four classes of the same Saudi intermediate school (age M =14.1 years; SD = 0.41). A quasi-experimental design was applied whereby two classes (n = 43) were randomly assigned as the 'experimental group', and two classes (n= 41) were selected as the 'control group'. Both groups were taught by the same mathematics teacher. This study was implemented over the period of 9 to 30 September 2018.

Materials:

Topics:

'Rational numbers' was selected as the topic of study. The content of the topic, which was new to the students, included: comparing and ordering rational numbers; multiplying and dividing rational numbers; and adding and subtracting rational numbers. In the experimental group, the students were presented with two worked examples in each classroom lesson, followed by collaborative reflection. They were then required to solve various problems (ranging from 2 to 6 problems). The worked examples and problems were similar to the worked example in structure, but had different context (see examples in Figure 1). The control group students were treated exactly the same as the students in the experimental group, except that no collaborative refection was conducted for the control group. All students received the same content (that is, the same set examples and problems). The instructions lasted for almost two weeks. They were carried out during 7 x 45-minute classroom sessions, with four sessions per week totaling 5.25 hours for each group.

Collaborative reflection

The reflective questions were prepared by the author and based on a literature review. The questions were designed for the students to obtain knowledge from worked examples and store this knowledge into their long-term memory. This would require reflection on what they had just learned, comparison between the worked examples they were exposed to, and the ability to implement the knowledge obtained into similar problems in the future. The students had to document their reflective thought by written text and visual representations, such as diagrams, graphs, charts, maps, and pictures), which the teachers could then view and provide feedback. Before implementation, the reflective questions were sent to a team of five experts for checking. The experts gave their opinions on the adequacy, clarity and relevance of the questions. The feedback received from the experts was considered and included in the preparation of the final version of the questions. (See Reflective Questions in Appendix 1).

Mathematics Test:

The mathematics test contained 16 questions: 8 questions for ‘the knowing domain’ domain comprising 6 multiple-choice questions and 2 short answer questions, and 8 questions for ‘the applying’ domain comprising 5 multiple-choice questions and 3 short answer questions. All 16 questions were posed before the experiment was carried out (pre-test), and after the intervention (post-test). The mathematics test items were adopted from released items TIMSS 2015 (published on the following website: https://timssandpirls.bc.edu/timss2015/international-database/assessment-items.html), which has been verified for reliability and credibility. After the tests had been prepared they were presented to 4 arbitrators for checking. The arbitrators gave their opinions on the adequacy, clarity and relevance of the content. The evaluations of the arbitrators were considered and included in the preparation of the final version of the tests. The reliability by retests was .91; the internal consistency for the sub-scale of the tests was .89**.** For the pre and post-test marks, each item scored either one or zero. See examples of the items in Figure 2.

Training for students and teachers :

The teachers were trained by the author to implement the strategy, prior to its introduction. The training course was for two days, and for two hours each day. He then implemented both strategies for other classes in his school under the supervision of the author. The teacher demonstrated good performance before he implemented the strategies for the main study. He was able to explain examples, as well as manage collaborative reflection and coach problem solving. In both groups, the teacher trained the students for one session. He explained how they would be taught over the following two weeks. The students in the experimental group showed good interaction with others during collaborative reflection.

Procedures:

All the participants gave their consent and agreement to take part in the study. They were informed that they could withdraw at any time, without having to provide any reason for their decision to leave the study. The author and the teacher designed the content of the topics together. The content was designed for both the 'worked examples - collaborative reflection - problem solving' and the 'worked examples - problem solving' instructions. The validity of the content was verified by three experts in the field. The teacher was trained to implement the strategy prior to its introduction. Experimental and control groups were randomly selected. Pre-tests were conducted for both groups before the instruction took place. In the experimental group, the students were trained for one session by the teacher. It explained by the teacher how they would be taught over the following two weeks.

In the experimental group, the worked examples - collaborative reflection - problem solving instruction strategy was implemented for almost two weeks. The strategy started with two worked examples being explained in detail by the teachers. The worked examples were similar in structure but with different contexts. This was followed by collaborative reflection. Once the students had reflected on the worked examples, they were presented with similar problems. It was not expected that all students would be able to solve all the problems, due to their individual differences.

The students in the control group were taught via worked examples - problem solving instruction. After two weeks, post-tests were carried out for both groups. The data collected was analysed.

Results:

'Knowing' domain :

The t-test results indicated no significant differences in the pre-test scores for both the experimental and control groups ($t = -1.75$).

Table 1 shows that there was a significant difference in the post-test $F(1, 82) = 6.27$, $p < .05$, with a moderate effect size. Students who were in the experimental group had higher post-test scores ($M = 2.93$, $SD = 2.22$) than students in the control group ($M = 1.63$, $SD = 1.53$).

Table 1. Summary of ANCOVA test results in the knowing domain

Variable	Group	N	Mean	SD	Adjusted mean	F	η²
Post-test	Experimental	43	2.93	2.22	2.74	6.27*	.072
	Control	41	1.63	1.53	1.83		

Note: *p<.05

'Applying' domain:

The t-test results showed no significant difference in pre-test scores for the experimental and control groups ($t = -1.89$). From Table 2, the one-way ANCOVA results showed that there was a significant difference on the post-test $F(1, 82) = 11.03.$, $p < .01$, with a moderate effect size.

Students who were in the experimental group had higher post-test scores ($M = 2.53$, $SD = 2.53$) than students in the control group ($M = .93$, $SD = 1.15$).

Table 2. Summary of ANCOVA test results of in the applying domain test

Variable	Group	*N*	Mean	SD	Adjusted mean	*F*	*η²*
Post-test	Experimental	43	2.53	2.53	2.26	11.03**	.120
	Control	41	.93	1.15	1.21		

Note: **p<.01

Discussion:

The results revealed that the students' average scores in the experimental group were significantly higher than the students' average scores in the control group, with medium effect size in both the knowing and 'applying' domain post-tests.

In the experimental group, the students were presented with multiple worked examples. The examples had different contexts (different problem stories). Theoretically, the students should have sufficient working memory capacity which can be devoted to formatting a schema for later problem solving (Paas & van Gog, 2006; Sweller, van Merriënboer & Paas, 1998). The students were then divided into groups to collaboratively reflect on what they had already learned from the worked examples. They had to make comparisons between the examples and get ready to be presented with similar problems that would have to be solved individually.In this stage (collaborative reflection), students construct the schema or gain obstructing principles by reflection. To construct the schema, research asserts that construction and abstraction are promoted when learners are provided with various examples, particularly when they are simultaneously taught to make comparisons between these examples (Gentner, Loewenstein & Thompson, 2003; Holyoak, 2005; Rittle-Johnson and Star, 2009; Silver et al. 2005). Reflection can work as a tool to transform knowledge within concrete examples into abstracting principles. Kolb's (1984) stressed that abstract concepts can be transformed from concrete experience by reflection.

During reflection processes, the students frequently remember information from their memory which may explain the improvement that they experienced in the 'knowing' domain test when compared to students in the control group. Reflection can increase students' awareness of their thoughts and actions and increase their recall of prior knowledge (Larsen, London & Emke, 2016). It is believed that by conducting reflections, students may increase their retention of the experience as a result of repeatedly retrieving the information from memory (see, Chang, 2019). Gibbs stated that 'It is not sufficient to have an experience in order to learn. Without reflecting on the experience, it may be forgotten quickly or the learning potential is lost' (Gibbs, 1988, p.9). Therefore, reflecting seems to help students in constructing a schema and increasing retention.

In the last stage (problem solving), the students encounter similar problems to be individually solved which, in theory, they should be ready for. They should use their new knowledge (which should be already constructing) to apply in new and similar situations. In other words, in this stage, the students should encounter problems with sufficient working memory because they have already obtained the necessary knowledge.

They should only need to focus on the knowledge required for the particular problem. This is in contrast to learning which relies on problem solving only, when the majority of the working memory's resources are dedicated to solving the problem, rather than obtaining new knowledge about the domain (Kirschner, Sweller & Clark, 2006; Sweller et al., 2011). Therefore, solving problems could be practical steps followed collaborative refection to improve problem solving skills ('applying' domain skills). This may explain why the students outperformed other students in the control group in the 'applying' domain test.

Limitations:

The limitations of this study are that the results of this study can be generated only to similar contexts. This study is limited to male students due to a gender segregation system that is operational in Saudi Arabia.

Recommendations and Conclusion:

A quasi-experimental study investigated the effectiveness of worked examples -collaborative reflection - problem solving instruction, on mathematical students' achievements in the 'knowing' and 'applying' domains in comparison to using worked examples - problem solving instruction. The findings found that integrating collaborative reflection to worked examples instruction improves students' learning. This study recommends that in-service and pre-service intermediate school mathematics teachers should receive training in how implement worked examples -collaborative reflection - problem solving instruction. More research is needed to assess this at different academic levels, and in different subjects. This study also paves the way for further studying the effect of this strategy on other different domains such as reasoning, and different levels of applying, such as far and near transfer. Additionally, students' attitudes to this intervention could be studied and the effect of this intervention for high-and low-achieving students could be examined. Furthermore, it can also be studied with multiple worked examples with high-variability.

References:

Agouridas, V. & Race, P. (2007). Enhancing knowledge management in design education through systematic reflection in practice. Concurrent Engineering, 15 (1), 63-76.

Aldahmash, A. H., Alshmrani, S. M., &Almufti, A. N. (2017). Secondary school science teachers' views about their reflective practices. Journal of Teacher Education for Sustainability, 19(1), 43-53.

Atkinson, R. K., Renkl, A. & Merrill, M. M. (2003). Transitioning from studying examples to solving problems: Combining fading with prompting fosters learning. Journal of Educational Psychology, 95, 774–783.

Bagley, T. & Gallenberger, C. (1992). Assessing students' dispositions: Using journal to improve students' performance. Mathematics Teacher, 85, 660-663.

Ball, D. (1994). Developing mathematics reform: What don't we know about teacher learning-but would make good working hypothesis? Paper presented at the Conference on Teacher Enhancement K-6, Arlington, VA.

Barnert, M. (2006). Effects of reflection prompts when learning with hypermedia. Journal of Educational Computing Research, 35(4), 359e375.

Chang, B. (2019). Reflection in Learning. Online Learning, 23(1): 95-110.

Chen, N. S., Wei, C. W., Wu, K. T., & Uden, L. (2009). Effects of high level prompts and peer assessment on online learners' reflection levels. *Computers & Education*, *52*(2), 283-291.

Chi, M. T. H., de Leeuw, N., Chiu, M.-H., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. Cognitive Science, 18(3), 439–477. Doi: 10.1207/s15516709cog1803_3

Collins, A. (1991). Cognitive apprenticeship and instructional technology. Educational values and cognitive instruction: Implications for reform, 1991, 121-138.

Dunlosky, J., Rawson, K. A., Marsh, E. J., Nathan, M. J., & Willingham, D. T. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest*, *14*(1), 4-58.

Freeman, B., Higgins, K. N., & Horney, M. (2016). How students communicate mathematical ideas: An examination of multimodal writing using digital technologies. Contemporary Educational Technology, 7(4), 281-313.

Gentner, D., Loewenstein, J. & Thompson, L. (2003). Learning and transfer: A general role for analogical encoding. Journal of Educational Psychology, 95, 393¡405.

Gentner, D., Loewenstein, J., Thompson, L., & Forbus, K. D. (2009). Reviving inert knowledge: Analogical abstraction supports relational retrieval of past events. Cognitive Science, 33, 1343–1382

Gibbs, G (1988). Learning by doing: a guide to teaching and learning methods. Oxford: Further Education Unit, Oxford Polytechnic.

Hausmann, R. G. M., Van de Sande, B. & Van Lehn, K. (2008). Are self-explaining and coached problem solving more effective when done by pairs of students than alone? In B. C. Love, K. McRae & V. M. Sloutsky (Eds.), Proceedings of the 30th annual conference of the cognitive science society (pp. 2369–2374) Austin, TX: Cognitive Science Society.

Helyer, R. (2015). Learning through reflection: The critical role of reflection in work-based learning (WBL). Journal of Work-Applied Management, 7(1), 15-27. Retrieved from https://doi.org/10.1108/JWAM-10-2015-003.

Helyer, R., & Kay, J. (2015). Building capabilities for your future. In Helyer, R. (Ed.), The work based learning student handbook (2nd ed., pp. 31-50). Palgrave.

Inoue, N., &Buczynski, S. (2011). You asked open-ended questions, now what? Understanding the nature of stumbling blocks in teaching inquiry lessons. The Mathematics Educator, 20(2), 10–23.

Kızılkaya, G. (2009). The effect of web-based learning environments supported with reflective thinking activities to problem solving. Unpublished Doctoral Dissertation. Ankara: Hacettepe University.

Kohen, Z., &Kramarski, B. (2012). Developing self-regulation by using reflective support in a video-digital microteaching environment. Journal for Education Research International, 3, 1-35.

Kolb, D. A. (1984). Experiential learning: Experience as the source of learning and development (Vol. 1). Englewood Cliffs, NJ: Prentice-Hall.

Krutka, D. G., Bergman, D. J., Flores, R., Mason, K., & Jack, A. R. (2014). Micro blogging about teaching: Nurturing participatory cultures through collaborative online reflection with pre-service teachers. Teaching and Teacher Education, 40, 83-93.

Larsen, D. P., London, D. A., & Emke, A. R. (2016). Using reflection to influence practice: Student perceptions of daily reflection in clinical education. Perspectives on Medical Education, 5(5), 285-291. Doi: 10.1007/s40037-016-0293-1

Liu, K. 2017. "Creating a Dialogic Space for Prospective Teacher Critical Reflection and Transformative Learning." Reflective Practice. doi:10.1080/14623943.2017.1361919.

Mason, A., & Singh, C. (2010). Helping students learn effective problem solving strategies by reflecting with peers. American journal of physics, 78(7), 748-754.

McNaught, K. (2010). Reflective writing in mathematics education programmes. Reflective Practice, 11(3), 369–379.

Nievelstein, F., Van Gog, T., Van Dijck, G., & Boshuizen, H. P. (2013). The worked example and expertise reversal effect in less structured tasks: Learning to reason about legal cases. Contemporary Educational Psychology, 38(2), 118-125.

Paas, F., & Van Gog, T. (2006). Optimising worked example instruction: Different ways to increase germane cognitive load.

Pagano, M., & Roselle, L. (2009). Beyond reflection: Refraction and international experiential education. *Frontiers: The Interdisciplinary Journal of Study Abroad*, *18*(1), 217-229.

Phan, H. P. (2009). Exploring students' reflective thinking practice, deep processing strategies, effort, and achievement goal orientations. Educational Psychology, 29(3), 297-313.

Renkl, A. (2002). Worked-out examples: Instructional explanations support learning by self-explanations. Learning and instruction, 12(5), 529-556.

Renkl, A. (2014). Toward an instructionally oriented theory of example-based learning. Cognitive science, 38(1), 1-37.

Renkl, A. (2017). Learning from worked-examples in mathematics: students relate procedures to principles. ZDM, 49(4), 571-584.

Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. Child Development, 77(1), 1–15. Doi: 10.1111/ j.1467-8624.2006.00852.x

Rittle-Johnson, B., &Loehr, A. M. (2017). Eliciting explanations: Constraints on when self-explanation aids learning. Psychonomic bulletin & review, 24(5), 1501-1510.

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples: Influences on conceptual and procedural knowledge of equation solving. Journal of Educational Psychology, 101(4), 836–852.

Rittle-Johnson, B., Loehr, A. M. & Durkin, K. (2017). Promoting self-explanation to improve mathematics learning: A meta-analysis and instructional design principles. ZDM Mathematics Education. Doi: 10.1007/s11858-017-0834-z.

Rourke, A. &Sweller, J. (2009). The worked-example effect using ill-defined problems: learning to recognize designers' styles. Learning and Instruction, 19, 185–199.

Schleppegrell, M. J. (2010). Language in mathematics teaching and learning: A research review. In J. Moschkovich (Ed.), Language and mathematics education: Multiple perspectives and directions for research (pp. 73-112). Charlotte, NC: Information Age Publishing, Inc.

Schön, D (1983). The reflective practitioner: how professionals think in action. New York: Basic Books.

Schwarz, B., Dreyfus, T., &Hershkowits, N. H. R. (2004). Teacher guidance of knowledge construction. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 4, 169-176. Retrieved from http://www.kurims.kyotou.ac.jp/EMIS/proceedings/PME28/RR/RR175_Schwarz.pdf

Schwonke, R., Renkl, A., Krieg, K., Wittwer, J., Aleven, V. &Salden R. (2009). The worked-example effect: Not an artefact of lousy control conditions. Computers in Human Behaviour, 25, 258–266.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. The Journal of Mathematical Behaviour, 24, 287–301.

Simamora, R. E., Sidabutar, D. R., & Surya, E. (2017). Improving learning activity and students' problem solving skill through problem based learning (PBL) in junior high school. International Journal of Sciences: Basic and Applied Research (IJSBAR), 33(2), 321-331.

Suhaimi, Z., Shahrill, M., Tengah, K. A., & Abbas, N. A. H. (2016). Incorporating the use of writing-to-learn strategy in grade 10 mathematics lessons: The students' perspectives. Journal of Mathematics Education at Teachers College, 7(2), 11-20.

Sweller, J. & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. Cognition and Instruction, 2, 59–89.

Sweller, J., Ayres, P. & Kalyug, A., S. (2011). Cognitive load theory. New York: Springer.

Taggart, G. L., & Wilson, A. P. (2005). Promoting reflective thinking in teachers: 50 action strategies. California: Corwin Press.

Tok, S. (2008). Fen bilgisi dersinde yansıtıcı dusunme etkinliklerinin ogrencilerin akademik basarılarına ve fen bilgisi dersine yonelik tutumlarına etkisi. Elementary Education Online, 7(3), 557-568

Vygotsky, L. (1981). The instrumental method in psychology. In J. V. Wertsch (Ed.), The concept of activity in Soviet psychology (pp. 134-143). Armonk, NY: M. E. Sharpe

White, B., &Frederiksen, J. R. (1998). Inquiry, modelling, and meta-cognition: making science accessible to all students. Cognition and Instruction, 16(1), 3e117.

Wille, A. M. (2017). Imaginary dialogues in mathematics education. Journal fürMathematik-Didaktik, 38(1), 29-55.

Winitzky, N. (1992). Structure and process in thinking about classroom management: An exploratory study of prospective teachers. Teaching and Teacher Education, 8(1), 1-14. Doi: 10.1016/0742-051X (92)90036-3

Ziegler, E., & Stern, E. (2016). Consistent advantages of contrasted comparisons: Algebra learning under direct instruction. Learning and Instruction, 41, 41-51.

Appendix:

Appendix 1. Reflective questions for each groups

1. What have you learned from the couple of worked examples that explained by the teacher?, please summarise.
2. Are the solutions' steps of the worked-examples fully understood? What are unclear steps?
3. What are the similarities and differences between the worked-examples?
4. How would you implement knowledge that you have just learned in similar problems?

Figures

Example of worked examples
(The following two worked examples were explained step-by-step to students by the teacher)

Worked example 1:

The ratio of 1st grade middle school students who participated in the school festival was 5/6;, the ratio of 2nd grade middle school students who participated in the festival was 3/4;,and the ratio of 3rd grade middle school students who participated in the festival was 4/5. Which class had the highest participation rate?

Worked example 2:

Ali has a bunch of nut wrenches; in inches their measurements are: 3/8, 1/4, 5/16, 1/2, 3/4. Can you arrange the wrenches by their measurements, from the largest to the smallest?

An example of the worked examples and collaborative reflection problems which students were required to solve

The United States produces 9/50 of the world's energy and consumes 6/25 of the world's energy. Which is greater: production or consumption? Please explain your answer.

Figure 1. Examples of worked examples and problems

Example of mathematics questions

'Knowing' domain:
Which number is equal to 3/5?

a) 0.8
b) 0.6
c) 0.53
d) 0.35

'Applying' domain:
Which shows the correct method for finding 1/3-1/4?

a) 1-1/4-3
b) 1/4-3
c) 3-4/3-4
d) 4-3/3×4

Figure 2: Examples of mathematics items

Appendix:

Appendix 1. Reflective questions for each groups:

5. What have you learned from the couple of worked examples that explained by the teacher?, please summarise.
6. Are the solutions' steps of the worked-examples fully understood? What are unclear steps?
7. What are the similarities and differences between the worked-examples?
8. How would you implement knowledge that you have just learned in similar problems?

Figures :

Example of worked examples
(The following two worked examples were explained step-by-step to students by the teacher)

Worked example 1:

The ratio of 1st grade middle school students who participated in the school festival was 5/6;, the ratio of 2nd grade middle school students who participated in the festival was 3/4;,and the ratio of 3rd grade middle school students who participated in the festival was 4/5. Which class had the highest participation rate?

Worked example 2:

Ali has a bunch of nut wrenches; in inches their measurements are: 3/8, 1/4, 5/16, 1/2, 3/4. Can you arrange the wrenches by their measurements, from the largest to the smallest?

An example of the worked examples and collaborative reflection problems which students were required to solve

The United States produces 9/50 of the world's energy and consumes 6/25 of the world's energy. Which is greater: production or consumption? Please explain your answer.

Figure 1. Examples of worked examples and problems

Example of mathematics questions

'Knowing' domain:

Which number is equal to 3/5?

e) 0.8
f) 0.6
g) 0.53
h) 0.35

'Applying' domain:

Which shows the correct method for finding 1/3-1/4?

e) 1-1/4-3
f) 1/4-3
g) 3-4/3-4
h) 4-3/3×4

Figure 2: Examples of mathematics items