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Time-Cost Optimization Model with Probabilistic Path and Float Consumption Impact

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ABSTRACT

In construction projects, achieving their main objectives is an urgent goal, Getting the optimum duration point which is associated with the minimum total cost with the lowest risk of delays. As the results of the calculations of critical-path-method are denoted by “floats” (De La Garza). This paper acknowledges the fact that “time is money” as time consumption is costly and time savings can supply interests to all parties of the project. Time-cost optimization technique result in reducing the available total float for noncritical activities, thus decrease the schedule flexible and increase the network criticality. Float consumption impact in noncritical activities is one of the complicated delays to assess on a project’s duration and cost. As a shortage of deterministic critical path method cannot sense the impact of that consumption unless they overpass the values of total float. This paper examines the relevance of a nonlinear-integer programming model with the impact of total float consumption, and a probabilistic model with control the risks of float consumption using Monte Carlo simulation.

Keywords: Time–Cost Trade-off, Float Consumption, Deterministic Path, Probabilistic Path, Monte Carlo Simulation.

INTRODUCTION

In construction projects, achieving their main objectives is an urgent goal, time is one of the main objectives of it. So that scheduling stage is very important in time management. Critical path method (CPM) has been used widely as a scheduling technique in construction projects. CPM was applied since 1950s to schedule and monitor the projects. In the 1980s has been a steady increase in using that technique. Critical and noncritical activities and paths are the outputs of its calculations. All parties involved in the project seek to adapt with that paths and its durations to protect their purposes. As time is a critical element in the construction process, the contractors, and owners incur additional costs in case of delays of contract dates (Schumacher 1996; Householder and Rutland 1990) and additional costs of schedule accelerations.
As the project accelerates, the shorter the duration of the project, the lower the indirect cost, but the direct cost increases as a result of that acceleration. As a result, critical activities and the risk of delay also increased, and project flexibility are reduced. So, a need to study the optimization of time and cost taking into account the increased risk.
As the results of the calculations of critical-path-method are denoted by “floats” (De La Garza 1991). Contractors or owners may misuse these floats. Therefore, the project lacks flexibility and This may affect the entire project. This has led to the difficulty of using the CPM as a tool of monitoring progress. Consequently, it was suggested that the probabilistic path for project scheduling taking into consideration the probability of all activities for implementation or delay.
Time is money. In addition, float is also money. It must be recognized here that it must have a financial value to deal with it as a commodity that can be traded between the parties (De La Garza 1991) and as an effective factor in the calculation of cost during the optimization.

So, Rana A. Al Haj and Sameh M. El-Sayegh (2014) presented a model for optimization between time and cost, taking into account the impact of total float consumption of noncritical activities.

LITERATURE REVIEW

Deterministic Path (CPM)

The "scheduling" science as defined in the Critical Path Method (CPM) celebrated its 60th anniversary in 2017. In 1956/1957, Morgan R. Walker and James E. Kelley began to develop algorithms that became the Activity-on-Arrow (AOA) or Arrow Diagramming Method (ADM) methodology. From late 1956 to April 1957, Morgan R. Walker helped James E. Kelley et al. to Identify the scope of a viable project. Their challenge was to solve the time-cost trade-off. They were trying to prove that by intensifying activities by labor or excessive effort to restore lost time, concentrating efforts on "right" activities could lead to less time. The problem was to define the "right" activities! Their first paper was published on Critical Path Scheduling in March (1959). Evolution of CPM scheduling Closely Developed by computer evolution in the 1970s and 1980s. Kelly attributed the term "critical path" to program developers and review techniques Which was developed almost simultaneously by Booz Allen Hamilton and US Navy (1957).
Estimations in deterministic scheduling require experience and involvement of previous data. The chances of effectively finishing the activity, according to the schedule, extraordinarily rely upon the estimations that are deterministic in nature. The number of basic activities might be less or equal to all project activities. The deterministic Scheduling is of a certain nature. This type of scheduling is used where one knows exactly what will happen. This method is useful when projects are similar or repetitive. The project manager will have to see the end of the projects. The project manager experience enables him to conduct a reliable risk assessment. During scheduling, consider all these factors, giving him confidence about the project plan.

Probabilistic Path

The weaknesses of the CPM motivate many researchers to do many researches that can adapt to the new needs of scheduling techniques. Ang et al. (1975) presented the probabilistic network evaluation technique to assess the uncertainties of activities. Diaz and Hadipriono (1993) investigated the hypothetical foundation of some scheduling techniques. Cox (1995) introduced an approximate simplified mathematical model for normal distribution of achievement time of combining activities. Senior and Halpin (1998) introduced another scheduling technique called project integrated cyclic analysis of serial systems operations. Alarcon and Ashley (1996) displayed an execution control demonstrate dependent on a calculated, qualitative mode structure and a mathematical model structure that utilizes ideas of cross-impact analysis and probabilistic inference to deal with the uncertainties. Bubshait and Cunningham (1998) examined three delay measurement techniques. Diekmann and Featherman (1998) researched the capacity of undertaking project personnel to foresee the size of cost development. Wang and Demsetz (2000a, b) displayed the simulation-based model NETCOR (networks under correlated uncertainty), which joins the impact of connection in schedules and gives factors-sensitivity information to help schedule risk management. Barraza et al. (2000) built up a Stochastic S curve (SS curve) to monitor and control project performance in substitution of the deterministic S curve. Simulation techniques were utilized in building up the SS curves.
Isidore and Back (2001) extended the optimum-cost schedule concept to be utilized with probabilistic scheduling techniques. Costing simulation activity was utilized by Isidore and Back in their research. Isidore and Back (2002) introduced an integrated cost and schedule technique based on simulation to acquire true project costs and durations. Isidore and Back utilized multiple simulation analysis technique to evaluate the relationship that exists between probabilistic scheduling and cost range assessing. Barraza et al. (2004) extended their earlier SS-curve system to be utilized in estimating project performance at any predefined period in the life cycle of a project. Gong and Hugsted (1993) built up a merge-event time-estimation technique to join the uncertainties of both critical and noncritical paths into the time-risk analysis of a project network. They utilized a back-forward uncertainty-estimation technique in their study.
The only difference between Deterministic Scheduling and Probabilistic Scheduling lies in the estimation of duration, and the statistical technique used to develop the schedule. Estimating the three points in the Probabilistic scheduling - the best case, the worst case and most likely.

Statistical techniques such as Program evaluation and review technique PERT, Monte Carlo simulation MCS, Graphical Evaluation and Review Technique GERT, and Gantt chart are developed as Probabilistic Scheduling techniques.
PERT analysis has the advantage of predicting the probability of project completion on time, or conversely, and what would be the project completion time, with a certain probability. Monte Carlo simulation MCS is a comprehensive set of computer algorithms that are determined by repeated random sampling to achieve numerical results. This method focuses on scheduling a realistic schedule by taking into account risk factors that may have a positive or negative impact on the project. Try to capture the risks and uncertainties associated with project activities and the project as a whole. The schedule is created with some floats to deal with risks and uncertainties.
In probabilistic scheduling, risks are stochastic procedures having probabilistic results. The project duration is not a fixed value, but a certain value determined from the probability distribution with a certain associated confidence level. This kind of scheduling is utilized whereas more the project uncertainties. While scheduling, the project manager needs to consider about different factors, which are doubtful in themselves. Probabilistic scheduling gives a realistic perspective on the project plan, helping project managers foresee the uncertainties and its impact on the schedule.

Time–Cost Optimization

By improvement modeling, the utilization of mathematical techniques in finding issues supported sure characteristics, together with the aim of integrated business coming up with: (Linear programming (LP), Mixed integer programming (MIP), Nonlinear programming (NLP), and Constraint programming (CP)). Over the last four centuries, several mathematicians made important contributions to the mathematical sciences, who soon began to define many famous problems e.g., Monte Carlo simulations. Implementing mathematical optimization on construction project issues includes a few troubles. In such cases the importance of finding a suitable (good) solution is more than enough getting the optimal (ideal) one using some heuristics (Taha 1997). So, recently the researchers have developed many techniques taking various objective functions especially in the last quarter of the twentieth century.
The time–cost trade-off technique has been researched since the 1960s with the purpose of getting up techniques that can improve the activity cost and duration, and the best interaction between the least cost and a certain deadline. Even so it may be important to complete the project in a particular time to: (finish the task in a specific date, recover early delays, avoid liquidated damages, free key resources ahead of schedule for different activities, avoid harmful climate conditions that may influence productivity, receive an early completion reward, and improve project cash flow). So, to shorten the project period planners perform time-cost trade-off analysis. This can be done by choosing some activities on the critical path to shorten their duration. As the direct price for the project equals the total of the direct costs of its activities, then the project direct cost can increase by decreasing its duration. On the opposite hand, the indirect cost can decrease by decreasing the project duration, because the indirect cost is virtually a linear perform with the project duration. The project total time-cost relationship is often determined by adding up the direct cost and indirect cost values along. The optimum project duration is often determined because the project duration that ends up in the smallest amount project total cost.
Researchers categorized Time–cost trade-off techniques into the following three categories: manual, mathematical (optimization), and metaheuristic methods. The mathematical techniques include: (linear (Kelly 1961; Hendrickson and Au 1989; Pagnoni 1990), integer (Meyer and Shaffer 1963; Patterson and Huber 1974; Ammar 1992; and Burns et al. 1996), and Dynamic programming (Robinson 1975; Elmaghraby 1993; De et al. 1995) are used to solve the optimization problem (Laptali et al. 1997). The heuristic methods include (Prager (1963), Siemens (1971), Moselhi (1993), and Elbeltagi (2005)).
Heuristic approaches are used to solve the time/cost tradeoff problem such as the cost-lope method used in this paper. In explicit, an easy approach is to initial apply critical path scheduling with all activity durations assumed to be at minimum value. Next, the planner will examine activities on the critical path and reduce the scheduled duration of activities that have the lowest resulting increase in costs. In essence, a list of activities is developed by the planner on the critical path ranked with their cost slopes. The heuristic answer takings by shortening activities within the order of their lowest cost slopes. As the duration of activities on the shortest path are shortened, the project duration is additionally reduced. Eventually, another path becomes critical, and a new list of activities on the critical path must be prepared. Using this fashion, sensible however not

essentially optimum schedules are often known. Time–cost trade-off techniques consume the total floats of non-critical activities, and hence, decrease the schedule flexibility and increase the probabilities of delays. Most of previously time–cost trade-off techniques ignore the impact of that float consumption. That technique provides a modified logical approach for decision making to account for risks associated with total float consumption in non-critical activities and improves the reliability and effectiveness of the crashing decision. So that, there is a need in using other model for time–cost trade off that taking the impact of float consumptions.

Float consumption

The planning stage is a process where it is difficult to ignore the certainties, or the uncertainties (as it has the same weight). If the uncertainties and risks are ignored, there may be a possibility of having an emergency cost to modify the deviation. Also, it causes in new critical paths. Briefly, as the float is considered as an emergency time, its traded value has to be determined and therefore the need to use a pricing model Such as any source of the project (De La Garza 1991). During float pricing, the scheduler should consider all the factors affecting its value, as, some floats is more valuable than others, because the total float is shared by all the activities on the same path, early float will be more expensive because of the side effects of its consumption on the following activities.

This high cost of early float can be justified on the basis of the level of confidence during the project. Furthermore, the delay of any activity makes the field management to abandon the opportunities to achieve early dates to end the project on track and thus, make some concessions in the quality required and the quantities of resources or solutions that lead to increased cost. All factors should be taken into account, including (but not limited to) the amount of flexibility consumed as a result of total float consumption, the result of float consumption, the loss of early finish bonuses of contractors (if any), the possibility of liquidated damages (if any), type of activity, acceleration costs in resources, resource constraints and inefficiency before float consumption.

Householder, and Rutland (1990) raised the issue of float ownership. Garza et al. (1991) recommended dealing with float as a commodity. Gong and James (1995) introduced a safe float consumption range for the activity to take the impact of float consumption and uncertainties of non-critical activities into account. Zhong and Zhang (2003) exhibited another strategy to calculate the path float in the PERT to adapt to the uncertainties and decrease confusing information. Isidore and Back (2002) used and extended a multiple simulation analysis technique (MSAT), to create reliable project costs and durations at various estimations of values of float consumptions in concerned activities. Monte Carlo simulation is utilized to produce these sets of durations and costs.

Method

This paper examines the relevance of a nonlinear-integer programming model with the impact of total float consumption, and a probabilistic model with control the risks of float consumption using Monte Carlo simulation. That proposed method enables the schedulers to quantify the impact of float consumption on project duration and cost. The suggested analysis method consists of five main stages. These stages are:

1. Deterministic critical path method (p50%); **2.** Probabilistic path (p80%); **3.** Getting the objective function; **4.** Identifying the problem variables; **5.** Identifying the problem constraints; and **6.** Running the optimization model and Getting the results.

For the purpose of demonstrating this approach, an example is implemented here its data are listed in Table 1. In this example, variability is assumed in schedule to be normally distributed. However, in real life projects, using the historical data the scheduler can round it into the nearest probability function.

In this example, the total project cost includes the normal direct cost, the indirect cost, the extra direct cost due to crashing critical activities, and the extra cost resulting from float loss in noncritical activities. The indirect cost is estimated to be $90/day.

Table 1. Activities, Durations, and Costs of Example Project

Act.	Predec.	Opt. Dur. (a) days	N. Dur. (m) days	Pess. Dur. (b) days	τ crashing Dur. (Days)	NC ($)	CC ($)	EFC	LFC
A	---	8	10	12	1	160	200	---	---
B	---	6	7	9	2	400	430	400	860
C	---	3	3	4	2	350	350	350	986
D	A	10	20	30	5	1020	1100	---	---
E	C	6	7	8	3	610	700	610	1040
F	B, D, E	9	10	11	2	1250	1300	---	---
G	B, D, E	6	7	10	2	525	660	525	855
H	F	14	15	16	3	825	895	---	---
I	F	10	11	13	3	390	410	390	470
J	G, H	6	7	8	2	430	490	---	---
K	I, J	4	7	8	2	250	350	---	---
L	G, H	1	2	4	2	615	765	615	855

1. Deterministic critical path method

Fig. 1 shows the network diagram for the example project. The model, set up in Microsoft Excel, based on normal duration for all project activities, the deterministic CPM showed that path **ADFHJK** is the only critical path in the project with a total duration of **68.67** days and a total cost of $ **6,825**. The calculations of the durations, the total floats, the crashing unit cost, the float consumption unit cost for all the activities in the project are listed in Table 2, where according to PERT technique the expected date of project completion (T_e) is equal to the sum of normal expected times of the critical activities. Which its probability that the project will be finished before it is 50%, that means if te_1, te_2, ……., te_n are the expected durations of the critical activities (te), then

$$Te = \sum_{i=1}^{n} tei \, , i = 1, 2, 3, \dots \dots, n$$

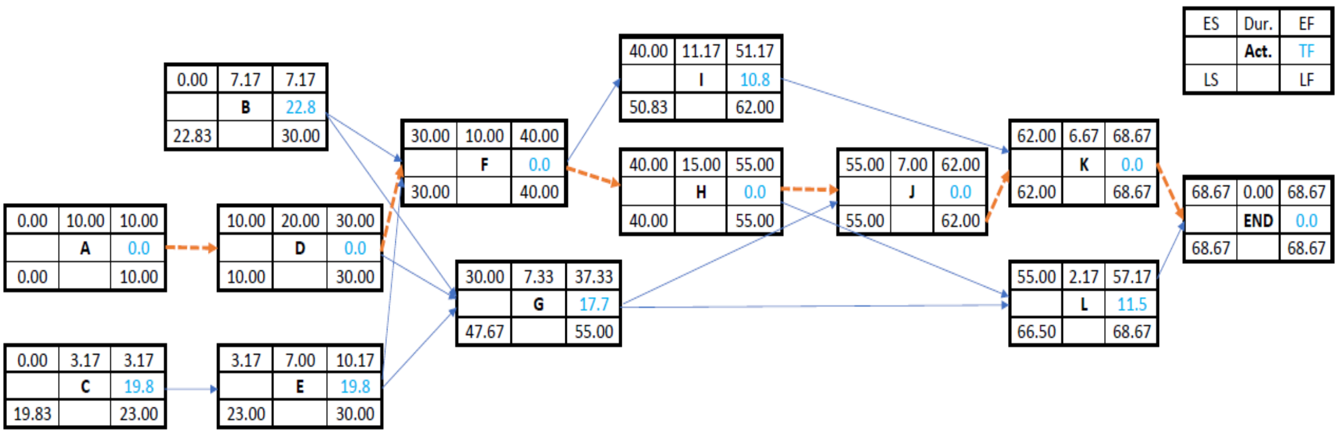


Fig. 1. Application Example Original Network Diagram

Table 2. Normal Expected Durations, Total Float, Crash unit cost, Float consumption unit cost of Activities

Act.	te (p50%)	σ	τ	NC	CC	Δc	EFC	LFC	Fo	δf
A	10	0.67	1	160	200	40	---	---	0	0
B	7.2	0.5	2	400	430	15	400	860	22.8	20.1
C	3.2	0.17	2	350	350	0	350	986	19.8	32.1
D	20	3.33	12	1020	1100	6.7	---	---	0	0
E	7	0.33	3	610	700	30	610	1040	19.8	21.8
F	10	0.33	2	1250	1300	25	---	---	0	0
G	7.3	0.67	2	525	660	67.5	525	855	17.7	18.7
H	15	0.33	8	825	895	8.75	---	---	0	0
I	11.2	0.5	3	390	410	6.7	390	470	10.8	32.3
J	7	0.33	2	430	490	30	---	---	0	0
K	6.2	0.67	2	250	350	50	---	---	0	0
L	2.2	0.5	2	615	765	75	615	855	11.5	20.9

$$te = \frac{a + 4m + b}{6} \quad , \quad \sigma = \frac{b - a}{6} \quad , and \quad v = \sigma^2$$

2. Probabilistic path (MCS)

In PERT technique to estimate the probability of a certain project meeting a specific schedule time can be described as follows:

$$Z = \frac{Td - Te}{\sigma\ proj.}$$

$$\sigma = \sqrt{v\ proj.}$$

$$v\ proj. = \sum_{i=1}^{n} vi$$

$$vi = (\frac{b - a}{6})^2$$

Here, Z is the number of standard deviations of the specific date (Td) from the mean or expected date, $\sigma_{proj.}$ is the project standard deviation, and $v_{proj.}$ project variance is the sum of critical activities variances.

Now consider the time to perform each of the activities along this path as independent random variables, the same assumption made during the process of collecting the a, m, and b activity time estimates. Furthermore, the sum of these random variables, which shall be denoted by Te, is itself a random variable which is governed by the Central Limit Theorem. and finally, the Central Limit Theorem enables one to assume that the shape of the distribution of Te is approximately normal. This probability can be read from the table of normal curve areas, this procedure was modified of the hypothetical activity performance time distributions, rather than the endpoints of the distributions. also, based on the Central Limit Theorem, estimates of the mean and variance in activity performance times were then used to compute a probability of meeting arbitrary scheduled times for special network events. It was recognized that it is difficult to obtain accurate estimates of the activity performance times, and procedures for improving the estimation by feedback of past estimation performance plotted on a standardized control chart was outlined. Hence, it is essential that the correct sign is placed on the z.
In this paper, an 80% confidence level (probability) was chosen for all activities to select new set of durations of the activities and then rescheduled again with these new durations and direct

costs. From the table of normal curve areas, the probability of 80% corresponding with z= .84, then we can recalculate new te (p80%) for each activity for example activity F (te @ p50% = 10)

$$Z = \frac{EF(P80\%) - EF(P50\%)}{\sigma F\ proj.}$$

$$.84 = \frac{EF(P80\%) - 40}{3.42}$$

$$\sigma F\ proj. = \sqrt{vA + vD + vF} = \sqrt{0.44 + 11.11 + 0.11} = 3.42$$

(for the critical path activities till F)

So, EF (P80%) = 42.9

Table 3 shows the new calculations of activities duration, variances, and standard deviations. Fig. 2 illustrates the new network of activities the path **ADFHJK** is also the only critical path in the project with a total probabilistic project duration of **72** days and a total cost of **$13,305**($6,825+$6,480).

Table 3. New Calculations of Activities Duration, New Original Floats, Variances, and Standard Deviations

Act	a	m	b	te	v	σ	σ pro	EF80	te*	FLo*	b*
A	8	10	12	10.0	0.44	0.67	0.67	10.6	11	0	18.0
B	6	7	9	7.2	0.25	0.50	0.50	7.6	8	25	14.0
C	3	3	4	3.2	0.03	0.17	0.17	3.3	4	39	9.0
D	10	20	30	20.0	11.11	3.33	3.40	32.9	22	0	42.0
E	6	7	8	7.0	0.11	0.33	0.37	10.5	7	39	8.0
F	9	10	11	10.0	0.11	0.33	3.42	42.9	10	0	11.0
G	6	7	10	7.3	0.44	0.67	3.46	40.2	8	17	14.0
H	14	15	16	15.0	0.11	0.33	3.43	57.9	15	0	16.0
I	10	11	13	11.2	0.25	0.50	3.45	54.1	12	10	18.0
J	6	7	8	7.0	0.11	0.33	3.45	64.9	7	0	8.0
K	4	7	8	6.7	0.44	0.67	3.51	71.6	7	0	10.0
L	1	2	4	2.2	0.25	0.50	3.47	60.1	3	11	9.0

3. Getting the objective function

The objective of the optimization model is to minimize the total cost of the project. The total project cost includes the normal direct cost, the indirect cost, the extra direct cost due to crashing critical activities, and the extra cost resulting from float consumption in noncritical activities. In this research, a method that is based on A nonlinear-integer programming (NLIP) model was developed by Rana and El-Sayegh (2015) to solve the optimization problem while taking into account the total float-loss cost. The total float cost per day for each noncritical activity is calculated according to Eq. (1) established by De La Garza et al. (1991)

$$\text{Total float cost per day} = \frac{\text{LFC} - \text{EFC}}{TF} \quad (1)$$

where EFC = early finish cost; LFC= late finish cost; and TF = total float.

The EFC based on (a perfect world) the most efficient method and crew, and all the flexibility to resources. The LFC based on (inflexible world) As the float is consumed, schedule loses its risks assimilation, and less desirable quality (De La Garza et al. 1991). Table 4. demonstrates all the symbols and notations which are used throughout the model.

Act.	P. N. Dur.	max τ	PNC	δc	S	di	τi	df	F	CC	δf	Flo*	Fci	FC	FCC
A	11	1	160	40	0	11	0	11	11	0	0.0	0	0	0	0
B	8	2	400	15	0	8	0	8	8	0	20.1	25	25	0	0
C	4	2	350	0	0	4	0	4	4	0	32.1	39	39	0	0
D	22	12	1020	16	11	22	0	22	33	0	0.0	0	0	0	0
E	7	3	610	30	4	7	0	7	11	0	21.8	39	39	0	0
F	10	2	1250	25	33	10	0	10	43	0	0.0	0	0	0	0
G	8	2	525	68	33	8	0	8	41	0	18.7	17	17	0	0
H	15	8	825	23	43	15	0	15	58	0	0.0	0	0	0	0
I	12	3	390	7	43	12	0	12	55	0	32.3	10	10	0	0
J	7	2	430	30	58	7	0	7	65	0	0.0	0	0	0	0
K	7	2	250	50	65	7	0	7	72	0	0.0	0	0	0	0
L	3	2	615	75	58	3	0	3	61	0	20.9	11	11	0	0
finish	0	0	0	0	72	0	0	0	72	0	0.0	0	0	0	0
			6825												

SITE OVERHEAD $/DAY 90

DC	6,825.0
IDC	6,480.0
CC	-
FCC	-
TC	13,305.0

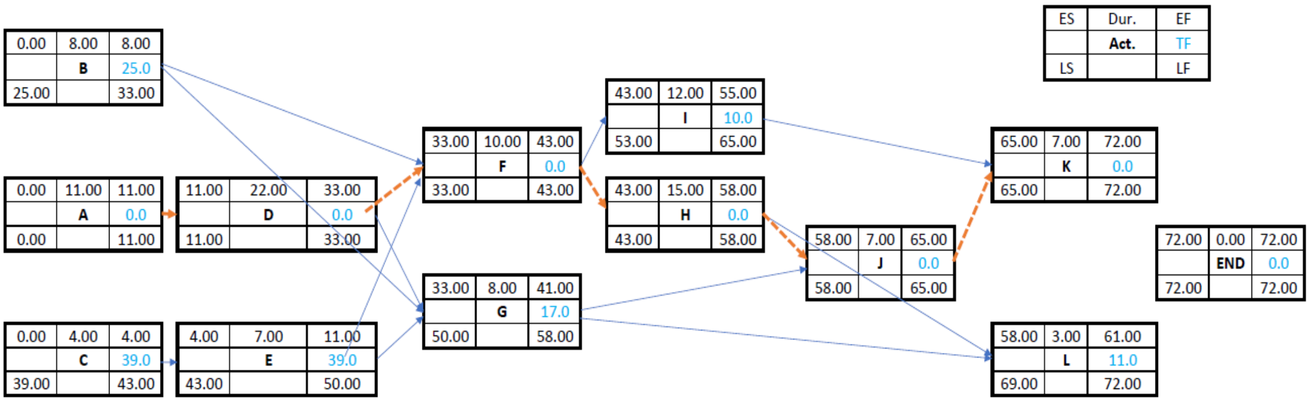


Fig. 2. Probabilistic Network Diagram

Table 4. Symbols and Notations of the Model

Symbol	Description	Symbol	Description
ND	Normal duration	**EF**	Early finish
CD	Crashed duration	**LS**	Late start
TC	Total cost	**LF**	Late finish
DC	Direct cost	δ_f	Float unit cost
IDC	Indirect cost	**EFC**	Early finish cost
C ov.	Overhead cost	**LFC**	Late finish cost
CC	Crashed cost	**di**	Duration of activity i
FCC	Float consumption cost	$d_{i.min}$	Crash duration of activity i
FL_o	Original float	$d_{i.max}$	Normal duration of activity i
FL	Current float	L_{ij}	Lag time between activity I & j
FC	Float consumption	**Si**	Start date of activity i
PD	Project duration	**Fi**	Finish date of activity i
CDT	Contract date	τ	Crashing days
ES	Early start	δ_c	Crashing unit cost

The objective of the optimization model is to minimize the total cost of the project. The total project cost includes the indirect cost, the normal direct cost, the extra direct cost due to crashing critical activities, and the extra cost resulting from float consumption in noncritical activities.

I. Direct cost

Eq. (2) is used to calculate the direct cost

$$DC = \sum_{i=1}^{n} DC\,i \quad (2)$$

II. Indirect cost

Eq. (3) is used to calculate the indirect cost

$$IDC = Cov. \times PD \quad (3)$$

III. Crashing cost

Eq. (4) is used to calculate the extra cost due to crashing (CC)

$$CC = \sum_{i=1}^{n} \delta c \times \tau \quad (4)$$

$$\tau \in \{0: (CD - ND)\} \quad (5)$$

$$\delta c = \frac{CCi - NCi}{di.max - di.min} \quad (6)$$

IV. Float consumption cost

Eq. (7) is used to calculate the extra cost due to float consumption (FCC)

$$FCC = \sum_{i=1}^{n} \delta f \times FC \quad (7)$$

$$\delta f = \frac{LFCi - EFCi}{FLo} \quad (8)$$

$$FC = FLo - FL \quad (9)$$

Now the objective function [Eq. (10)] can be written by combining Eqs. (2), (3), (4), and (7), which results in an optimization equation for the total project cost.

Minimize TC

$$TC = \sum_{i=1}^{n} DC\,i + (Cov. \times PD) + \sum_{i=1}^{n} \delta c \times \tau + \sum_{i=1}^{n} \delta f \times FC \quad (10)$$

4. Identifying the problem variables

The problem variables are $\tau$is for activities when they change, the durations of the activities are changed, and this affect the model. table 5. illustrates the numbers of the allowable crashing days of the activities.

Table 5. No. of Crashing Days of Activities

Act.	A	B	C	D	E	F	G	H	I	J	K	L
$\tau \in \{0: ...\}$	1	2	2	12	3	2	2	8	3	2	2	2

5. Identifying the problem constraints

The problem constraints could be classified to three types, Fig. 3 shows these types:

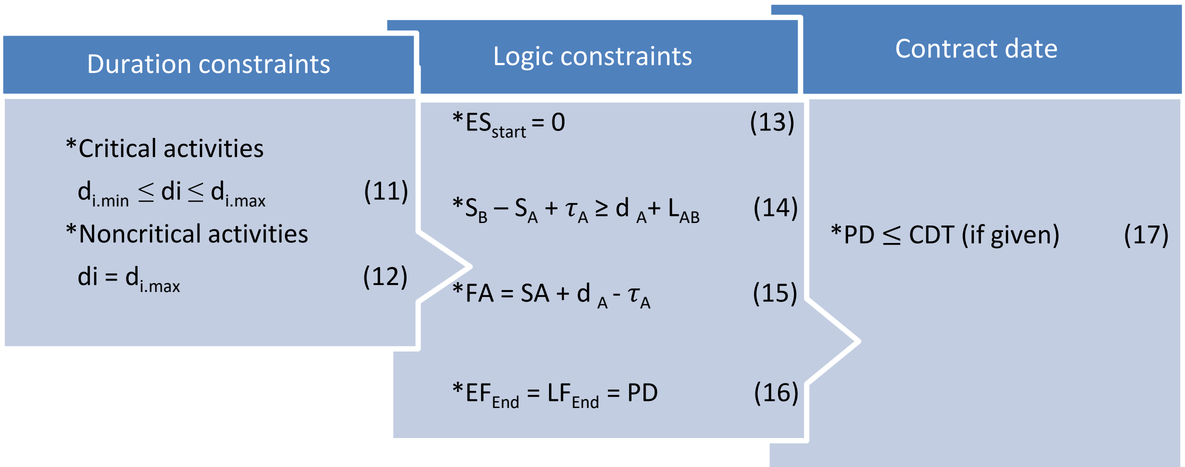


Fig. 3. Types of the Model Constraints

The constraints of activity-duration must be satisfied. These constraints are shown in Fig. 3. Eq. (11) is used to set up the duration constraints for critical activities, whereas Eq. (12) is used to set up the duration constraints for noncritical activities. The activities' duration (di) are set to be adjustable parameters and linked to the network. As the network changes, activities' duration changes and the calculations are done accordingly through the model.
The logic constraints are developed. the starting point of the network, the early start (ES_{start}) of the first activity, should be set to 0 according to Eq. (13).
For activity B, for example, it has activity A as a predecessor with FS relationships. Applying the precedence constraint for the FS relationship based on Eq. (14), (15) as shown in Fig. 3,4.

- FS relationship:

$$SB \geq SA + dA + LAB - \tau A$$

$$SB - SA + \tau A \geq dA + LAB \quad (14)$$

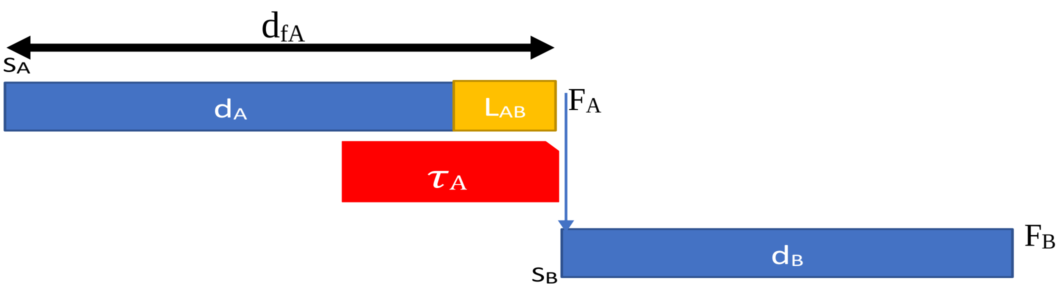


Fig. 4. The Precedence Constraint for the FS Relationship

The project duration is calculated as the EF of the last activity according to Eq. (16). At the same time, the LF of the last activity is set as its EF. The project duration must satisfy the targeted contract date, if specified Eq. (17).

6. Running the optimization model and Getting the results.

The table above Fig. 2. Shows the cost calculations. The normal duration and original total float, for all activities, are kept constant throughout the calculations. The current float column will vary according to the model calculations at each crashing scenario. The values in the current total float (FLo) column have to be linked to the total float of each activity in the network. As the network changes, activities' total float changes, and the current float values change accordingly. The values in the crashing unit cost (δc) column are calculated using Eq. (6). The values in the float unit cost (δf) column are calculated using Eq. (8). The values in the crashing cost (CC) and the float cost (FCC) columns are calculated using Eqs. (4) and (7), respectively. The total float-loss cost should be constrained to be ≥ 0 for each activity.
The objective function is then set in the summary table above Fig. 2. The direct cost is calculated using Eq. (2) as the sum of all the direct (normal) cost of all activities. From Table 1, the sum of all the normal costs is US$6,825. The overhead cost (C ov.) is assumed to be US$90=day for this example project. The indirect cost is calculated using Eq. (3) and is equal to US $6,480 (72 days × US$90=day). At the normal probabilistic case, there are no crashing costs nor float cost. Thus, the normal total cost is calculated to be US$13,305.
After running the model using solver, the optimum duration, considering the total float cost impact, is found to be 63 days. The associated minimum total cost is US$13,253.1. Fig. 5 presents the final solution using the proposed model. According to the model, the solution was reached after activity D was crashed by 5 days, activity J was crashed by 2 days, and activity K was crashed by 2 days resulting in an extra direct crashing cost of US$240 and a total float loss in activity B by 5 days, activity C by 5 days, activity E by 5 days , activity I by 2 days, and activity L by 4 days resulting in a total float consumption cost of US$518.1.

Act.	P. N. Dur.	max τ	PNC	δc	S	di	τi	df	F	CC	δf	Flo*	Fci	FC	FCC
A	11	1	160	40	0	11	0	11	11	0	0.0	0	0	0	0.0
B	8	2	400	15	0	8	0	8	8	0	20.1	25	20	5	100.7
C	4	2	350	0	0	4	0	4	4	0	32.1	39	34	5	160.3
D	22	12	1020	16	11	22	5	17	28	80	0.0	0	0	0	0.0
E	7	3	610	30	4	7	0	7	11	0	21.8	39	34	5	108.9
F	10	2	1250	25	28	10	0	10	38	0	0.0	0	0	0	0.0
G	8	2	525	68	28	8	0	8	36	0	18.7	17	17	0	0.0
H	15	8	825	23	38	15	0	15	53	0	0.0	0	0	0	0.0
I	12	3	390	7	38	12	0	12	50	0	32.3	10	8	2	64.6
J	7	2	430	30	53	7	2	5	58	60	0.0	0	0	0	0.0
K	7	2	250	50	58	7	2	5	63	100	0.0	0	0	0	0.0
L	3	2	615	75	53	3	0	3	56	0	20.9	11	7	4	83.5
finish	0	0	0	0	63	0	0	0	63	0	0.0	0	0	0	0.0
			6825												

SITE OVERHEAD $/DAY 90

DC	6,825.0
IDC	5,670.0
CC	240.0
FCC	518.1
TC	13,253.1

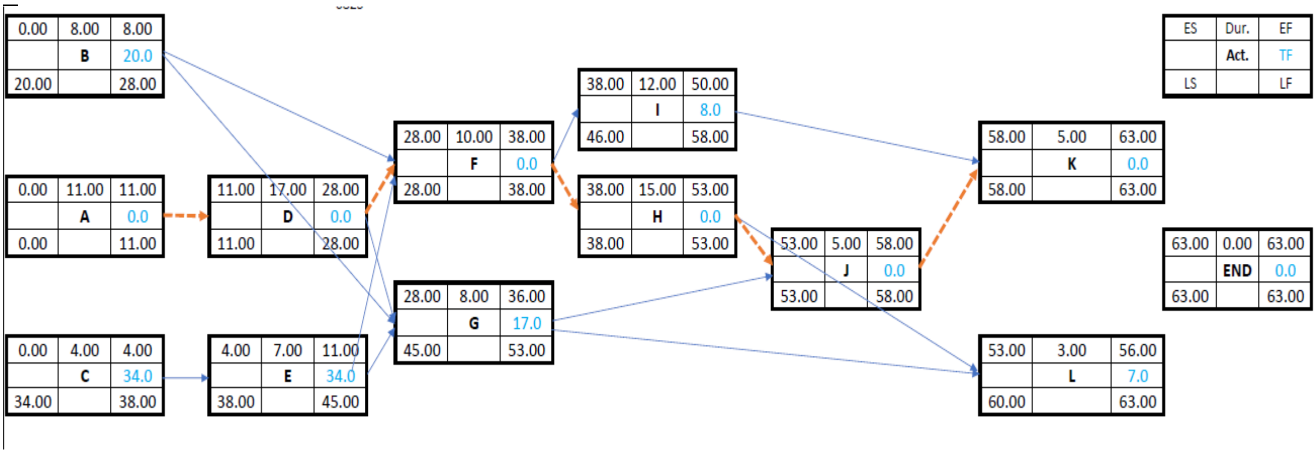


Fig. 5. Probabilistic Crashed Network Diagram with Float Consumption

To prove the point of view of the NLIP model the optimization is implemented with neglecting the float consumption cost on the example. After running the model that time, the optimum duration is found to be 57 days. The associated minimum total cost without the float consumption cost is US$11,503.7, but in fact it is US$13,597.4. Fig. 6 presents that case, according to that model, the solution was reached after activity A was crashed by 1 days, activity D was crashed by 12 days, activity F was crashed by 2 days, activity H was crashed by 8 days, activity J was crashed by 2 days, and activity K was crashed by 2 days resulting in an extra direct crashing cost of US$628,7, although that crashing resulted in a total float loss in activity B by 13 days, activity C by 23 days, activity E by 23 days, activity G by 10 days, activity I by 10 days, and activity L by 4 days resulting in a total float consumption cost of US$2,093.8. hence, that consumption of the total floats of non-critical activities causes decrease in the schedule flexibility and increase in the probabilities of delays.

Act.	P. N. Dur.	max τ	PNC	δc	S	di	τi	df	F	CC	δf	Flo*	Fci	FC	FCC
A	11	1	160	40	0	11	1	10	10	40	0.0	0	0	0	0.0
B	8	2	400	15	0	8	0	8	8	0	20.1	25	12	13	261.9
C	4	2	350	0	0	4	0	4	4	0	32.1	39	16	23	737.5
D	22	12	1020	16	10	22	12	10	20	192	0.0	0	0	0	0.0
E	7	3	610	30	4	7	0	7	11	0	21.8	39	16	23	501.0
F	10	2	1250	25	20	10	2	8	28	50	0.0	0	0	0	0.0
G	8	2	525	68	20	8	0	8	28	0	18.7	17	7	10	186.8
H	15	8	825	23	28	15	8	7	35	186.67	0.0	0	0	0	0.0
I	12	3	390	7	28	12	0	12	40	0	32.3	10	0	10	323.1
J	7	2	430	30	35	7	2	5	40	60	0.0	0	0	0	0.0
K	7	2	250	50	40	7	2	5	45	100	0.0	0	0	0	0.0
L	3	2	615	75	35	3	0	3	38	0	20.9	11	7	4	83.5
finish	0	0	0	0	45	0	0	0	45	0	0.0	0	0	0	0.0
			6825												

SITE OVERHEAD 90

DC	6,825.0		
IDC	4,050.0	TCNOFC	
CC	628.7	11,503.7	
FCC	2,093.8		
TC	13,597.4		

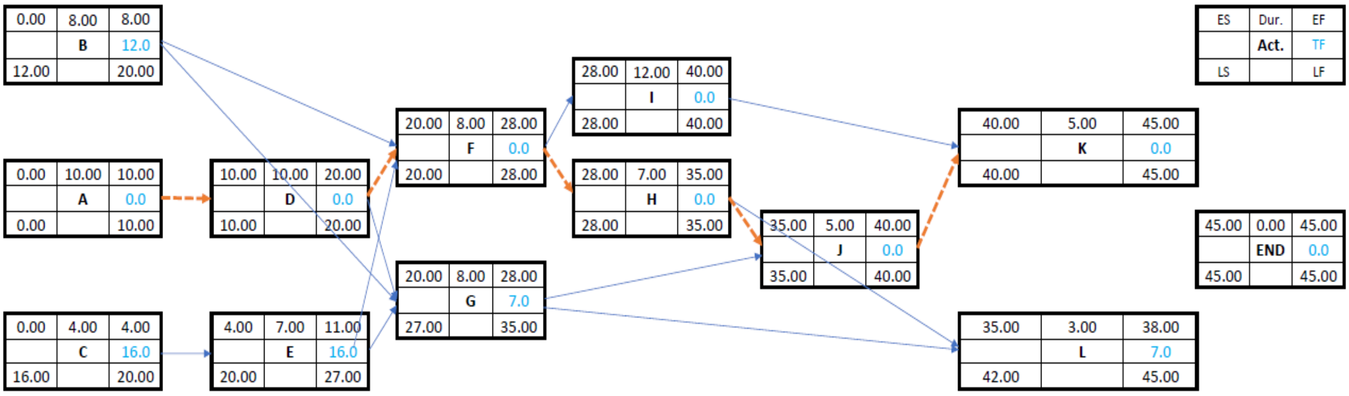


Fig. 6. Probabilistic Crashed Network Diagram without Float Consumption

Table 6. shows a comparison of remaining total float between the traditional crashing for the probabilistic model and the NLIP probabilistic model, it`s clearly that the traditional model causes in all non-critical activities became critical. The NLIP probabilistic model saves some of the total float, that gives the project schedule some flexibility.

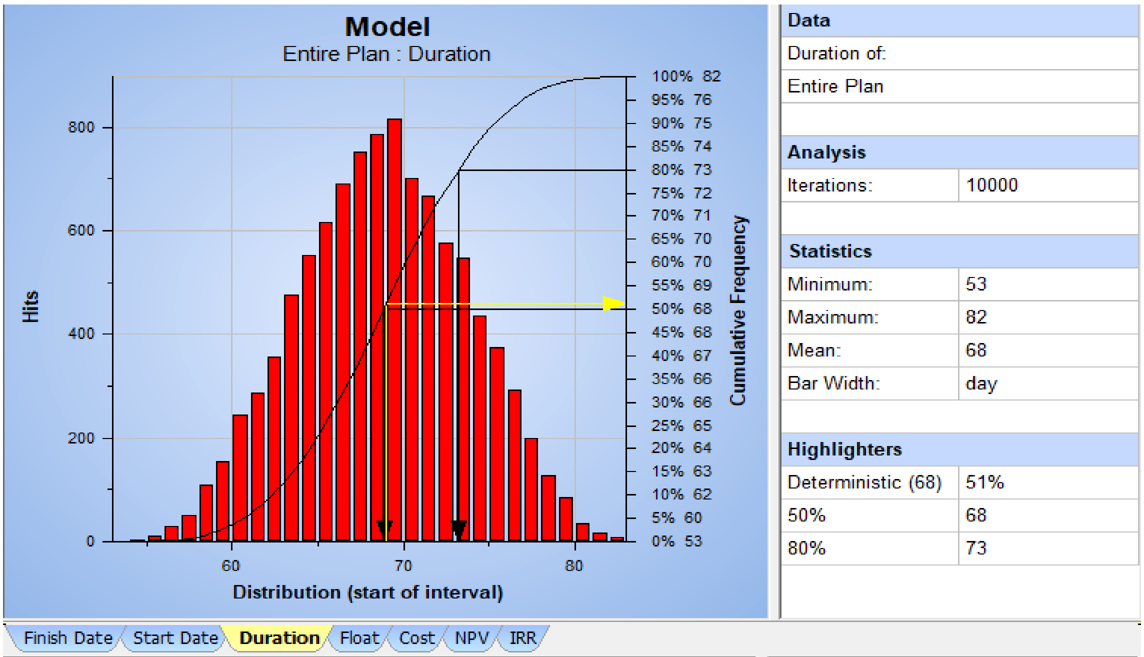
Table 5. Comparison of TF between Cases

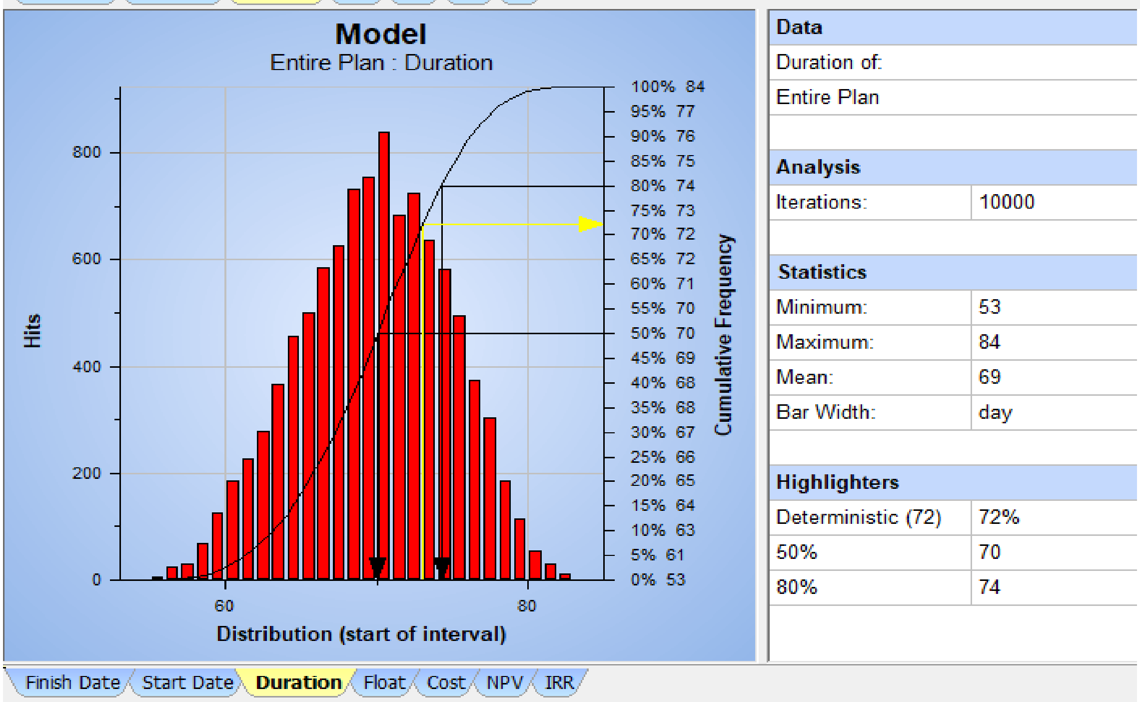
Act.	Normal prob. case	Traditional prob. case	NLIP prob. case
A	0	0	0
B	25	12	20
C	39	16	34
D	0	0	0
E	39	16	34
F	0	0	0
G	17	7	17
H	0	0	0
I	10	0	8
J	0	0	0
K	0	0	0
L	11	7	7

The importance of incorporating probabilistic scheduling into the optimization problem highlights when a comparison is performed between the results of the NLIP framework and the probabilistic scheduling method in terms of the probability of finishing (POF) the project on time. Monte Carlo simulation, using Primavera Risk Analysis, was run on the baseline schedule (p50%), probabilistic schedule (p80%), NLIP crashed schedule, and traditional crashing with no delays in any activity taking into account the statistical data of mean and standard deviation of duration as its measure of uncertainty in the event. The simulation was run with 10,000 iterations. The simulation results are produced in Table 6 and Fig 7. It shows the POF of the project on time using the models. This proves that the new proposed model magnifies the probability of finishing, and reduces the risk of project delays compared with the NLIP probabilistic framework.

Table 5. probability of finishing (POF) the project on time

Cases	POF
Normal det. case	51%
Normal prob. case	72%
Traditional prob. case	56%
NLIP prob. case	13%





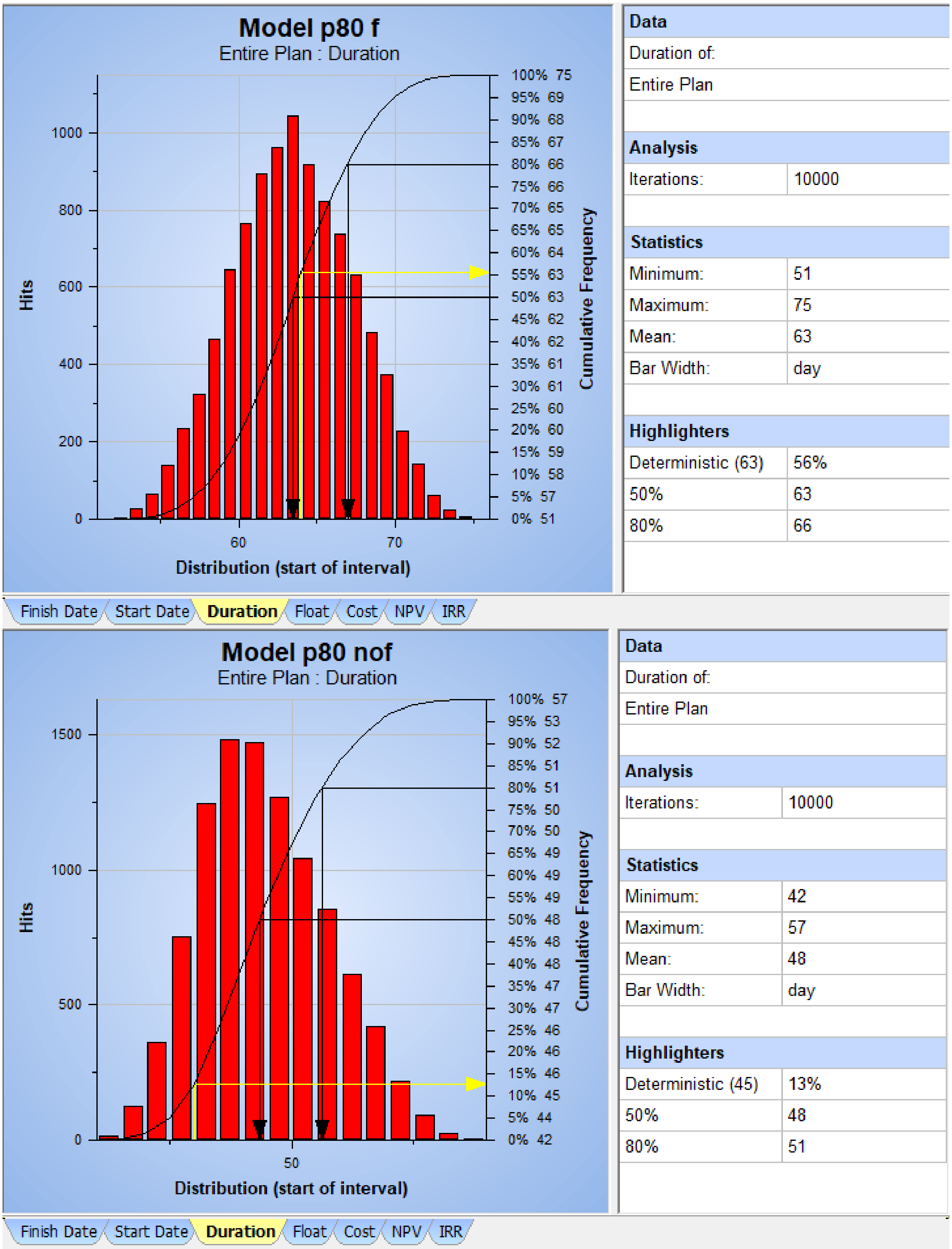


Fig. 7. The Probability of Finishing (POF) for Cases

Conclusion

This research illustrates the effect of scheduling a realistic schedule by taking into account risk factors that may have a positive or negative impact on the project, attempt to deal with the risks and uncertainties related with the project activities data specially activities durations, and helping project managers foresee the uncertainties and its impact on the schedule.
The relevance of a nonlinear-integer programming probabilistic model with the impact of total float consumption with control the risks of float consumption can produce a more realistic optimum duration that accounts for risks arising from future potential delays, and taking total float

consumption into consideration, while crashing, will assist in providing a more realistic optimized schedule with the least cost. The outcomes demonstrate that the proposed model decreases the risks resulted from the total float consumption due to crashing. Delaying noncritically to finish beyond their early finish date makes a construction projects schedule less flexible and can lead to an increase in project duration and/or cost.

Even though the optimum duration and cost are higher than those gotten from traditional strategies, the schedule risk is lower. That model provides planners with a better tool to deal with the time– cost trade off issue with the least conceivable risks related with delays and expenses from total float consumption. The proposed model could be beneficial, if implemented, as it can help management in providing an efficient solution to the time–cost trade-off problem while maintaining a flexible schedule that meets the project needs with less inhabited risk, and gives contractors and owners with a planning tool to evaluate and measure the effect of any project duration delays and cost. This can decrease the conflicts, disputes, and litigations between all construction stakeholders. That is a result of replacing personal evaluation with objective and quantitative identification that are based on float loss cost.

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