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Inter Nuclear Energy Levels of ^{100}Ru and ^{102}Ru Modeling Approach

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Nuclear energy level distribution has been discussed and some mathematical models which describes the energy level distribution of the protons and neutrons for the even protons and even neutrons number, also the Yrast line has been introduced. Ruthenium isotopes ^{100}Ru and ^{102}Ru energy level distribution have been calculated by the nuclear softness model, variable moment of inertia model, Anharmonic vibrator- model (AVM1 and AVM2), exponential model (exp- model) and interaction boson model (IBM) in O6. Experimental results have been used to calculate the unknown factors of the used equations. Finally, the energy levels have been calculated for both isotopes and the higher energy levels which needs very high energetic bombardment to the nuclei if we need to get it experimentally.

Keywords: Nuclear modeling/ Ruthenium isotopes/ Anharmonic vibration model/ Nuclear softness model/ Boson model.

Introduction

Nuclei might be classified to two major categories one is the spherical nuclei and the second is the non-spherical nuclei. The non-spherical nuclei constructed of such non-spherical distribution in its nuclear density, that could be tend to what so called “deformed nuclei”. Deformed nuclei lie in the atomic number regions around $A\approx 24$, and in between $150 \leq A \leq 180$ finally the region greater than $A \geq 224$. Otherwise, any deformed nuclei exhibit rotational and vibrational nuclear spectra. Rotational spectrums of any deformed nuclei consist of different rotational bands, ground state band and excited as super band.

Sometimes the least vitality state band called the ground state band or the Yrast band. The word Yrast is a Swedish word means in English "the fastest rotation".

Rotation motion can be watched as it were nuclei with non-spherical equilibrium shapes (which is the case of vibrational motion).

Plotting the excitation energy of each energy level of the ground state band against its angular momentum, which is known as Yrast line. The presence of different crossed Yrast lines at certain spin quantum number have been introduced by Johnson et. al. as a back bending phenomenon.

There are several models describing the proton and neutron distribution inside such nucleus. One form those models is the Collective Nuclear Model which is suitable for that nuclei which includes even number of protons and even number of neutrons.

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One from the factors activated the need for new abroach is the significant failure [1-3] of the shell model to nuclear structure is its inability to explain the huge quadrupole moments that have been observed for nuclei in certain region of periodic table.

By the mid of the last century J. Rainwater [2] suggests that the single particle deforms the whole nucleus, and the observed quadrupole results from the collective deformation of many orbits. Hence it could be considered that nuclear quadrupole moment gives the opportunity to calculate the deviation of nuclear shape from a sphere. The mathematical development of the modeling ideas [4-7] is the basis for the collective model which has been particularly successful for A=24, 150<A<190 and A>230.

The collective model could describe both the rotational states and the vibrational states, in molecular spectra, [8-10] one could easily see the evidences about three types of electronic excitation [11-13] through the electronic transitions, vibrational levels transitions and/or rotational states. On the other hand, inside the nuclei, one could be easily notice that the nucleons suffer the same transitions.

Flexing the surface of such nucleus causes vibrations affecting the nuclear levels and forming vibrational states which have more complicated nature than that outer molecular vibrational states. The rotational motion is more complicated than the rotation of rigid body. It could be regarded as such rotation of deformed body surface enclosing free particles.

Both nuclear vibrational motion and rotational motion involve orderly displacements of many nucleons, and both types are therefore classified as nuclear collective motion.

The rotational energy level distribution could be interpreted by the following equation,

$$\boldsymbol{E = \frac{\hbar}{2\Phi}\left(J(J+1) - K^2\right)}$$

Where ϕ is the ground state moment of inertia for the nucleus, K is the band head angular momentum, and J is the total angular momentum.

While the nucleus is not a pure sphere such correction factor (B) should be included in the previous equation, hence the new form of the rotational energy levels will be

$$\boldsymbol{E = \frac{\hbar}{2\Phi}\left(J(J+1) + B(J^2(J+1)^2\right)}$$

The collective model is not the only model but there are other different models like nuclear softness and variable moment of inertia also the anharmonic vibrator model. The challenge in computing the energy level like any modeling work is always the selection of the appropriate model.

The aim from this work is to calculate the energy level of even-even nucleus in particular Ru^{100} and Ru^{102}.

Methods:

In this work the variable moment of inertia model (VMI1, VMI2), nuclear softness (NS3), exponential model (exp- model) and interaction boson model (IBM) in O6 were utilized to calculate the energy levels.

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A computer subroutine was developed to calculate the unknown parameters and computing the back pending data.

Results and discussion:

Following the prescription given before concerning the VMI- model [4], nuclear softness [14] model (NS3), Anharmonic vibrator- model [15] (AVM1 and AVM2), exponential model [16] (exp- model) and interaction boson model [17] (IBM) in O6, respectively we have re-evaluated the level energies E_J, by means of the well-known equations:

$$E_J = [J(J+1)/2\Phi_J]\{1+[J(J+1)/4C\Phi_J^3]\} \quad \text{(VMI)}$$

$$E_J = A J(J+1)/(1+\sigma_1 J+\sigma_2 J^2) \quad \text{(NS3)}$$

$$E_J = \Phi_o^{-1}(J+X J(J-2)) \quad \text{(AVM1)}$$

$$E_J = \Phi_o^{-1}(J+X J(J-2)+Y J(J-2)(J-4)) \quad \text{(AVM2)}$$

$$E_J = (\hbar^2/\Phi_o) J(J+1) \, \mathrm{Exp}[\Delta_o (1-J/J_c)^{0.5}] \quad \text{(exp)}$$

$$E_{([N],\sigma,\tau,\beta,A,C,J)} = (A/4)(N-\sigma)(N+\sigma+4)+\beta\tau(\tau+3)+C J(J+1) \quad \text{(IBM-O6)}$$

For the ground state bands of an even-even ^{100}Ru and ^{102}Ru, a computer program is designed where the parameters in each model were determined by means of a least square fitting procedure involving the experimentally known level energies.

The evaluations have been made for ^{100}Ru and ^{102}Ru. Making use of computer program, the constants of equations VMI and NS models were calculated and presented in table (1), while the constants of both AVM1 and AVM2 were calculated and presented in table (2), and the constants IBM-O6 and Exp. models are calculated and presented in table (3).

The results of the ground state energy levels up to spin J=30 is presented in tables (4) and (5), where the corresponding experimental energies from various data are also listed.

Table (1) the parameters of the VMI-model and NS3-model as calculated by least squares fitting using a computer program and the known experimental data.

	VMI-model		**NS3-model**		
Isotope	**C (Kev^2)**	**φ_o (Kev^{-1}) ×10^{-3}**	**A (Kev) ×10^{-3}**	**σ_1**	**σ_2 ×10^{-3}**
^{100}Ru	**7289726**	**1.528387**	**2.673**	**0.5905**	**-2.075**
^{102}Ru	**5099606**	**1.649654**	**3.574**	**0.42551**	**-9.041**

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Table (2) the parameters of the AVM1 and ANM2-model as calculated by least squares fitting using a computer program and the known experimental data.

isotope	φ_o (Kev $^{-1}$)	X	Y ×10^{-3}
^{100}Ru	269.753	0.0682	1.227
^{102}Ru	237.57	0.0822	-1.807

Table (3) the parameters of the IBM (O6) and exp.-model as calculated by least squares fitting using a computer program and the known experimental data.

	IBM (O6)		Exp.-model		
isotope	B	C	φ_o (Kev $^{-1}$)	Δ_o	Jc
^{100}Ru	171.6325	-24.504	0.62997	5.518379	16
^{102}Ru	143.174	-16.26951	1.54799	4.445898	14

The calculated equations parameters in table 1,2 and 3 have been employed to calculate the energy levels of the nuclei isotopes of 100Ru in table 4 and 102Ru isotope in table 5.

Table (4). Experimental and calculated energy levels of ^{100}Ru by different models, the energy is given in Kev.

^{100}Ru	[19]	[18]	Present work					
Spin(J)	Exp	VMI	VMI	NS3	AVM1	AVM2	exp	O6
2	539.506	539.506	527.65	534.9	539.5	539.5	659.6	539.50
4	1226.245	1226.245	1231.7	1234.7	1226.2	1226.2	1499.2	1226.2
6	2076.1	2028	2060.9	2069.7	2060.2	2076.1	2076.1	2060.2
8	3062.2	2919	2986.9	3063.9	3041.4	3104.9	2245.5	3041.4
10	4085.2	3884	3993.4	4260	4170	4328.7	2033.8	4169.9

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12		**4914**	**5069.7**	**5724.2**	**5445.5**	**5763**	**1551.5**	**5445.5**
14		**6002**	**6207.8**	**7554.5**	**6868.4**	**7424**	**930.8**	**6868.4**
16		**7142**	**7401.8**	**9906.7**	**8438.6**	**9328**		**8438.6**
18		**8329**	**8647**	**13040**	**10156**	**11490**		**10155.9**
20			**9939.7**	**17420**	**12021**	**13926**		**12021**
22			**11276.6**	**23974.5**	**14032**	**16653**		**14032**
24			**12654.9**	**34854.7**	**16191.5**	**19686**		**16191.5**
26			**14072.5**	**56451**	**18498**	**23040**		**18497.75**
28			**15527.2**	**119952**	**20951**	**26733**		**20951.29**
30			**17017**	**4307580**	**23552**	**30779**		**23552.05**

Table (5). Experimental and calculated energy levels of ^{102}Ru by different models, the energy is given in Kev.

^{102}Ru	**[20]**	**[18]**	**Present work**					
Spin(J)	**Exp**	**VMI**	**VMI**	**NS3**	**AVM1**	**AVM2**	**exp**	**O6**
2	**475.079**	**475.079**	**471.08**	**469.21**	**475.08**	**475.08**	**569.5334**	**475**
4	**1106.35**	**1106.35**	**1097.6**	**1116.7**	**1106**	**1106**	**1326.313**	**1106**
6	**1873.21**	**1849.1**	**1834.9**	**1865.4**	**1893.8**	**1873.2**	**1873.21**	**1893.8**

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8	**2704.3**	**2678.3**	**2658**	**2706.4**	**2837**	**2755**	**2047.031**	**2837.5**
10	**3431.3**	**3579.3**	**3552.7**	**3645**	**3937**	**3731**	**1833.293**	**3937**
12	**4052.3**	**4542.5**	**4509**	**4694.7**	**5193**	**4781**	**1296.215**	**5193**
14	**4803.3**	**5561**	**5520**	**5871**	**6606**	**5884**		**6605.6**
16	**5717.3**	**6629.4**	**6581**	**7198**	**8174**	**7020**		**8174**
18		**7743.5**	**7687.6**	**8705**	**9898.6**	**8167.9**		**9898.6**
20			**8835.9**	**10428**	**11779**	**9307**		**11779.4**
22			**10024**	**12418**	**13816**	**10416**		**13816**
24			**11247.9**	**14743**	**16009.6**	**11477**		**16009.6**
26			**12507**	**17492**	**18359**	**12466.6**		**18359**
28			**13799**	**20793**	**20864.6**	**13365.1**		**20865**
30			**15123**	**24831**	**23526**	**14152**		**23526**

The comparison with the previous calculated values of VMI and the experimental values show identicality between the current VMI calculations and the previous calculations while the anharmonic calculations show more accuracy in the lower spin values while the higher spin values diverge more from the experimental values.

Conclusion

The nuclear models of even-even nuclei have been utilized to calculate the energy levels of the Ruthenium isotopes nuclei. The it is clear that the anharmonic model is identical with the experimental values in some levels while it deviates a little bit from the experimental values. In some cases, the interaction boson model is closer to the experimental values. It is clear that the need for such model which could calculate the energy level distribution of the nucleus make it easy to predict the higher excitation states without any need for doing experimental work with very high cost and very complicated steps. Understanding the energy level distribution leads to understanding how the radiation take place and how to control the radiation and the nuclear reactions. The results clarifies that we have to apply that models in

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other isotopes to see if the accuracy will be affected in other isotopes, also we still need to develop mathematical models to have more accurate values for the calculated energy levels.

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