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A proposed method for solving multiobjective linear fractional programming problems with rough coefficients

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Abstract: In this paper, a new method for solving multiobjective linear fractional programming problems with rough coefficient (MORLFP) is proposed. The MORLFP problem is considered by incorporating rough intervals in the coefficients of the objective functions. It is provided that a MORLFP problem is converted to an optimization problem with rough interval valued objective functions which it their bounds are four multiobjective linear fractional functions. The rough efficient solutions are characterized by using a new proposed algorithm. A numerical example is given for the sake of illustration

Key words: Multi objective linear fractional programming problems, Rough interval, Rough interval function.

1. Introduction

Fractional programming concerns with the optimization problems of one or several ratio functions subject to some constraints. Decision makers sometimes, may face up with the decision to optimize actual cost/standard cost, output/employee, etc with respect to some constraints. In management problems, both the ratio functions profit, cost and quality to be optimization are conflicting in nature. Such types of problems are inherently multiobjective fractional programming problems.

Pawlak [11] defined rough set theory as a new mathematical approach to imperfect knowledge. Kryskiewice [8] uses rough set theory to incomplete has found many interesting applications. the rough set approach seems to be of fundamental importance to cognitive sciences, especially in the areas of machine learning, decision analysis, and expert systems Pal [13]. Rough set theory, introduced by Pawlak [12], expresses vagueness, not by means of membership, but employing a boundary region of a set. The theory of rough set deals with the approximation of an arbitrary subset of a universe by two definable or observable subsets called lower and upper approximations. Tsumoto [19] used the concept of lower and upper approximation in rough sets theory, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules. The concept of rough interval will be introduced by Lu and Huang [9] to represent dual uncertain information of many parameters.The associated solution method will be presented to solve rough interval fuzzy linear programming problems.

Chakraborty and Gupta [3] a different methodology had been proposed for solving multiobjective linear fractional programming (MOLFP) problems always yielding an efficient solution and reduces the complexity in solving the (MOLFP) problems.. Tantawy [18] proposes a new method for solving linear fractional programming problems. Effati and Pakdaman [5] introduce an interval valued linear fractional programming (IVLFP) problem. They convert an IVLFP to an optimization problem with interval valued objective function which its bounds are linear fraction function. Sulaiman and Abulrahim [14] use transformation technique for solving multiobjective linear fractional programming problems to single objective linear fractional programming problem through a new method using mean and median and then solve the problem by modified simplex method. Guzel [6] proposes a new solution to the multiobjective linear fractional programming (MOLFP) problem. Thus MOLFP problem is reduced to linear programming problem. Sulaiman and Abulrahim [17] uses a new transformation technique for solving multiobjective linear fractional programming problems to single objective linear fractional programming problem through a new method using arithmetic average and new arithmetic average technique and then solve the problem by modified simplex method.

This paper deals with a new method for solving MORLFP problem. The MORLFP problem is considered by incorporating rough intervals into coefficient of the objective functions of the problem. The MORLFP problems are converted to four optimization problems. An algorithm is proposed for characterizing the solutions concept of the MORLFP problems. A numerical example is given for the sake of illustration.

2. Preliminaries

2.1 Linear fractional programming problem:

The general linear fractional programming (LFP) problems are defined as follows:

$$\text{Max} \quad \frac{N(x)}{D(x)}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n : \; Ax \le b \,, \; x \ge 0 \},$$

$$c^T,\; d^T \in \mathcal{R}^n \; \alpha,\; \beta \in \mathcal{R} \,, b \in \mathcal{R}^m \,, A \in \mathcal{R}^{m\times n}$$

Where $N(x) = c^T x + \alpha$, $D(x) = d^T x + \beta$ are real valued and continuous functions on $X \; and \;\; d^T x + \beta \ne 0$

Theorem 1. [6] $z^* = \frac{N(x^*)}{D(x^*)} = Max \; \frac{N(x)}{D(x)}$ if and only if

$$F(z^*, x^*) = Max\{N(x) - z^* D(x) \,, x \in X \} \; = 0 \,.$$

2.2 Multi objective linear fractional programming problem

The general multi objective linear fractional programming (MOLFP) problems written as:

$$Max \;\; z(x) = \{z_1(x), z_2(x), \ldots\ldots z_k(x)\}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n : \; Ax \le b \,, \quad x \ge 0 \},$$

where $z_i(x) = \frac{c_i x + \alpha_i}{d_i x + \beta_i} = \frac{N_i(x)}{D_i(x)}$,

$c_i \,, d_i \in \mathcal{R}^n \,, \alpha_i \,, \beta_i \in \mathcal{R} \;\; D_i(x) > 0,$

$for \; all \; i = 1,2, \ldots, k.$

Definition 1. $x^* \in \mathcal{R}^n$ is an efficient solution for MOLFP problems if there is no $x \in \mathcal{R}^n$ such that

$\frac{N_i(x)}{D_i(x)} \ge \frac{N_i(x^*)}{D_i(x^*)}$, $i = 1,2,$ ...k and

$\frac{N_i(x)}{D_i(x)} > \frac{N_i(x^*)}{D_i(x^*)}$, for at least one i .

Theorem 2. If $\tilde{x}$ is an optimal solution of

$$Max\left\{\sum_{i=1}^{k} w_i (N_i(x) - (z_i)^* \left(D_i(x\,)\right)\right),\; x \in X \}$$

where is $(z_i)^* = \frac{N_i(x^*)}{D_i(x^*)} = Max \;\; \frac{N_i(x)}{D_i(x)}$ for all $i = 1,2, \ldots k$,

$$w_i \in W = \left\{w_i \in \mathcal{R}^n : w_i \ge 0 \,, \sum_{i=1}^{k} w_i = 1\right\}$$

then $\tilde{x}$ is an efficient solution of MOLFP problems.

The proof of this theorem is much similar to the proof given by Guzel in [6].

2.3 Rough interval linear fractional programming

Definition 2. Suppose I is the set of all compact intervals in the set of all real numbers $\mathcal{R}$. If $A \in I$ then we write $A = [a^L \,, a^U]$with $a^L \le a^U$ and the following holds: [5]

i. $A \ge 0$ iff $x_i \ge 0$ for all $x_i \in A$.

ii. $A \leq 0$ iff $x_i \leq 0$ for all $x_i \in A$.

Definition 3. Let X be denote a compact set of real numbers. A rough interval X^R is defined as:

$X^R = [\, X^{(LAI)} : X^{(UAI)}]$ where $X^{(LAI)} and\ X^{(UAI)}$ are compact intervals denoted by lower and upper approximation intervals of X^R with $X^{(LAI)} \subseteq X^{(UAI)}$.

Definition 4. For the rough interval X^R the following holds:

i. $X^R \succcurlyeq 0$, iff $X^{(LAI)} \geq 0$ and $X^{(UAI)} \geq 0$

ii. $X^R \preccurlyeq 0$, iff $X^{(LAI)} \leq 0$ and $X^{(UAI)} \leq 0$.

In this paper we denote by I^R is the set of all rough intervals in $\mathcal{R}$. Suppose A^R , $B^R \in I^R$ we can write $A^R = \left[A^{(LAI)} \ : \ A^{(UAI)}\right]$ and also $B^R = [\, B^{(LAI)} \ : \ B^{(UAI)}\,]$ where $A^{(LAI)} = [a^{-L} , \ a^{+L}]$, $B^{(LAI)}$ $[b^{-L}, \ b^{+L}]$ a^{-L} , a^{+L}, b^{-L} , and $b^{+L} \in \mathcal{R}$.

Similarly we can defined $A^{(UAI)}$, $B^{(\,UAI)}$.

Definition 5. [9] For two rough intervals A^R, B^R when $A^R \succcurlyeq 0 \ and \ B^R \succcurlyeq 0$ we can define the following operations on rough intervals as follows:

1) $A^R + B^R = \left[\, \left[A^{(LAI} + B^{(LAI)}\right] : \left[A^{(UAI)} + B^{(UAI)}\right] \right]$

Such that:

$\left[A^{(LAI)} + B^{(LAI)}\right] = [a^{-L} + \ b^{-L} , a^{+L} + \ b^{+L}]$ and

$\left[A^{(UAI)} + B^{(UAI)}\right] \ = [a^{-U} + \ b^{-U} , a^{+U} + \ b^{+U}]$.

2) $A^R - B^R = \left[\, \left[A^{(LAI} - B^{(LAI)}\right] : \left[A^{(UAI)} - B^{(UAI)}\right]\right.$

Such that:

$\left[A^{(LAI)} - B^{(LAI)}\right] = [a^{-L} - \ b^{+L} , \ a^{+L} - \ b^{-L}]$ and

$\left[A^{(UAI)} - B^{(UAI)}\right] = [a^{-U} - \ b^{+U} , \ a^{+U} - \ b^{-U}]$.

3) $A^R \times B^R = \left[\, \left[A^{(LAI)} \times B^{(LAI)}\right] : \left[A^{(UAI)} \times B^{(UAI)}\right] \right]$

Such that:

$\left[A^{(LAI)} \times B^{(LAI)}\right] = [a^{-L} \times \ b^{-L} , \ a^{+L} \times \ b^{+L}]$ and

$\left[A^{(UAI)} \times B^{(UAI)}\right] = [a^{-U} \times \ b^{-U} , \ a^{+U} \times \ b^{+U}]$.

4) $A^R \,/\, B^R = \left[\, \left[A^{(LAI)} \,/\, B^{(LAI)}\right] : \left[A^{(UAI)} \,/\, B^{(UAI)}\right] \right]$

Such that:

$\left[A^{(LAI)} \,/\, B^{(LAI)}\right] = [a^{-L} \,/\, \ b^{+L} , \ a^{+L} \,/\, \ b^{-L}]$ and

$\left[A^{(UAI)} \,/\, B^{(UAI)}\right] = [a^{-U} \,/\, \ b^{+U} , \ a^{+U} \,/\, \ b^{-U}]$.

Definition 6.[5] Let I be the set of all closed and bounded intervals in $\mathcal{R}$.

A function $f : \mathcal{R}^n \rightarrow I$ is called an interval valued function with $f(x) = [f^L(x) , \ f^U(x) \ ]$ where for every $x \in \mathcal{R}^n$, $f^L(x)$, $f^U(x)$ are real valued function ,with $f^L(x) \preccurlyeq f^U(x)$.

Definition 7. A function $f : \mathcal{R}^n \ \rightarrow I^R$ is called a rough interval function with $f^R(x) = \left[f^{(LAI)}(x) : f^{(UAI)}(x)\right]$ where for every $x \in \mathcal{R}^n$, $f^{(LAI)}(x)$, $f^{(UAI)}(x)$ are lower and upper approximation interval valued functions, with $f^{(LAI)}(x) \ \preccurlyeq f^{(UAI)}(x)$

Proposition: [10] Let f be a rough interval function defined on $X \subset \mathcal{R}^n$ and $x_0 \in X$. Then f is continuous at x_0 if and only if $f^{(LAI)}(x)$ and $f^{(UAI)}(x)$ are continuous at x_0 .

3. Problem Formulation

The multiobjective linear fractional programming problems with rough coefficient (MORLFP) are defined as follows

$$Max \left\{ Z_i^R(x) = \frac{N_i^R(x)}{D_i^R(x)} = \frac{c_i^R x + \alpha_i^R}{d_i^R x + \beta_i^R} \quad i = 1,2, \dots k \right\}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n \colon \ Ax \leq b, \ \ x \geq o\,\}. \quad (1)$$

where c_i^R, d_i^R, α_i^R and $\beta_i^R \in I^R$, A is an $m \times n$ constraint matrix, $b \in \mathcal{R}^m$, $k \geq 2$.

We can rewrite problem (1) as follows:

$$Max \left\{ Z_i^R(x) = \frac{[c_i^{LAI} x + \alpha_i^{LAI} : \ c_i^{UAI} x + \alpha_i^{UAI}\,]}{[d_i^{LAI} x + \beta_i^{LAI} : \ d_i^{UAI} x + \beta_i^{UAI}\,]} \quad i = 1,2 \dots , k \right\}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n \colon \ Ax \leq b, \ \ x \geq o\,\}. \quad (2)$$

The objective function in (2) is a quotient of two rough interval functions. Using the definition of operations on a rough intervals we have

$$Z_i^R(x) = [\frac{c_i^{LAI}x+\alpha_i^{LAI}}{d_i^{LAI}x+\beta_i^{LAI}} : \frac{c_i^{UAI}x+\alpha_i^{UAI}}{d_i^{UAI}x+\beta_i^{UAI}} \quad i = 1,2 \ldots k \quad (3)$$

Now equations (3) can be written into the form:

$$Z_i^R(x) = [\quad z_i^{LAI}(x) : \; z_i^{UAI}(x)\,]$$

Where $z_i^{LAI}(x)$, $z_i^{UAI}(x)$ lower and upper multiobjective approximation interval valued linear are fractional functions defined as:

$$z_i^{LAI}(x) = \frac{[c_i^{-L}x+\alpha_i^{-L},\ c_i^{+L}x+\alpha_i^{+L}]}{[d_i^{-L}x+\beta_i^{-L},\ d_i^{+L}x+\beta_i^{+L}]}$$

and $z_i^{UAI}(x) = \frac{[c_i^{-U}x+\alpha_i^{-U},\ c_i^{+U}x+\alpha_i^{+U}]}{[d_i^{-U}x+\beta_i^{-U},\ d_i^{+U}x+\beta_i^{+U}]}$,

for all $i = 1,2 \ldots, k$

Using the theorem (2-1) in [5] we can write equation (3) as the following:

$$Z_i^R(x) = \left[\,[\; z_i^{-L}(x)\ ,\ z_i^{+L}(x)] : [\; z_i^{-U}(x),\ z_i^{+U}(x)]\right], \quad (4)$$

where $z_i^{-L}(x)$, $z_i^{+L}(x)$, $z_i^{-U}(x)$ and $z_i^{+U}(x)$, for all $i = 1,2 \ldots., k$ are multiobjective linear fractional functions defined as:

$$z_i^{-L}(x) = \frac{c_i^{-L}x+\alpha_i^{-L}}{d_i^{+L}x+\beta_i^{+L}} \quad , \quad z_i^{+L}(x) = \frac{c_i^{+L}x+\alpha_i^{+L}}{d_i^{-L}x+\beta_i^{-L}},$$

$$z_i^{-U}(x) = \frac{c_i^{-U}x+\alpha_i^{-U}}{d_i^{+U}x+\beta_i^{+U}} \text{ and } z_i^{+U}(x) = \frac{c_i^{+U}x+\alpha_i^{+U}}{d_i^{-U}x+\beta_i^{-U}}$$

For all $i = 1,2 \ldots., k$.

Now the problem (1) can be converted into multiobjective rough interval linear fractional programming (MORLFP) problems as follows:

$$Max \left\{Z_i^R(x) = [\,[\; z_i^{-L}(x),\ z_i^{+L}(x)] : [\; z_i^{-U}(x),\ z_i^{+U}(x)]]\right\},$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}. \quad (5)$$

For all $i = 1,2 \ldots., k$

By using the arithmetic operations and partial ordering relations, we decompose the MORLFP problem (5) can be the following four sub problems defines as:

P_1 :

$$Max \quad z_i^{+U}(x) = \frac{N_i^{+U}(x)}{D_i^{+U}(x)} = \frac{c_i^{+U}x+\alpha_i^{+U}}{d_i^{-U}x+\beta_i^{-U}},\ i = 1,2 \ldots, k$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

P_2 :

$$Max \quad z_i^{-U}(x) = \frac{N_i^{-U}(x)}{D_i^{-U}(x)} = \frac{c_i^{-U}x+\alpha_i^{-U}}{d_i^{+U}x+\beta_i^{+U}},\ i = 1,2 \ldots, k$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

$$z_i^{-U}(x) Maximize\ value\ of\ \ z_i^{+U}(x)$$

P_3 :

$$Max \quad z_i^{+L}(x) = \frac{N_i^{+L}(x)}{D_i^{+L}(x)} = \frac{c_i^{+L}x+\alpha_i^{+L}}{d_i^{-L}x+\beta_i^{-L}}\ i = 1,2 \ldots., k$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

$$max\ value\ of\ \ z_i^{-U}(x) \le\ z_i^{+L}(x) \le\ Max\ value\ of\ \ z_i^{+U}(x)$$

P_4 :

$$Max \quad z_i^{-L}(x) = \frac{N_i^{-L}(x)}{D_i^{-L}(x)} = \frac{c_i^{-L}x+\alpha_i^{-L}}{d_i^{+L}x+\beta_i^{+L}},\ i = 1,2 \ldots., k$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

$$maximum\ value\ of\ \ z_i^{-U}(x) \le\ z_i^{-L}(x) \le\ Maximize\ value\ of\ \ z_i^{+L}(x)$$

Now using Theorem (2) for socialization problems P_1, P_2, P_3 and P_4 which are MOLFP problems to the equivalent form which are linear programming (LP) problems (P_1', P_2', P_3' and P_4') as follows:

$\mathbf{P_1'}$:

$$\{\textstyle\sum_{i=1}^{k} \omega_i\ (N_i^{+U}(x) - (Z_i^{+U})^* D_i^{+U}(x)\,),\ i = 1,2, \ldots k\ \}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

$\mathbf{P_2'}$:

$$Max \{\textstyle\sum_{i=1}^{k} \omega_i\ (N_i^{-U}(x) - (Z_i^{-U})^* D_i^{-U}(x)\,), i = 1,2, \ldots, k\ \}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\,\}$$

$$z_i^{-U}(x) \le Maximize\ value\ of\ \ z_i^{+U}(x)$$

$\mathbf{P_3'}$:

$$Max \left\{\sum_{i=1}^{k} \omega_i(N_i^{+L}(x) - (Z_i^{+L})^* D_i^{+L}(x)\,), i = 1,2, \ldots, k\right\}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o\}$$

maximum value of $z_i^{-U}(x) \le\ z_i^{+L}(x) \le$ Maximize value of $z_i^{+U}(x)$

$\mathbf{P_4'}$:

$$Max \{\textstyle\sum_{i=1}^{k} \omega_i\ (N_i^{-L}(x) - (Z_i^{-L})^* D_i^{-L}(x)\,), i = 1,2, \ldots, k\ \}$$

Subject to:

$$x \in X = \{x \in \mathcal{R}^n:\ Ax \le b,\ \ x \ge o$$

$$maximum\ value\ of\ z_i^{-U}(x) \leq z_i^{-L}(x) \leq Maximize\ value\ of\ z_i^{+L}(x)$$

Where $\omega \in W = \{\omega_i : \sum_{i=1}^{k} \omega_i = 1,\ \omega_i \geq 0, i = 1,2,\dots,k\}$

Theorem 3.[4] If $x^* \in \mathcal{R}^n$ is an optimal solution for LP problems $P_i', i = 1,2,3,4$ then $x^* \in \mathcal{R}^n$ is an efficient solution of the corresponding MOLFP problems P_i , $i = 1,2,3,4$.

Definition 8. $x^* \in \mathcal{R}^n$ is a rough efficient solution of MORLFP problem (1) if there is no $x \in \mathcal{R}^n$ such that $\frac{N_i^R(x)}{D_i^R(x)} \geqslant \frac{N_i^R(x^*)}{D_i^R(x^*)}$, $i = 1,2,\dots,k$ and $\frac{N_i^R(x)}{D_i^R(x)} \succ \frac{N_i^R(x^*)}{D_i^R(x^*)}$ for at least one i

Theorem 4. If $x^* \in \mathcal{R}^n$ is an efficient solution of the problems P_i , $i = 1,2,3,4$ then $x^* \in \mathcal{R}^n$ is a rough efficient solution of problem (1).

4. Algorithm solution for MORLFP problem :

We construct the algorithm for solving a MORLFP problem as follows:

Step1. Convert the problem to the form of MORLFP problem (5).

Step2. Transfer the problem (5) to four problems on forms P_1, P_2, P_3 and P_4 which are MOLFP problems.

Step3. Find the maximum value of each objective function of P_1, P_2 , P_3 and P_4 as:

$$(z_i)^* = \frac{N_i(x^*)}{D_i(x^*)} = \underbrace{\text{Max}}_{x \in X} \frac{N_i(x)}{D_i(x)}$$

Step4. Use the weighting method to convert each problems P_1, P_2, P_3 and P_4 to single objective in the form P_1', P_2', P_3' and P_4' respectively.

Step5. Find the optimal solution of each linear programming LP problem P_1', P_2', P_3' and P_4'.

Step6. Using the results of step5, obtain a rough efficient solution to the given MORLFP problem by the Theorem 3 and Theorem 4. with objective value:

$$Z_i^R(x^*) = [\,[Z_i^{-L}(x^*),\ Z_i^{+L}(x^*)] : [Z_i^{-U}(x^*),\ Z_i^{+U}(x^*)]\,]$$

$$for\ all\ i = 1,2 \dots, k$$

The algorithm is illustrated with the following example.

5. Numerical example:

Consider the following MORLFP problem:

$$\text{Max } Z_1(x) = \frac{[[1.5,\ 2.5] : [1,3]]x_1 + [[2.5,\ 3.5] : [2,\ 4]]x_2}{[[1,\ 2] : [0.5,\ 3]]x_1 + [[2,\ 3] : [1,\ 5]]x_2 + [[2,\ 5] : [1,\ 7]]},$$

$$Z_2(x) = \frac{[[2,\ 4] : [1,5]]x_1 + [3,\ 5] : [2,\ 6]]x_2}{[[3,\ 5] : [1,\ 7]]x_1 + [[2,5] : [1,\ 6]]x_2 + [[2,\ 3] : [1,\ 4]]}$$

Subject to:

$$x_1 + x_2 \leq 5, \quad 3x_1 + x_2 \leq 10$$
$$2x_1 + x_2 \leq 7, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0$$

Now the decomposition problem of the given MORLFP problem as in the following form:

$$Max\left\{ z_1^{+U}(x) = \frac{3x_1+4x_2}{0.5x_1+x_2+1},\ z_2^{+U}(x) = \frac{5x_1+6x_2}{x_1+x_2+1} \right\}$$
$$Max\left\{ z_1^{-U}(x) = \frac{x_1+2x_2}{3x_1+5x_2+7},\ z_2^{-U}(x) = \frac{x_1+2x_2}{7x_1+6x_2+4} \right\}$$
$$Max\left\{ z_1^{+L}(x) = \frac{2.5x_1+3.5x_2}{x_1+2x_2+2},\ z_2^{+L}(x) = \frac{4x_1+5x_2}{3x_1+2x_2+2} \right\}$$
$$Max\left\{ z_1^{-L}(x) = \frac{1.5x_1+2.5x_2}{2x_1+3x_2+5},\ z_2^{-L}(x) = \frac{2x_1+3x_2}{5x_1+5x_2+3} \right\}$$

Subject to :

$$x_1 + x_2 \leq 5, \quad 3x_1 + x_2 \leq 10$$
$$2x_1 + x_2 \leq 7, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0$$

Now construct the four problems and solving as follows :

$\mathbf{P_1}$**:**

$$Ma\left\{ z_1^{+U}(x) = \frac{3x_1+4x_2}{0.5x_1+x_2+1},\ z_2^{+U}(x) = \frac{5x_1+6x_2}{x_1+x_2+1} \right\}$$

Subject to:

$$x_1 + x_2 \leq 5, \quad 3x_1 + x_2 \leq 10$$
$$2x_1 + x_2 \leq 7, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0$$

It is observed that $0 \leq z_1^{+U} \leq 3.71$ and $0 \leq z_2^{+U} \leq 5$.

This MOLFP problem is equivalent to the following LP problem can be written as:

$\boldsymbol{P_1'}$**:**

$$Max\ \{\omega_1(3x_1 + 4x_2 - 3.71(0.5x_1 + x_2 + 1)) + \omega_2(5x_1 + 6x_2 - 5(x_1 + x_2 + 1))\}$$

Subject to :

$$x_1 + x_2 \leq 5, \quad 3x_1 + x_2 \leq 10$$
$$2x_1 + x_2 \leq 7, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0$$

For $\omega_1 = \omega_2 = 0.5$

The optimal solution of the LP problem P_1' is obtained as: $x_1{}^* = 0$, $x_2{}^* = 5$

The efficient solution of MOLFP problem P_1 are: $x_1{}^* = 0$, $x_2{}^* = 5$ with objective value . $z_1^{+U} = 3.33$, $z_2^{+U} = 5$.

$\boldsymbol{P_2}$**:**

$$Max\left\{ z_1^{-U}(x) = \frac{x_1+2x_2}{3x_1+5x_2+7},\ z_2^{-U}(x) = \frac{x_1+2x_2}{7x_1+6x_2+4} \right\}$$

Subject to:

$$\frac{x_1+2x_2}{3x_1+5x_2+7} \leq 3.33, \quad \frac{x_1+2x_2}{7x_1+6x_2+4} \leq 5,$$
$$x_1 + x_2 \leq 5, \quad 3x_1 + x_2 \leq 10$$
$$2x_1 + x_2 \leq 7, \quad x_1 \leq 3, \quad x_1, x_2 \geq 0$$

It is observed that $0 \leq z_1^{-U} \leq 0.31$ and $0 \leq z_2^{-U} \leq 0.29$.

This MOLFP problem is equivalent to the following LP problem can be written as:

$\boldsymbol{P_2'}$**:**

$Max\,\{\omega_1(x_1+2x_2-0.31(3x_1+5x_2+7))+\omega_2\,(x_1+2x_2-0.29(7\,x_1+6x_2+4))\}$

Subject to: $\frac{x_1+2x_2}{3x_1+5x_2+7} \le 3.33$, $\frac{x_1+2x_2}{7x_1+6x_2+4} \le 5$

$x_1+x_2 \le 5,\quad 3x_1+x_2 \le 10$

$2x_1+x_2 \le 7,\quad x_1 \le 3\,,\quad x_1,\ x_2 \ge 0$

For $\omega_1=\omega_2=0.5$

The optimal solution of the LP problem P_2' is obtained as: $x_1{}^*=0$, $x_2{}^*=5$

The efficient solution of MOLFP problem P_2 are:

$x_1{}^*=0$, $x_2{}^*=5$ with the objective value $z_1^{-U}=0.31$, $z_2^{-U}=0.29$.

$\boldsymbol{P_3}$ **:**

$Max\left\{z_1^{+L}(x)=\frac{2.5x_1+3.5x_2}{x_1+2x_2+2},\ z_2^{+L}(x)=\frac{4x_1+5x_2}{3x_1+2x_2+2}\right\}$

Subject to :

$$0.31 \le \frac{2.5x_1+3.5x_2}{x_1+2x_2+2} \le 3.33\,,$$

$$0.29 \le \frac{4x_1+5x_2}{3x_1+2x_2+2} \le 5\,,$$

$x_1+x_2 \le 5\,,\quad 3x_1+x_2 \le 10\,,$

$2x_1+x_2 \le 7,\quad x_1 \le 3\,,\quad x_1\,,x_2 \ge 0$

It is observed that $0 \le z_1^{+L} \le 1.57$ and $0 \le z_2^{+L} \le 2.08$.

This MOLFP problem is equivalent to the following LP problem can be written as:

$\boldsymbol{P'_3}$ **:**

$Max\,\{\omega_1(2.5\,x_1+3.5x_2-1.57(x_1+2x_2+2))+\omega_2\,(4x_1+5x_2-2.08(3\,x_1+2x_2+2))\},$

Subject to :

$$0.31 \le \frac{2.5x_1+3.5x_2}{x_1+2x_2+2} \le 3.33\,,$$

$$0.29 \le \frac{4x_1+5x_2}{3x_1+2x_2+2} \le 5\quad,$$

$x_1+x_2 \le 5\,,\quad 3x_1+x_2 \le 10\,,$

$2x_1+x_2 \le 7,\quad x_1 \le 3\,,\quad x_1\,,x_2 \ge 0$.

For $\omega_1=\omega_2=0.5$

The optimal solution of the LP problem P_3' is obtained as: $x_1{}^*=0$, $x_2{}^*=5$

The efficient solution of MOLFP problem P_3 are $x_1{}^*=0$, $x_2{}^*=5$, with objective value $z_1^{+L}=1.46$, $z_2^{+L}=2.08$

$\mathbf{P_4}$ **:**

$Max\left\{z_1^{-L}(x)=\frac{1.5x_1+2.5x_2}{2x_1+3x_2+5}\ ,\ z_2^{-L}(x)=\frac{2x_1+3x_2}{5x_1+5x_2+3}\right\}$

Subject to :

$$0.31 \le \frac{1.5x_1+2.5x_2}{2x_1+3x_2+5} \le 1.46\,,$$

$$0.29 \le \frac{2x_1+3x_2}{5x_1+5x_2+3} \le 2.08\,,$$

$x_1+x_2 \le 5\quad 3x_1+x_2 \le 10\,,$

. $2x_1+x_2 \le 7,\quad x_1 \le 3\,,\quad x_1\,,x_2 \ge 0$

It is observed that $0 \le z_1^{-L} \le 0.625$ and $0 \le z_2^{-L} \le 0.54$.

This MOLFP problem is equivalent to the following LP problem can be written as:

$\boldsymbol{P'_4}$:

$Max\,\{\omega_1(1.5\,x_1+2.5x_2-0.625(2x_1+3x_2+5))+\omega_2(2x_1+3x_2-0.54(5\,x_1+5x_2+3))\}$

Subject to :

$$0.31 \le \frac{1.5x_1+2.5x_2}{2x_1+3x_2+5} \le 1.46\,,$$

$$0.29 \le \frac{2x_1+3x_2}{5x_1+5x_2+3} \le 2.08\ ,$$

$x_1+x_2 \le 5\,,\quad 3x_1+x_2 \le 10\,,$

$2x_1+x_2 \le 7,\quad x_1 \le 3\,,\quad x_1\,,x_2 \ge 0$

For $\omega_1=\omega_2=0.5$

The optimal solution of the LP problem P_4' is obtained as: $x_1{}^*=0$, $x_2{}^*=5$

The efficient solution of MOLFP problem $P_4$are: $x_1{}^*=0$, $x_2{}^*=5$, with objective value $z_1^{-L}=0.625$, $z_2^{-L}=0.54$.

The rough efficient solution of original MORLFP problem is $x_1{}^*=0$, $x_2{}^*=5$ with the rough objective value

$z_1^{\mathrm{R}}=[[0.625\,,\ 1.46]\ :\ [0.31\,,\ 3.33]]\,,$

$z_2^{\mathrm{R}}=[[0.54\,,\ 2.08]\ :\ [0.29\,,\ 5]]\,.$

6. Conclusion

A new approach is proposed for solving multiobjective linear fractional programming problems with rough coefficients (MORLFP) problem. For treating the problems use the method of Effati and Pakdaman to convert the MORLFP problem into four multi objective linear fractional programming MOLFP problems. By the method of Dinkelbach, the MOLFP problems is convert to linear programming LP problems . An algorithm is established for characterizing the solutions concept of MORLFP problems .

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الملخص العربي :

في هذه الورقة تناولنا طريقة جديده لحل المشاكل الامثليه الخطيه الكسريه متعددة الاهداف حيث تكون معاملات دوال الهدف Rough intervals تتلخص الطريقة في تحويل المشكلة الامثلية المعطاه الي اربعة مشاكل امثليه خطيه كسرية متعدده الاهداف في صوره ابسط حيث تكون معاملات دوال الهدف اعداد حقيقيه . استعرضنا بعض التعريفات والنظريات واقترحنا خوارزمية لايجاد الحل الامثل للمشكله واعطينا مثال عددي من اجل التوضيح.