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ATTAINABILITY TECHNIQUE BASED ON POLYHEDRAL APPROACH: APPLICATION TO HYBRID TEMPERATURE CONTROL SYSTEM

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This paper presents a new attainability technique based on a polyhedral approach for hybrid systems. The main aim of the proposed technique is to obtain a robust-control closed loop system with attainable behavior under all possible and allowable disturbances. Moreover, and in contrast to existed approaches, this technique considers a new switching mechanism that allows the exploration of all different sub-models of piecewise affine systems that represent the interaction between continuous and discrete parts in the hybrid systems. The proposed technique is applied to a temperature control system to examine its ability in providing a reliable attainability.

1. INTRODUCTION

The natures of many current industrial systems that incorporate heterogeneous dynamical systems of both continuous and discrete systems have made the use of hybrid systems of special interest to many researchers. This trend of using hybrid systems is ascending and can be found in many control applications such as control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control. Various approaches have been proposed to model hybrid systems (Branicky *et al*., 1998), such as Automata, Petri nets, Linear Complementary (LC), Piecewise Affine (PWA) (Sontag, 1981), Mixed Logical Dynamical (MLD) models (Bemporad and Morari, 1999a). Different techniques are used to control hybrid systems, for example Model Predictive Control (MPC) (Schutter and van den Boom, 2004; Thomas *et al*, 2003; Olaru *et al*, 2003; Olaru *et al*, 2004) and optimal control (Bemporad, and Morari, 1999a).

However, two main challenging areas in the above field are still open for research namely dealing with hybrid systems subject to uncertainties (parameters or disturbance), and switching between operating modes these include problems like safety, reachability, attainability and robust control.

This paper presents a polyhedral approach that generates the state space regions for which a robust control drives the plant to a desired behavior despite the possible disturbances. This is achieved mainly by adopting the attainability concept which determines a sequence of admissible control laws such that a certain region sequence can be followed. This concept is mainly based on backward reachability and

1511

safety analysis that will be also considered. Moreover, this paper introduces a new technique that allows the control system to switch between different operating modes. This approach is introduced to a temperature control system to examine its efficiency in reaching certain target spaces.

2. PIECEWISE AFFINE SYSTEMS SUBJECT TO BOUNDED DISTURBANCES

The main characteristic of this class is that the continuous dynamics are described by linear difference equations, the discrete dynamics by finite automata, and the interaction between the continuous and the discrete part is defined by piecewise linear maps. Piecewise affine systems are powerful tools for describing or approximating both nonlinear and hybrid systems, and represent a straightforward extension from linear to hybrid systems. First, we present some basic and necessary notations that are used in the modeling formalism.

$$P^i : \left\{ \mathbf{x}_{k+1} = \mathbf{A}^i \mathbf{x}_k + \mathbf{B}^i \mathbf{u}_k + \mathbf{E}^i \mathbf{d}_k^i + \mathbf{f}^i \right\}, \text{ For } \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \in \chi_i \qquad (1)$$

where $\mathbf{x}_k \in \mathrm{X}$, is the state, $\mathbf{u}_k \in \mathrm{U}$, is the input, and $\mathbf{d}_k^i \in \mathbf{D}$, is the disturbance vector at instant *k for the* i^{th} model) while **X**, **U**, **D** denote polytopes, and $\mathbf{A}^i$, $\mathbf{B}^i$, $\mathbf{E}^i$ are real matrices of appropriate dimensions and $\mathbf{f}^i$ is a real vector for all $i = 1 \cdots p$. In the meantime, $\left\{ \chi_i^p \right\}$ is the polyhedral coverage of the state and input spaces $\mathbf{X} \times \mathbf{U}$, *p* being the number of subsystems. Each χ_i is given by

$$\chi_i = \left\{ \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \middle| \; \mathbf{Q}^i \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} \le \mathbf{q}^i \right\} \qquad (2)$$

Each subsystem P^i defined by $\left\{ \mathbf{A}^i, \mathbf{B}^i, \mathbf{C}^i, \mathbf{f}^i, \mathbf{Q}^i, \mathbf{q}^i \right\}, i \in I = (1, 2, \cdots, p)$ is a component of the global hybrid system where *I* is the collection of all subsystems $\mathbf{A}^i \in \Re^{n \times n}$, $\mathbf{B}^i \in \Re^{n \times m}$, $\mathbf{C}^i \in \Re^{n \times r}$, $\mathbf{Q}^i \in \Re^{s_i (n+m)}$ and $\mathbf{q}^i \in \Re^{s_i}$ is a suitable constant vector, where *n*, *m*, *r* are respectively the dimension of state, input and disturbance vectors, and s^i is the number of hyperplanes defining the χ_i polyhedral.

It should be mentioned that the sets $\left\{ \chi_i^p \right\}$ are assumed to be not disjoint in the sense that the desired model dynamics can be chosen by the bias of switching decision variables. This model may has logical inputs that are considered by developing an affine model (Equation 1) for each input value (1/0), defining linear inequality constraints linking the model with relevant input (Equation 2).

3. DIRECT REACHABILITY, SAFETY, AND ATTAINABILITY: A POLYHEDRAL APPROACH

This section presents the main objective of this paper where a new technique based on a polyhedral approach is developed to calculate the reachable set of states for the considered class. This technique is also used to examine the reachability, safety and attainability properties for the given region of states.

3.1 Reachability

This is described by considering the target region in the global state space **X**, which is $\mathbf{R}_k$, $k > 1$. The reachability concept can be easily defined in terms of the robust one-step control region $\mathbf{R}_{k-1}$ as the region in the state space for which there exists a feasible mode and an admissible control signal that is able to take the states from $\mathbf{R}_{k-1}$ to $\mathbf{R}_k$ in one-step despite all possible disturbance, i.e

$$\mathbf{R}_{k-1} = \left\{ \begin{array}{l} \mathbf{x}_{k-1} \in \mathbf{X} \,\middle|\, \exists\, i \wedge \mathbf{u}_{k-1} \in \mathbf{U}, \begin{bmatrix} x_{k-1} \\ u_{k-1} \end{bmatrix} \in \chi_i \\ s.t. \\ \mathbf{A}^i \mathbf{x}_{k-1} + \mathbf{B}^i \mathbf{u}_{k-1} + \mathbf{C}^i \mathbf{d}^i_{k-1} + \mathbf{f}^i \in \mathbf{R}_k, \forall \mathbf{d}^i_{k-1} \in \mathbf{D} \end{array} \right\} \tag{3}$$

This region can be calculated using the following polyhedral approach. First, consider the global state space defined by the following constraints:

$$\mathbf{X} := \left\{ \mathbf{F}_s \mathbf{x} \le \mathbf{g}_s, \mathbf{F}_s \in \Re^{s \times n}, \mathbf{g}_s \in \Re^{s} \right\} \tag{4}$$

while the control input supposed to be bounded :

$$\mathbf{U} := \left\{ \mathbf{m}\mathbf{u} \le \mathbf{n},\ \mathbf{m} \in \Re^{s \times m},\ \mathbf{n} \in \Re^{s \times 1} \right\} \tag{5}$$

Let the target region, which the system states have to go into it, is defined by the following constraints:

$$\mathbf{R_k} := \left\{ \mathbf{F}\mathbf{x_k} \le \mathbf{g} \right\} \tag{6}$$

Considering the i^{th} model and using Equation 1, the region $\mathbf{R}^i_{k-1}$ for the model i from which the states can be driven in only one step to the target region $\mathbf{R}_k$ despite any possible existing disturbance $d^i_{k-1} \in D$, can be defined as following:

$$\mathbf{R}^i_{k\text{-}1} = \left\{ \begin{array}{l} \mathbf{x}_{k-1} \in \Re^n \,\middle|\, \forall \mathbf{d}^i_{k-1} \in \mathbf{D}, \mathbf{F}(\mathbf{A}^i \mathbf{x}_{k-1} + \mathbf{B}^i \mathbf{u}_{k-1} + \mathbf{E}^i \mathbf{d}^i_{k-1} + \mathbf{f}^i) \le \mathbf{g}, \\ s.t. \quad \mathbf{Q}^i \begin{bmatrix} \mathbf{x}_{k-1} \\ \mathbf{u}_{k-1} \end{bmatrix} \le \mathbf{q}^i, \quad \left\{ \mathbf{F_s} \mathbf{x}_k \le \mathbf{g_s} \right\} \end{array} \right\} \tag{7}$$

The presence of the disturbances can be ignored in the first step, this leads to the following set

$$\tilde{\mathbf{R}}^i_{k\text{-}1} = \left\{ \begin{bmatrix} \mathbf{FA}^i & \mathrm{F}\mathbf{B}^i \\ \mathbf{F_s} & \mathbf{0} \\ \multicolumn{2}{c}{\mathbf{Q}^i} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k-1} \\ \mathbf{u}_{k-1} \end{bmatrix} \le \begin{bmatrix} \mathbf{g - Ff}^i \\ \mathbf{g_s} \\ \mathbf{q}^i \end{bmatrix} \right\} \tag{8}$$

and to the expression of the maximal admissible region for the mode *i* in the absence of the disturbances can be found by making a projection on the state space dimensions as following:

$$\hat{\mathbf{R}}^i_{k\text{-}1} = \mathrm{Pr}_{\mathbf{X}}\, \tilde{\mathbf{R}}^i_{k\text{-}1} \tag{9}$$

The projection of polyhedral sets can be efficiently handled in a double representation (generators/constraints) and related tools can be found as for example - POLYLIB (Wilde, 1994) and The Multi-Parametric (MPT) Toolbox (Kvasnica *et al*., 2004).

For having a robust control strategy against any possible disturbances within the allowable disturbances for mode *i*, Equation (9) can be modified to be:

$$\mathbf{R}^i_{k\text{-}1} = \hat{\mathbf{R}}^i_{k\text{-}1} - \mathbf{C}^i\mathbf{D} \tag{10}$$

where the subtraction is computed considering an exact geometric operation. The set $\mathbf{C}^i\mathbf{D}$ is the image of $\mathbf{D}$ by the linear mapping

$$f(\mathbf{d}) = \mathbf{C}^i\mathbf{d} \tag{11}$$

The global one-step robust controllable regions of states in the state space under all modes are thus given by:

$$\mathbf{R}_{k-1} = \bigcup_{i=1}^{p} \mathbf{R}^i_{k-1} \tag{12}$$

The above procedures can be repeated in a recursive way to obtain the domain for any limited *N* steps horizon. Using a dynamic programming approach, after defining the target region $\mathbf{R}_{k+N}$, the state space domain $\mathbf{R}_k$ can be recursively calculated in a manner that includes all the states which have a feasible control policy that can in *N* step derived to the target region $\mathbf{R}_{k+N}$ despite all possible disturbances.

The other two properties crucial issues in this paper, namely safety (static specification) and attainability (dynamic specification) will be considered in the following sections.

3.2 Safety

This section addresses the safety specification which is classified as a static specification. The region is called to be safe (control invariant) when the evaluation of the system states inside this region will not go out of this region. A well-known geometric condition for a set to be safe (control invariant) is the following (Lin and Antsaklis, 2002):

the set $\mathbf{R}_{\mathrm{k}}$ *is safe if only and only if* $\mathbf{R}_{\mathrm{k+1}} \subseteq \mathbf{R}_{\mathrm{k}}$

3.3 Attainability

It is normally classified as dynamic specification of a control system. Given a finite number of regions $(\mathbf{R}_k, \mathbf{R}_{k+1}, \cdots, \mathbf{R}_{k+N}) \in I \times \chi$, the attainability for this sequence of regions is equivalent to the following two properties, namely the direct reachability from region $\mathbf{R}_{k+j}$ to $\mathbf{R}_{k+j+1}$ for $0 \le j \le N-1$ and the safety for the final region $\mathbf{R}_{k+N}$.

3.4 Region exploration and sub models switching

One of the main concerns in controlling PWA systems, with many sub-models *p* and long horizon *N*, is the identification of reachability regions. For simplicity many researchers are biased to consider "no switch" between sub-models over the *N* steps horizon (Figure 1), for example in (Koutsoukos and Antsaklis, 2003), but this leads to more conservatism solution. In this paper we adapt a different technique that considers all possible switching between models over the horizon N (Figure 2), which leads to the calculation of exact regions (non conservatism solution).

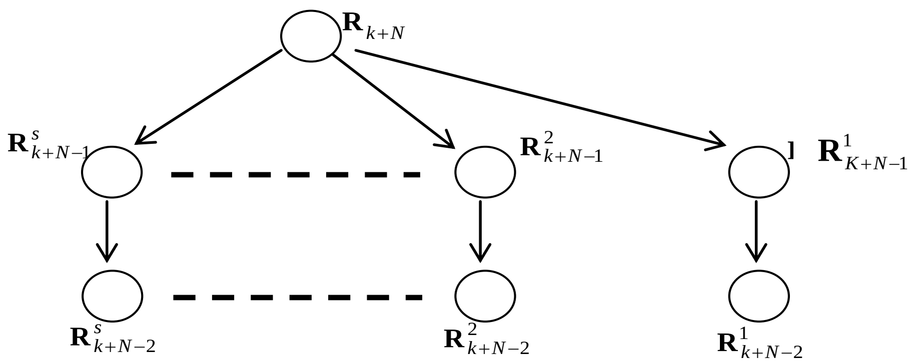

Figure 1: Exploration with no switch over the N steps.

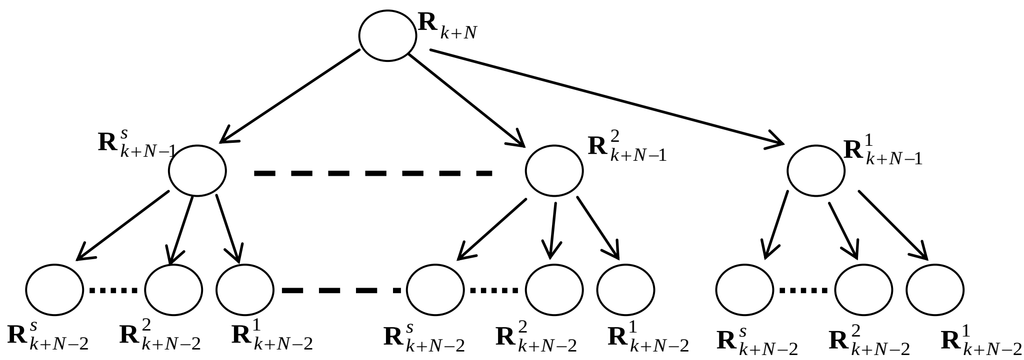

Figure 2. Complete regions exploration

4. TEMPERATURE CONTROL SYSTEM

To examine the ability of the proposed technique in achieving the above specifications, this algorithm is introduced to a temperature control system. The temperature is represented by using its analogous to electric voltage, heat quantity to current, heat capacity to capacitance, and thermal resistance to electrical resistance (Koutsoukos and Antsaklis, 2003). This system includes a furnace that can be switched on and off. The furnace has two different equivalent circuits based on the operation mode. In the on-mode (its equivalent electric circuit is shown in Figure 3), a continuous input controls the produced heat. The specific control objective is to control the temperature at a point B of the system by applying the heat input at a different point A. In the meantime, the environment temperature at point C affects the temperature at point B, which is the external disturbance that affects the system.

In this model, states x_1 and x_2 refer to voltages (temperatures) across the capacitors C_1 and C_2 which can be controlled by changing the current (heat) input u. The temperature x_2 is also affected by the environment temperature d which is modeled as a continuous disturbance. Applying the well known Kirchhoff's laws to Figure 3, the system can be represented in the following state-space form for mode q_1 (on-mode):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/R_1C_1 & 1/R_1C_1 \\ 1/R_1C_2 & -(R_1+R_2)/(R_1R_2C_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/C_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1/R_2C_2 \end{bmatrix} d \qquad (13)$$

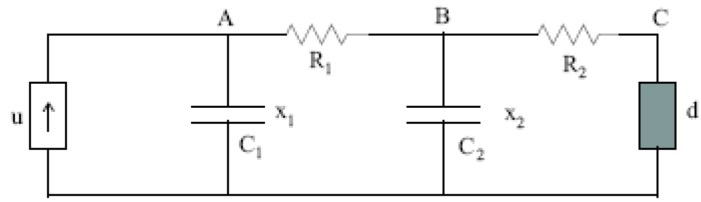

Figure 3: Electric circuit when the furnace is on.

Considering the second mode when the furnace is off, the temperature is decreasing and the behavior of the system can be described by the electrical circuit shown in Figure 4. The values of the resistors and the capacitors model the time constants of the system. The time constants are, in general, different depending on whether the temperature is increasing or decreasing. The state-space representation of the system for the mode q_0 (off-mode) takes the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/R_3C_3 & 1/R_3C_3 \\ 1/R_3C_4 & -(R_3+R_4)/(R_3R_4C_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/R_4C_4 \end{bmatrix} d \qquad (14)$$

The voltages (temperatures) x_1 and x_2 can be affected either by the continuous control input u or by switching on or off the furnace using the control input events B_{on} and B_{off}. For protecting a point A from over heating, it is assumed that if the temperature x_1 exceeds a prescribed level u_b the furnace would be switched off automatically using a relief switch

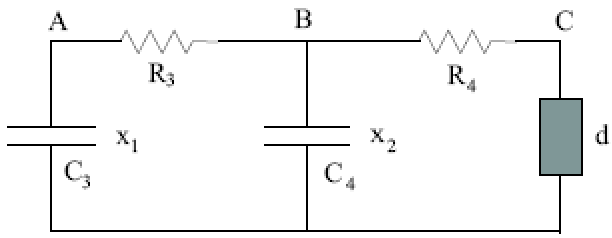

Figure 4: Electric circuit when the furnace is off

The control objective for the system is to maintain the temperature x_2 within predefined temperatures that described by the interval $[L_T; H_T]$.

4.1 Attainability Application

The proposed technique will be illustrated through the hybrid temperature control system. The values of the temperature control system are considered to be in the on-mode and off-mode as following:

$$R_1 = 2, R_2 = 1, C_1 = 1, C_2 = 1, , \; R_3 = 10, R_2 = 2, C_3 = 0.5, C_4 = 1,$$

Discretizing the system with sampling time T_s=1, the continuous system can be represented by the following difference equation, when the furnace is on (on-mode):

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(k)$$

Similarly in the off-mode (furnace off), the following difference equation :

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} d(k)$$

The global state space constraints can be given by:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \leq \begin{bmatrix} 40 \\ 30 \\ 20 \\ 30 \end{bmatrix}$$

The control signal is assumed to be bounded and defined by

$$0.5 \leq u \leq 5$$

The disturbance $\mathbf{d}$ is assumed to take values in the polyhedral and pounded sets $\mathbf{D}$; for on-mode $\mathbf{d} \in \mathbf{D_1} = [0,1]$, and for off-mode where the furnace is off $\mathbf{d} \in \mathbf{D}_0 = [-1,0]$.

Reachability test

In the first step, we will examine the reachability concept for the hybrid temperature control system considering regions P_1 and P_2, which are defined as follows

$$P_1 = \{x \in \Re^2 | (0 \le x_1 \le 20) \wedge (-20 \le x_2 \le 0)\},$$
$$P_2 = \{x \in \Re^2 | (0 \le x_1 \le 20) \wedge (0 \le x_2 \le 5)\}$$

In this step the main concern is to examine if every state in region P_1 can be driven to region P_2 in finite steps without entering a third region. To investigate this issue, we start from region P_2 applying the robust one-step reachability technique presented above in order to compute the set of states in the region P_1 that can be driven to P_2 using appropriate control inputs. The result for just one-step is presented in Figure 5, it is clear that one step is far from covering the region P_1, this indicates that only few states of region P_1 are able to enter region P_2 in only one step.

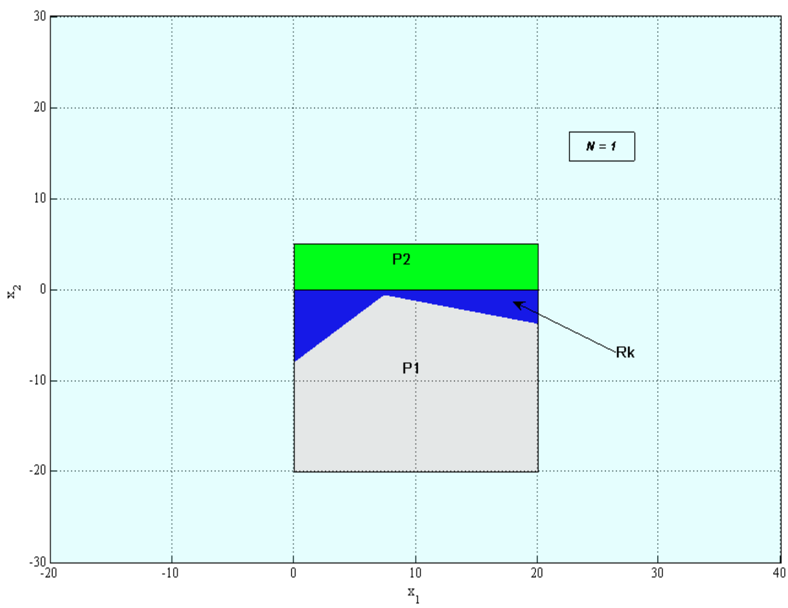

Figure 5: The computed set of states inside region P_1 for one step

For two steps ($N=2$) and three steps ($N=3$), the results are shown in Figure 6 and Figure 7 respectively. It can be easily seen that there is still a part of region P_1 which is not covered; this means that there are states in P_1 that cannot be driven to region P_2 in two or three steps.

We mention that the regions called R_k shown in Figures 5:9 are only the intersection part with P_1 i.e. $R_k \cap P_1$ and not the whole regions calculated according to the technique presented above, equation (12), as our interest in this example is to show which region of P_1 is cover in each step.

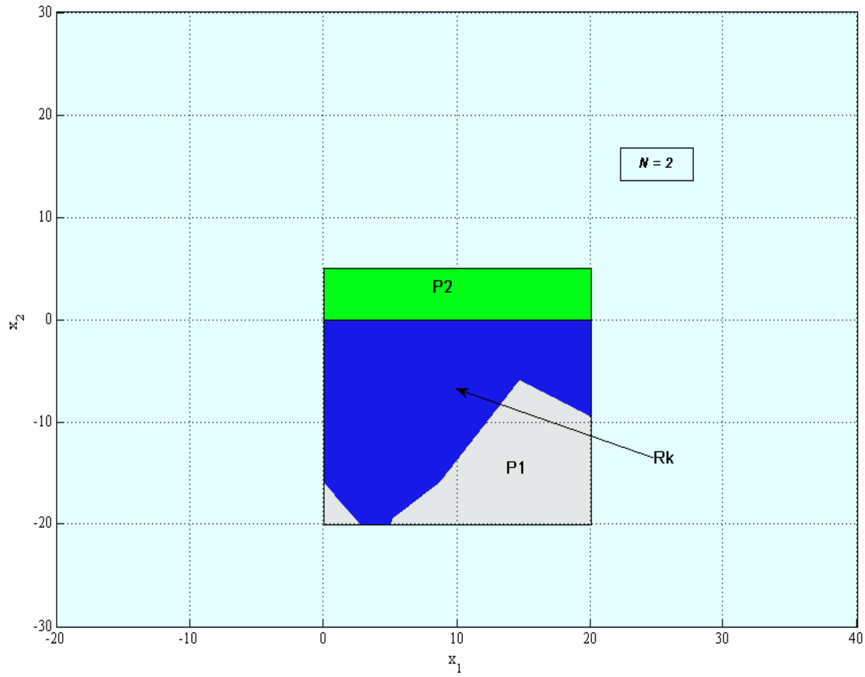

Figure 6: The computed set of states inside region P1 for two steps (*N*=2)

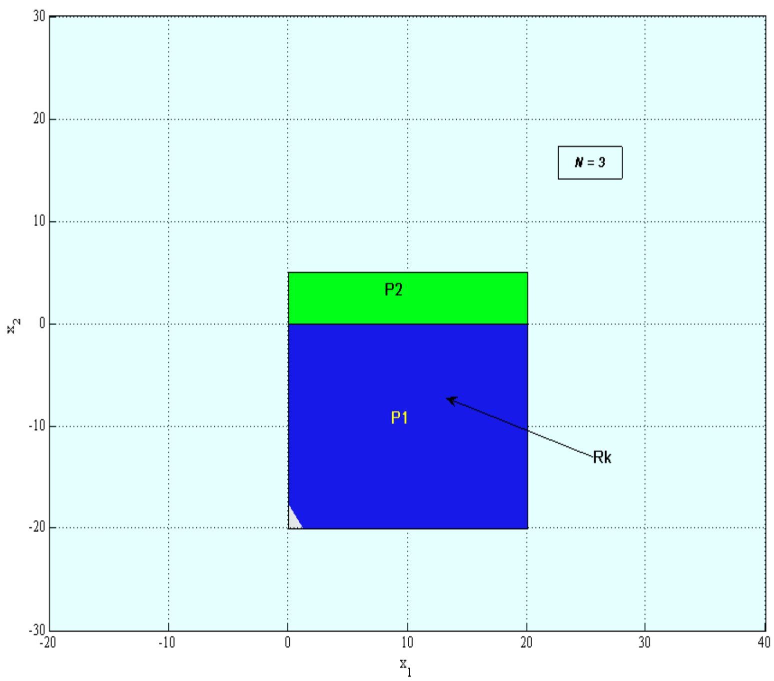

Figure 7: The computed set of states inside region P1 for three steps (*N*=3)

Figure 8 shows that after four iteration ($N = 4$) the computed set of states covers the region P_1, which indicates that all the states in P_1 can be driven to region P_2 in four steps without entering a third region using the allowable control signals. The global set of regions can be easily seen in Figure 9, where the progress and evolution of the developed regions from *N=1* to *N=4* are given.

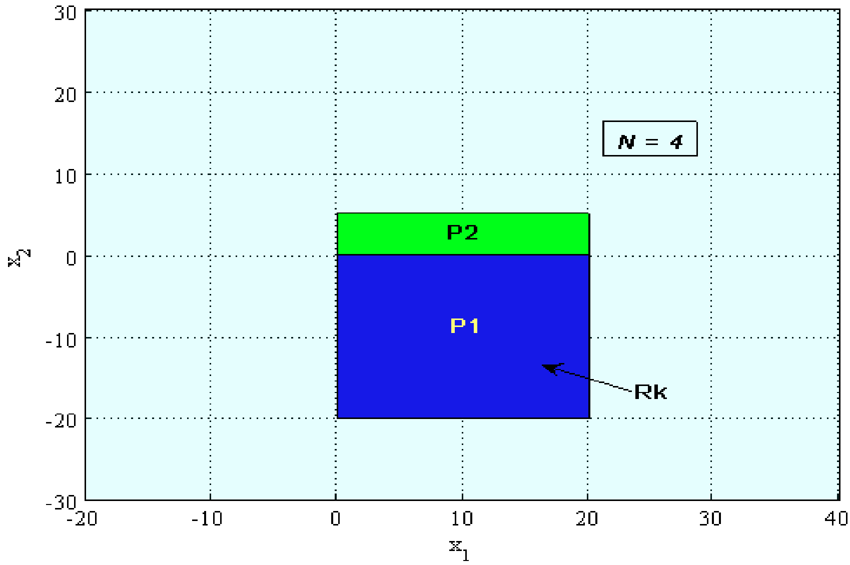

Figure 8: The computed set of states inside covering the region P1 in four steps (*N=4*)

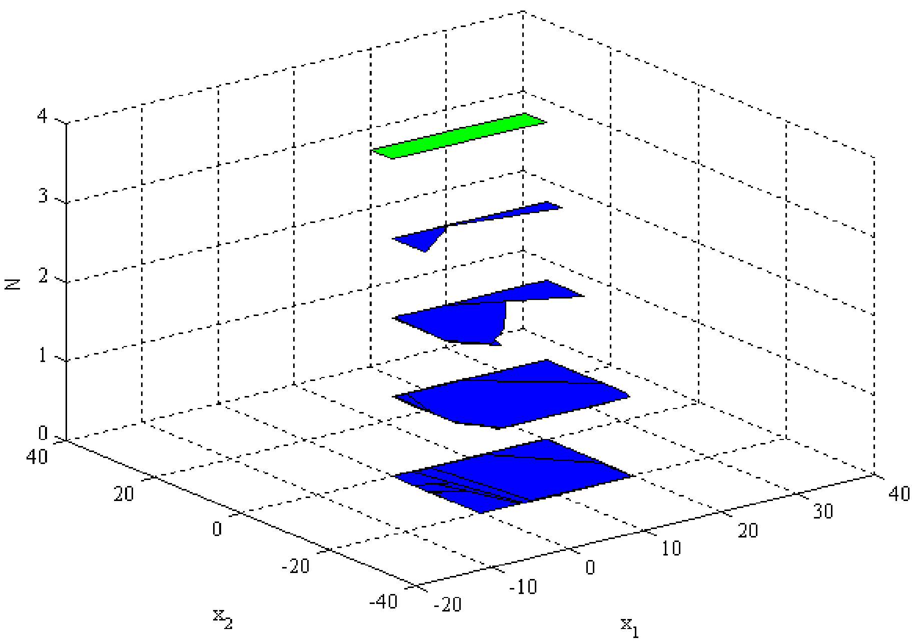

Figure 9: The reachable regions in four steps (*N=4*)

Safety test

In the second step, the safety property (control invariant) is investigated for the region P_2 (see: section 3.2). Figure 10 shows the result for just one-step, it is obvious that the region P_2 is not a safe (control invariant) region. There is a very small region (*R unsafe*) not covered which means that the condition $\mathbf{R}_{k+1} \subseteq \mathbf{R}_k$ is not valid in this case. However, examining the safety properties in two steps (see Figure 11) shows that $\mathbf{R}_{k+2} \subseteq \mathbf{R}_k$. This means that the states in region P_2 may go out of the region P_2 but it will return again to the same region in two steps, i.e. the region P_2 is safe in two steps. In conclusion the region P_2 is reachable from region P_1 in four steps ($N = 4$), and when the states reach the region P_2 it will be safe (control invariant), i.e. the states will rest inside P_2 for $N \geq 2$. The region P_2 is attainable from the region P_1. It should be mentioned that in contrast to the shown regions in Figure 5:9, in Figures10 and 11, the whole calculated regions are shown.

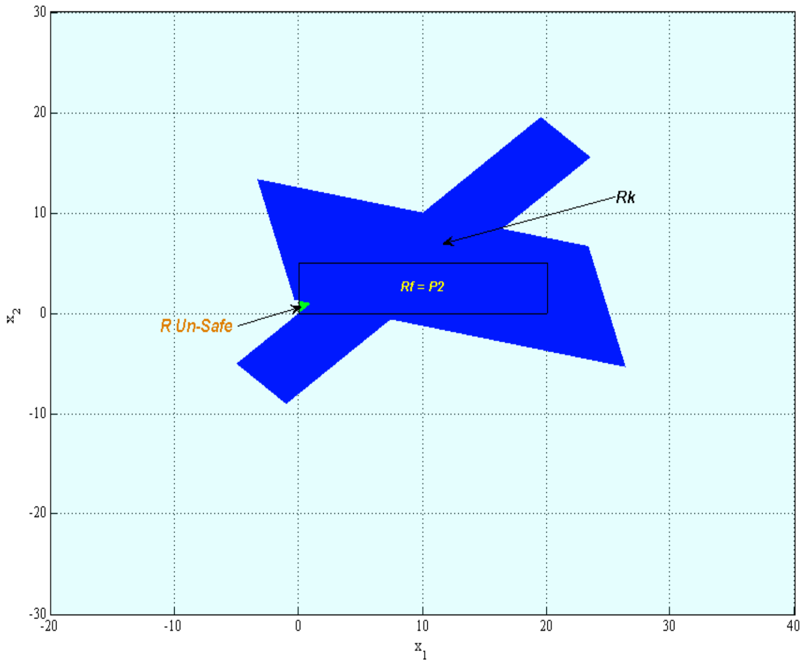

Figure 10: The safety property checking result, the region P2 is not safe.

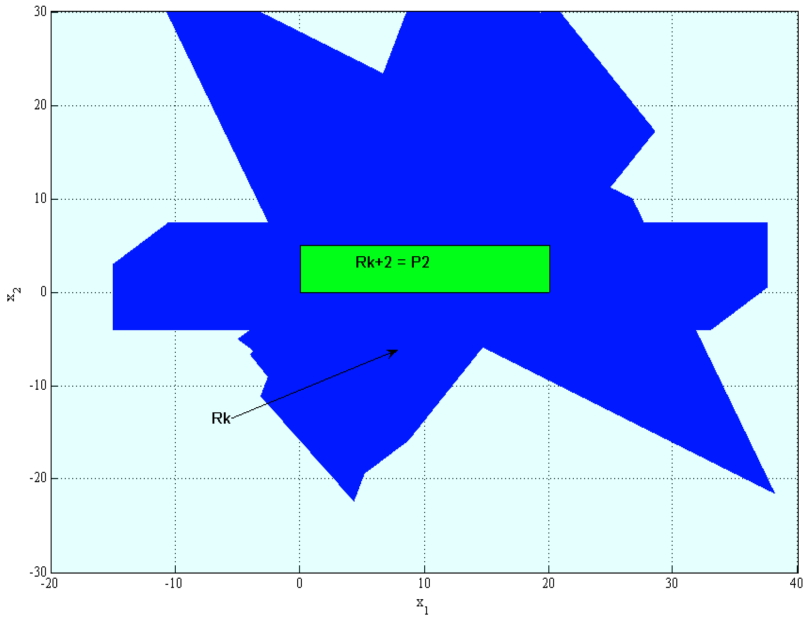

Figure 11: The safety property checking result for two steps, the region P_2 is safe for $N \geq 2$.

5. CONCLUSION

This paper has introduced a polyhedral approach to a temperature control system. The new technique has the ability to generate state space regions that have a robust control which can drive the system to a desired behavior despite the existing possible disturbances. The proposed technique has shown ability to examine the reachability, safety and attainability properties. This has been achieved in a fashion that allows the control system to switch between different operating modes rather than the conventional techniques which using the no-switch principle. In addition, based on this new technique a sequence of admissible control laws such that a certain region sequence can be followed could be developed.

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تقنية إحراز مبنية على نظم السطوح المتعددة: تطبيق على نظام تحكم حراري هجين

يقدم هذا البحث تقنية إحراز جديدة للنظم الهجينة مبنية على طريقة السطوح المتعددة. ويعتبر الهدف الرئيسي من هذه التقنية هو الحصول على نظم تحكم قوية ذات قدرة إحرازية ضد جميع احتمالات التشويش . بالإضافة إلى ذلك وعلى عكس تقنيات التصميم المتاحة في هذا المجال يقدم هذا البحث تقنية أخرى تتيح فرصة التبديل بين الأنظمة الفرعية المكونة للنظم الأصلية المصاحبة والمستخدمة في تمثيل العلاقات التفاعلية بين القطاعات المستمرة والرقمية في النظم الهجينة . وقد تم تطبيق هذه التقنية على نظام تحكم حراري لاختبار مدى قدرته على الوصول إلى نتائج موثوق فيها .