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صفحة

Toward a Consistent Estimate of the Substitutability between Money and Near Monies: An Application of the Fourier Flexible Form

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Abstract

Accurate estimates of elasticities of substitution require not only arbitrarily accurate approximations of the consumer's true indirect utility function, but also its derivatives. Because of their flexibility and consistency with both the theory of utility maximization and the theory of statistics, the truncated versions of the Fourier form (Gallant [1981]) are tractable for modeling consumer behavior in general and in yielding a consistent estimate of Allen's elasticity of substitution (AES) in particular. On the other hand, the problem of estimating the asymptotic standard errors of the AES is almost ignored in the literature, this may be because of the nonlinearity of the function or because of the substantial econometric sophistication required in their estimation. In this paper the methodology of computing the AES and its asymptotic standard errors for the entire sample period are presented.

Equally important is the issue to which we apply the Fourier Flexible form; this is the substitutability among monetary assets. We conclude, in the empirical part of this paper, that for the demand for liquid assets (a) the Fourier model satisfies the restrictions of consumer theory, and the elasticities of substitutions, (b) are generally low, and (c) suggest that "like assets" are closer substitutes than unlike assets. We also confirm the existence of "institutional loyalty" among the holders of liquid assets.

"Toward a Consistent Estimate of the Substitutability between Money and Near Monies; An Application of the Fourier Flexible Form" *

I. Introduction

Recently, the issue of money and near monies substitutability/complementarity relationship has attracted a great deal of attention in the literature (see Feige and Pearce [21] for a general review). An important reason for this interest is the poor performance of the traditional model of the demand for money; in particular it has been (apparently) unstable over the 70's. Even a recent redefinition of money by the Federal Reserve has not worked well and the (apparent) inability of the Federal Reserve to conduct a strong monetary policy has induced further pressure and has called for a re-examination of the traditional analytical models and the methodology of defining money. One reasonable explanation of why these models have failed to accurately capture recent experience in the money market is that a simple unweighted summation measurement of money generally has been used. In fact potentially numerous assets possess some degree of moneyness and therefore, a more reasonable approach to the definition of money is to regard monetary assets as joint products with different degrees of moneyness. Based upon the degree of moneyness the quantity of money then can be defined as the weighted sum of the aggregate value of all monetary assets (see Friedman and Schwartz [25]). When a quantitative measure of the degree of moneyness is required it is very obvious that an appropriate candidate is the elasticity of substitution.

A number of recent works have examined the substitutability among different monetary assets. They include direct estimates of the elasticity of substitution using some constrained flexible functional forms

(Offenbacher [34]) or unconstrained versions used to test for theoretical and functional form restrictions (Ewis-Fisher [20]); furthermore there is interesting and attractive work defining aggregate monetary assets by Barnett [2].(1) In the present paper we will restrict our review of the literature to the three papers mentioned, because of their common methodology--the use of the theory of consumer demand--and because of their employment of the same data base. Of course this also permits us to establish some comparative properties of the four approaches; these comparisons enable us to illustrate the characteristics of the present approach more clearly.

Using the translog functional form, Offenbacher [34] estimates the elasticity of substitution between currency, demand deposits, and time deposits. His main finding is of low substitutability over M2 components in general and he also finds that the substitutability between currency and time deposits is larger--in magnitude--than that between currency and demand deposits. Apart from the fact that imposing symmetry and linear homogeneity without testing it represent, essentially, unlikely and even unnecessary restrictions, his work is,nontheless, important because it suggests that we consider - among other empirical topics - the disaggregation of narrow money (M1) among our research agenda.

Barnett's [2] theoretical model of the monetary aggregates is innovative and is a very comprehensive study. In this work the monetary assets group is broken down into transactions balances, and a second group includes various types of passbook savings accounts (and time deposits at different institutions) for which an exact aggregator function is estimated. The main finding is of high substitutability among the various passbook savings accounts (and time deposits accounts) but low substitutability between transaction balances and the nested "like assets" group. In this study, Barnett also applies aggregation and index number theory to the selection and construction of money aggregates. He explores the implications of statistical index number theory for the construction of a monetary quantity index number and advocates the use of the Tornquist-Theil Divisia index to measure the quantity of money. His finding is that a simple sum index fails to internalize the long-run substitution effects that have occurred in the money markets during the past decade, and that either a Tornquist-Theil Divisia index or a Fisher Ideal index does. This work has been followed by a rapidly growing quantity of research on the use of aggregation and index number theory (see [3], [4], [5], [6], and [7]).

Estimates of a quantitative measure of the degree of substitutability of monetary assets in the context of a flexible functional form (with no undesirable restrictions) comprises the central part of the Ewis-Fisher [20] paper. In their model, instead of making assumptions (like symmetry-linear homogeneity) to justify the aggregation over individuals, the authors test for the existence of the representative utility function by testing for the integrability conditions. In so doing, they avoid any bias which could result from imposing restrictions that may be inappropriate.(2) The explicit derivation and treatment of technological change in the utility function and the extension of the model to include foreign monetary assets, in addition to the stochastic specification within a non-linear estimation technique, make their study distinct from the other two studies that are most similar in approach. Their findings, however, do not differ significantly from the main finding of Feige-Pearce [21] (and confirmed by Barnett [2] and many others). This is for low substitutability among the monetary assets. Three points need to be underscored about the Ewis-Fisher paper, however, to establish the point of departure for the present effort.

First, they actually obtained several cases in which symmetry was accepted statistically; this is not a common result and it does not generally obtain for highly aggregated studies of "consumer" behavior.(3) Second, aside from their main finding that confirmed the low substitutability among liquid assets, the elasticity of substitution between foreign monetary assets (currency) and all near monies is high (in magnitude). This has policy implications concerning the possibility of conducting an independent monetary policy (see Miles [33]).(4) Third, linear homogeneity and the Hicks-neutrality of monetary innovation were rejected as assumptions in all the empirical investigations; this suggested strongly that such restrictions, when imposed without testing, could easily produce bias. Nonetheless, this study shares with all the other studies the problem that it ignores the statistical aspects of the AES estimates; in particular there is no estimation of the standard errors of the elasticity of substitution. In the context of flexible functional forms the AES is a non-linear function of random variables and thus estimates of its standard errors are relatively difficult and require econometrically sophisticated techniques. Indeed, from the record, the cost of calculating such estimates seems to have been generally binding.(5)

Returning to the question of the monetary aggregates and the problem of measuring the degree of substitutability among alternative liquid assets one is immediately confronted by a choice between functional forms that exhibit good behavior globally and those that possess flexibility in that they impose no prior restrictions, especially on the behavior of the elasticity of substitution (see [29]). Certain simple forms, such as the CES, satisfy certain regularity conditions globally, but place unnecessarily stringent conditions on the possible values of the estimated AES; this is unfortunate since this is the main point of such work. Flexible functional forms-a Taylor expansion such as the translog for example--possess flexibility and impose no undesirable restrictions; Taylor's theorem, however, only applies locally. Indeed the local applicability of the approximation suffices to transform propositions from the theory of demand into restrictions on the parameters of the approximating expenditure system. However, Taylor's theorem fails to provide a satisfactory means of understanding the statistical behavior of parameter estimates and test statistics (Gallant [27, P. 212]). In contrast, the Fourier series expansion used in this paper permits a natural transition from demand theory to statistical theory.

In this paper, two main points are of concern. In the context of the flexible functional form method we wish to establish that to estimate a demand system and (hence) to obtain the elasticity of substitution one needs not only to approximate the true indirect utility function but also to approximate its first and second derivatives. In particular, the classical Fourier sine/cosine series expansion of the indirect utility function leads directly to an expenditure system (and an estimate of the elasticity of substitution) with the property that the average prediction bias may be made arbitrarily small by increasing the number of terms in the expansion. Thus, the first main point is to apply the Fourier flexible form based on the argument that it is the function that can capture the true indirect monetary services utility function. In this regard, we simplify and explore the computation of the function in an effort to help guide future research with this model. Furthermore, the integrability conditions will be tested--with special emphasis on the symmetry condition--rather than assuming the existence of the underlying utility function. As an issue, this is an important matter, as noted by Jorgenson and Lau [31, p. 118]:

> "If the integrability conditions are valid, then the theory of individual consumer behavior is applicable to the analysis of aggregate consumer demand functions in per capita form".

The second main point concerns a common problem in the literature; this is "how large is large" when we refer to the elasticity of substitution. For reference consider the following from Feige and Pearce [21, p. 463], where the monetary topic of our paper was surveyed:

> "While most of the studies assert that the degree of substitution between money and near-monies is an important empirical issue, none have to date included an analytical framework capable of telling us how large, for example, the cross-elasticity must be before the definition of money should be broadened ..."

In our view, we believe that any analytical framework capable of solving such a problem also has to explore the distinction between "statistical significance" and "economic significance". Before statistical significance is established, speaking of the economic significance of any estimate of the AES is utterly meaningless; estimating the standard errors of the AES is therefore essential.

The objective of this paper is to investigate the degree of substitutability between monetary assets using a flexible form that can globally approximate the parameters of the underlying indirect utility function to within an arbitrarily small degree of bias and yield a consistent estimate of the AES. In the empirical model, the specification of the Fourier indirect utility function is argued to be the most accurate approximation of the true indirect utility function of consumer's monetary services. Within this model a consistent estimate of the elasticity of substitution is obtained and a method for computing the AES asymptotic standard errors is analyzed.

II. The Model

When one's goal is to measure the degree of substitutability between money and near monies, deriving a demand function for each from traditional consumer theory is the most attractive and most frequently used approach. The resulting method has the advantage of unifying the theoretical model of the demand for all monetary assets including money, without the need to explore how the household's decision has been made in the financial markets. To begin, then, we assume that assets in the portfolio may have non-pecuniary characteristics (i.e. liquidity, safety, convenience. . .); in particular, we assume that the flow of commodities consumed and the holding of real and financial assets provide utility to the household. A convenient approach is the following: rather than introducing all these service streams (pecuniary and non-pecuniary) explicitly into the analysis, they can be included implicitly by allowing utility to be a function of the holdings of assets.

The maximization problem of the representative household can be simplified to the following one-period maximization:[6]

$$\max_{\underline{C},\underline{Q}} U_t = U_t\,(C_1, C_2, \ldots, C_M, q_1, q_2, \ldots, q_N), \tag{1}$$

$$(t = 1, 2, \ldots, n \quad j = 1, 2, \ldots, M,\ i = 1, 2, \ldots, N)$$

Subject to

$$\sum_{j=1}^{M} P_j C_j + \sum_{i=1}^{N} P_i q_i = W_t \tag{2}$$

where U is assumed to satisfy the regularity conditions, C is an m-vector of quantities consumed in period t, q is an N-vector of quantities of monetary assets held in period t, W_t is the wealth constraint, and P_j is the j^{th} commodity prices (j=1,2,...,M). P_i is the price of the i^{th} monetary assets evaluated as[7]

$$P_{it} = \frac{R_t - r_{it}}{1 + R_t} \quad (3)$$

which denote the discounted interest foregone by holding a dollar's worth of the i^{th} asset, r_i is the market yield of the i^{th} monetary asset, and R is the yield available on a "benchmark" asset that is held only as a pure store of wealth. The "benchmark" asset is assumed to provide no monetary services (i.e. has a non-pecuniary return). It is held not because of its liquidity or other characteristics, but because it facilitates transferring wealth between periods in a general multi-period planning horizon (see Barnett [1]).

Explicit consideration of the commodities sector or the labor/leisure (and hence human capital) decision is, of course, beyond the scope of the current paper. Instead, a simplification is needed to specify the problem in its final form; this is known as the multistage maximization procedure. To permit the construction of a demand system involving only the opportunity costs and quantities of monetary assets, the utility function in (1) is assumed to be functionally separable in the monetary assets; then the individual's choice of the q's is the result of the second stage of a two-stage maximization. In the first stage, the consumer selects aggregate monetary assets expenditure and aggregate consumer goods expenditure for period t. The second-stage allocation decision over individual current period monetary assets is then to

$$\text{Max } U_t = U(q_{1t}, q_{2t}, \ldots, q_{Nt}), \quad (4)$$

$$i=1,2,\ldots,N,$$
$$t=1,2,\ldots,n.$$

Subject to

$$\sum_i^N P_{it} q_{it} - M_t = 0, \quad (5)$$

in which M_t is the value of the flow of monetary services.

The problem of maximizing the monetary services utility function (4) subject to the expenditure constraint (5) is more fruitfully analyzed using duality theory. If U(.) is assumed to be monotonically increasing, twice differentiable and quasi-concave, then there exists an indirect monetary services utility function,

$$g^* (x) = g(x_1, x_2, \ldots, x_N), \quad (6)$$

which is twice differentiable, strictly decreasing and quasi-convex in the x_i, where x_i is the income normalized price, (P_i/M). To put it differently, define an N x 1 normalized price vector x = P/M. The indirect utility function $g^*(x)$ corresponding to the maximization problem (4) - (5) is defined as

$$g^* (x) \equiv \text{Max } \{U(q); \; x^t q < 1; \; q > 0_n\} \quad (7)$$

with respect to q

Assuming that the direct monetary services utility function satisfies certain regularity conditions, then $g^*(x)$ will satisfy corresponding regularity conditions (let $g^*(x)$ denote the consumer's true indirect utility function hereafter).

Deriving the share equations of the demand system require application of Roy's Identity (Roy [35, p. 222]). According to this theorem, the consumer's utility is maximized when expenditures are allocated according to the expenditure system (Roy's Identity),

$$p_i X_i / M = [\sum_{i=1}^{N} x_i (\partial/\partial x_i) g^*(x)]^{-1} x_i (\partial/\partial x_i) g^*(x), \quad (i=1,2,\ldots,N). \tag{8}$$

It is assumed that $g^*(x)$ has continuous partial derivatives and that

$$(\partial/\partial x_i) g^*(x_i) < 0, \tag{9}$$

for all $x \in \bar{\chi}$ where χ is the region of approximation; the overbar denotes closure of a set.

III. Specification of the Fourier Flexible Form

A familiar method for obtaining an expenditure system for empirical work is to set forth an indirect utility function $g(x)$ which is thought to adequately approximate $g^*(x)$ and then apply Roy's Identity,

$$p_i X_i / M = [\sum_{i=1}^{N} x_i (\partial/\partial x_i) g(x)]^{-1} x_i (\partial/\partial x_i) g(x), \quad (i=1,2,\ldots,N) \tag{10}$$

to obtain the approximating expenditure system. One can see from Roy's Identity that if this approach is to succeed it is actually the partial derivatives of the indirect utility function which need to be accurately approximated by the partial derivatives $(\partial/\partial x_i)\, g(x)$ and not just the function $g(x)$. A global approximation over χ is said to be provided by a Fourier approximation (see Gallant [27]). When the Fourier flexible form is chosen the resulting expenditure system has a feature which distinguishes it from other flexible form expenditure systems. When estimated, it will approximate the true expenditure system to within an average prediction bias which may be made arbitrarily small by increasing the number of terms in the Fourier expension (verification of this argument is in Gallant [27, Section 4.]).

The Fourier flexible form of an indirect utility function may be written as

$$g_K(x) = a_0 + b'x + \tfrac{1}{2}x'Cx + \sum_{\alpha=1}^{A}\sum_{j=-J}^{J} a_{j\alpha} e^{ij\kappa'_\alpha x} \tag{11}$$

where

$$C = -\sum_{\alpha=1}^{A} a_{0\alpha}\kappa_\alpha\kappa'_\alpha, \tag{12}$$

$$a_{j\alpha} = \bar{a}_{-jx}$$

However, in an empirical investigation it is more convenient to work with a sine/cosine representation than with the exponential representations mentioned above. Therefore, an equivalent form may be obtained; setting

$$a_{0\alpha} = u_{0\alpha}, \qquad (\alpha=1,2,\ldots,A),$$

$$a_{j\alpha} = u_{j\alpha} + iv_{j\alpha}, \qquad (j=1,2,\ldots,J), \tag{13}$$

$$a_{-j\alpha} = u_{j\alpha} - iv_{j\alpha};$$

where i is the imaginary unit, we then have

$$g_K(x) = u_0 + b'x + \tfrac{1}{2}x'Cx$$

$$+ \sum_{\alpha=1}^{A}\{u_{0\alpha} + 2\sum_{j=1}^{J}[u_{j\alpha}\cos(j\kappa'_\alpha x) - v_j \sin(j\kappa'_\alpha x)]\} \tag{14}$$

where

$$C = -\sum_{\alpha=1}^{A} u_{0\alpha}\kappa_\alpha\kappa'_\alpha. \tag{15}$$

The derivatives of $g_K(x)$ are

$$(\partial/\partial x)\, g_K(x)$$

$$= b + Cx - 2 \sum_{\alpha=1}^{A} \sum_{j=1}^{J} j[u_{j\alpha} \sin(jk_\alpha' x) + v_{j\alpha} \cos(jk_\alpha' x)]\, k_\alpha \, . \qquad (16)$$

$$(\partial^2/\partial x \partial x')\, g_K(x)$$

$$= - \sum_{\alpha=1}^{A} \{u_{0\alpha} + 2 \sum_{j=1}^{J} j^2 [u_{j\alpha} \cos(jk_\alpha' x) - v_{j\alpha} \sin(jk_\alpha' x)]\}\, k_\alpha k_\alpha' \, . \qquad (17)$$

where k is a multi-index.

The construction of a sequence of elementary multi-indexes

$$K_N^* = \{k_\alpha : \alpha=1,2,\ldots,A\} \qquad (18)$$

is explained in [27,p. 215] and can be reproduced as follows. Let

$$K_N^* = \{k : |k|^* \leq K\}, \qquad (19)$$

be the set of multi-indexes of dimension N and length $|k|^* = \sum_{i=1}^{N} |k_i| \leq K$. First, delete from K_N^* the zero vector and any k whose first non-zero element is negative. Second, delete any k whose elements have a common integral divisor. Third, arrange the k which remains into a sequence,

$$K_N^* = \{k_\alpha : \alpha=1,2,\ldots,A\} \qquad (20)$$

such that $k_1, k_2, \ldots, k_N$ are the elementary vectors and $|k_\alpha|^*$ is non-decreasing in α. Finally, define J to be the smallest positive integer with

$$K_N^* \subset \{jk_\alpha : \alpha=1,2,\ldots,A;\ j=0,\pm 1,\pm 2,\ldots,\pm J\} \, . \qquad (21)$$

One should expect that in applications it would suffice to truncate at some K and fit the resulting expenditure system. How one goes about choosing K depends on whether the problem is hypothesis testing or estimation (see Gallant [28], Section 5). In general, K may be chosen according to either a deterministic procedure--using for example some fixed rule--or an adaptive rule--such as to increase K when a significance test rejects the current model. In either event, consistency obtains (see EL Badwi, Gallant, and Souza [18]). Also, note that A and J can be viewed as functions of K.

In the present study, we choose A=6 and J=1. The Fourier form for these values is presented in the Appendix.

Differentiating (14) and applying Roy's Identity (10), the following Fourier expenditure system is obtained for the household:

$$f_i(x,\theta) = \frac{(x_i b_i - \sum_{\alpha=1}^{A} \{u_{0\alpha} x' k_\alpha + 2 \sum_{j=1}^{J} j[u_{j\alpha} \sin(jk_\alpha' x) + v_{j\alpha} \cos(jk_\alpha' x)]\} k_{i\alpha} x_i)}{(b'x - \sum_{\alpha=1}^{A} \{u_{0\alpha} x' k_\alpha + 2 \sum_{j=1}^{J} j[u_{j\alpha} \sin(jk_\alpha' x) + v_{j\alpha} \cos(jk_\alpha' x)]\} k_\alpha' x)}, \quad (22)$$

where $i=1,2,\ldots,N-1$ and $b_N=-1$ (normalized). Three cost share equations (28) form the basis for our empirical estimations. A convenient arrangement of the parameters is obtained by setting

$$\theta_{(o)} = (b_1, b_2, \ldots, b_{N-1})',$$

$$\theta_{(\alpha)} = (u_{0\alpha}, u_{1\alpha}, v_{1\alpha}, u_{2\alpha}, v_{2\alpha}, \ldots, u_{j\alpha}, v_{j\alpha})'.$$

and

$$\theta = (\theta_{(o)}', \theta_{(1)}', \theta_{(2)}', \ldots, \theta_{(A)}')', \quad (23)$$

which is a vector of length $N-1 + A(1+2J)$.

IV. Elasticities: Derivation and the Computation of the Standard Errors

When parameter estimates are in hand the quantities typically of in-interest in a demand study may be obtained. For this paper the derivations of the elasticities are effected using Gallant's methods of computation (Gallant [27]); we will show this in the present section as well as the method for computing the standard errors of these elasticities. We will show this with respect to the elasticity of substitution, because of its importance, with the understanding that the other elasticities (and their standard errors) can be obtained in the same way.

Define the price elasticity of the i^{th} demand $q = q\ (P,M)$ with respect to a change in P as

$$\eta = Q^{-1}\ [(\partial/\partial P')\ q\ (P,M)]P\ , \tag{24}$$

where $q = q\ (P,M)$ denotes the Marshallian demand system.

$$\nabla g = (\partial/\partial x)\ g_K\ (x)$$

$$\nabla^2 g = (\partial^2/\partial x \partial x')\ g_K\ (x)$$

$$P = \text{diag}\ (P_1, P_2, \ldots, P_N) \tag{25}$$

$$Q = \text{diag}\ (q_1, q_2, \ldots, q_N)\ ,$$

and the income elasticities are

$$E = Q^{-1}\ [(\partial/\partial M)\ q\ (P,M)]\ M$$

$$= -\ Q^{-1}\ [(\partial/\partial P')\ q\ (P,M)]\ P\ . \tag{26}$$

In both (24) and (26), the following simplification is used since we are actually estimating the parameters of $g_K\ (x)$

$$(\partial/\partial P')\ q\ (P,M) = (P'\nabla g)^{-1}\ [\nabla^2 g - gx'\nabla^2 g - (x'\nabla g)\ qq']\ , \tag{27}$$

$$(\partial/\partial M)\ q\ (P,M) = -\ [(\partial/\partial P')\ q\ (P,M)]\ x\ . \tag{28}$$

These are all evaluated at the point $x = P/M$.

The Allen partial elasticity of substitution (AES) between two liquid assets i and j, σ_{ij}, can be derived from an indirect utility function as

$$\sigma_{ij} = \frac{[\Sigma_K x_K g_K] g_{ij}}{g_i g_{ij}} - \frac{\Sigma_K x_K g_{jK}}{g_i} - \frac{\Sigma_K x_K g_{iK}}{g_i} + \frac{\Sigma_m \Sigma_K x_K g_{Km} x_m}{\Sigma_n x_n g_n} \qquad (29)$$

where g_i and g_{ij} denote elements of $(\partial/\partial x)\, g(x)$ and $(\partial^2/\partial x \partial x')\, g(x)$, respectively (Diewart [16]).

Given that the AES is of primary concern to this study, two points must be emphasized. Firstly, as Gallant [1981, p. 221] argues, "global approximation to within arbitrary accuracy of the elasticities of substitution appears to us to be a far more appealing property than equality at a single point." That is, if we let $\sigma^*_{ij}(x)$ correspond to the true indirect utility function and let $\sigma_{ijK}(x)$ correspond to the Fourier function form, then Gallant shows that for $\varepsilon > o$ there is a K with

$$|\sigma^*_{ij}(x) - \sigma_{ijK}(x)| < \varepsilon \text{ , all } x \text{ .} \qquad (30)$$

Secondly, El-Badawi, Gallant, and Souza [18] show that if the Fourier flexible form is used with an estimation procedure that satisfies the identification condition then consistent estimation of price, income, and substitution elasticities is possible. They conclude that if we let $\hat{\sigma}_n(x)$ be an elasticity of substitution computed from $g_{Kn}(x|\hat{\theta}_n)$, if $\sigma^*(x)$ be computed from $g^*(x)$, and if $\lim_{n\to\infty} K_n = \infty$ then

$$\lim \operatorname{Sup}_{x \in X} |\hat{\sigma}_n(x) - \sigma^*(x)| = o \text{ .} \qquad (31)$$

A similar result holds for other elasticities.

Generally speaking, estimating the consistency of the AES requires not just approximating the true indirect utility function, but its first and second derivatives as well. This is the key behind our choice of the Fourier flexible form to fit the data for U.S. monetary assets. There is more, however, and to have economic inference of the AES, estimation of its asymptotic standard errors is also required. In the following, we explain how one can obtain such estimates.

Recall the first and second partial derivatives of the Fourier form

$(\partial/\partial x)\, g_K(x)$

$$= b + C x - 2 \sum_{\alpha=1}^{A} \sum_{j=1}^{J} j\, [u_{j\alpha} \sin (j k_\alpha' x) + v_{j\alpha} \cos (j k_\alpha' x)]\, k_\alpha \,, \tag{32}$$

$(\partial^2/\partial x \partial x')\, g_K(x)$

$$= -\sum_{\alpha=1}^{A} \{u_{0\alpha} + 2 \sum_{j=1}^{J} j^2 [u_{j\alpha} \cos (j k_\alpha' x) - v_{j\alpha} \sin (j k_\alpha' x)]\}\, k_\alpha k_\alpha' \,. \tag{33}$$

and let

$$\theta = (\theta_{(0)}', \theta_{(1)}', \theta_{(2)}', \ldots, \theta_{(A)}')' \,, \tag{34}$$

where $\theta_{(0)}$ and $\theta_{(\alpha)}$ are defined before. Then a first and second order partial are linear functions of the form

$$(\partial/\partial x_i)\, g_K(x|\theta) = g_i'\theta \tag{35}$$

$$(\partial/\partial x_i \partial x_j)\, g_K(x|\theta) = h_{ij}'\theta \tag{36}$$

where g_i, h_{ij} and θ are vectors of length $N - 1 + A\,(1+2J)$. Using the previous notation, an elasticity of substitution and its derivative with respect to θ are

$$\begin{aligned}\sigma_{ij}(\theta) = {} & [\Sigma_K x_K (g_K'\theta)]\, (h_{ij}'\theta)\, (g_i'\theta)^{-1} (g_j'\theta)^{-1} \\ & - [\Sigma_K x_K (h_{jK}'\theta)]\, (g_j'\theta)^{-1} \\ & - [\Sigma_K x_K (h_{iK}'\theta)]\, (g_i'\theta)^{-1} \\ & + [\Sigma_m \Sigma_K x_K x_m (h_{Km}'\theta)]\, (\Sigma_n x_n g_n'\theta)^{-1}\end{aligned} \tag{37}$$

$$
\begin{aligned}
&(\partial/\partial\theta)\,\sigma_{ij}\,(\theta) \\
&\quad = [\Sigma_K\, x_K\, g_K'']\,(h_{ij}'\theta)\,(g_i'\theta)^{-1}\,(g_j'\theta)^{-1} \\
&\quad + [\Sigma_K\, x_K\,(g_K'\theta)]\,h_{ij}'\,(g_i'\theta)^{-1}\,(g_j'\theta)^{-1} \\
&\quad - [\Sigma_K\, x_K\,(g_K'\theta)]\,(h_{ij}'\theta)\,(g_i'\theta)^{-2}\,(g_i')\,(g_j'\theta)^{-1} \\
&\quad - [\Sigma_K x_K\,(g_K'\theta)]\,(h_{ij}'\theta)\,(g_i'\theta)^{-1}\,(g_j'\theta)^{-2}\,(g_j') \\
&\quad - [\Sigma_K\, x_K\,(h_{jK}')]\,(g_j'\theta)^{-1} \\
&\quad + [\Sigma_K x_K\,(h_{jk}'\theta)]\,(g_j'\theta)^{-2}\,(g_j') \\
&\quad - [\Sigma_K x_K\,(h_{iK}')]\,(g_i'\theta)^{-1} \\
&\quad + [\Sigma_K\, x_K\,(h_{iK}'\theta)]\,(g_i'\theta)^{-2}\,(g_i') \\
&\quad + [\Sigma_m\,\Sigma_K\, x_K x_m\,(h_{Km}')]\,(\Sigma_n x_n\,(g_n'\theta))^{-1} \\
&\quad - [\Sigma_m\,\Sigma_K\, x_K x_m\,(h_{Km}'\theta)]\,(\Sigma_n x_n\,[g_n'\theta])^{-2}\,(\Sigma_n x_n g_n').
\end{aligned}
\tag{38}
$$

Now, let $\hat{\theta}$ denote the seemingly unrelated regression computed as in the next section. Its estimated variance - covariance matrix is $\hat{\Omega}$. Then an estimate of the AES, $\sigma_{ij}\,(\theta)$ at $\hat{\theta}$,

$$\hat{\sigma}_{ij} = \sigma_{ij}\,(\hat{\theta})\,, \tag{39}$$

and therefore, using the transformation method, its standard errors are computed as

$$SE\,(\hat{\sigma}_{ij}) = [\partial/\partial\theta')\;\sigma_{ij}\;(\hat{\theta})\;\hat{\Omega}\;(\partial/\partial\theta)\;\sigma_{ij}\;(\hat{\theta})]^{\frac{1}{2}} \tag{40}$$

The same technique can be applied in a straightforward manner to the other elasticities.

Finally, using the parameter estimates obtained from the Fourier indirect utility function, one can verify the restrictions on the form of this indirect utility function as implications of the theory of utility-maximization behavior; those restrictions are for non-negativity, monotonicity, and curvature (quasi-convexites). The positivity and monotonicity restrictions are checked by direct computation of the values of the fitted demand functions and the gradient vector of the estimated indirect utility function, respectively. The curvature conditions are tested by examining the computed (σ_{ij}) matrix. This requirement implies that the Allen-Uzawa elasticities of substitution provide a negative semi-definite matrix of rank equal to at most $(N-1)$. Negative semi-definiteness requires placing alternating sign restrictions on the first $N-1$ principle minors of the N-dimensional matrix. A necessary, but not sufficient, requirement of the curvature condition is that the own elasticities of substitution must all be non-positive.

V. Econometric Estimation, Hypothesis Testing, and Data Estimation

The market expenditure share equations of any N monetary assets demand equations are given by equation (22). The observed shares are assumed to deviate from the "true" shares by an additive disturbance term, $u_{i(t)}$, that is assumed to be due to errors in the utility maximizing process or in aggregation over either assets or consumers.

The system of direct Fourier expenditure share equations can be re-written for the i^{th} equation as

$$S_{i(t)} = F_i\,(x_{i(t)}, \theta_i) + U_{i(t)} \qquad (41)$$

$$i=1,2,\ldots,N$$
$$t=1,2,\ldots,n$$

Here the inputs $x_{i(t)}$ are the exogeneous variables in the i^{th} equation, θ_i represents the vector of unknown parameters mentioned above, and $U_{i(t)}$ is a random disturbance term which will be defined below. More compactly, the system can be written as

$$S_{(t)} = f_{(t)}\,(\theta^o) + u_{(t)}\ ,\qquad (42)$$

in which the error is specified as [8]

$$u_{(t)} = RU_{(t-1)} + e\,(t)\qquad (43)$$

and $R = \{R_{ij}\}$ is an NxN matrix of unknown parameters. $e\,(t)$ is assumed to be distributed normally, independent of the exogeneous variables. Furthermore, it is assumed that

$$E\,[(e_{(t)})] = o\qquad (44)$$

$$E\,[e_{(s)}\,e_{(t)}'] = \begin{cases} \Sigma & \text{for } s=t \\ o & \text{for } s\neq t \end{cases}$$

where Σ is an NxN symmetric and positive definite matrix. Here $u\,(t)$ is a vector that is assumed to follow a first order autoregressive process with

$$E\,(u_{(t)}) = o$$

$$E\,(u_{(t)}, u_{(t)}') = \Omega = \Sigma_{j=o}^{\infty} R^j\ e\,(t-j)\qquad (45)$$

where Ω_{ij} is the NxN covariance matrix $E\,(u_i u_j')$. This specification allows both contemporaneous and non-contemporaneous disturbance terms to be correlated.

Since the expenditure shares $S_i\,(t)$, by definition sum to unity at each observation, it follows that $\Sigma_{i=1}^{N} u_i\,(t) = o$ at each observation. Systems of equations having this property are "singular systems". In general, some constraints must be placed on the form of Ω if it is to be estimated. Indeed in a singular system with autocorrelation, the adding up property of the share equations imposes additional restrictions on the serial correlation parameters (Berndt and Savin [8]). When these additional restrictions are not imposed, any estimation and any hypothesis testing are conditional on the equation deleted. In the present study, to overcome this problem, the autoregressive coefficients $\{R_{ij}\}$ have been restricted to be equal across equations; that is, the R is R = diag $\{R_{11}, R_{22}, R_{33}\}$ in which $R_{11} = R_{22} = R_{33} = \bar{R}$.

Using the Iterative Non-linear Seemingly Unrelated Regression (INSUR-Gallant [1975]) method to estimate $(\theta, \bar{R})$ simultaneously produces the following final model is used, (9)

$$(1 - \bar{R}^2)^{\frac{1}{2}} S_{i(t)} - (1 - \bar{R}^2)^{\frac{1}{2}} f_i (x_{(t)} \mid \theta) = e_{i(t)} \qquad t=1$$

$$S_{i(t)} - \bar{R}S_{i(t-1)} - f_i (x_{(t)} \mid \theta) + \bar{R}f_i (x_{(t-1)} \mid \theta) = e_{i(t)}$$

$$t=2,\ldots,n \quad (46)$$

In the actual estimation, one share equation is arbitrarily dropped, and a truncated disturbance covariance matrix Σ is used. In the present study, we assume that the third equation is the deleted one in all empirical investigations. The INSUR method is invariant with respect to the equation to be deleted when the presented autoregressive coefficient is constrained (as in our case); verification of such an argument may be found in Tibibian [37].

Hypothesis Testing

To insure the consistency of the Fourier indirect utility function with the underlying theory, the parameters have to satisfy some restrictions. Equality and symmetry restrictions, in a Fourier indirect utility function, are examples of such restrictions. Using the Fourier functional form, one can argue that tests of symmetry and equality that are asymptotically free of specification bias can be constructed. In such a case, a significant test statistic can thus be attributed to violation of symmetry and equality rather than specification bias (see Gallant [27], Section 7]).

Following Gallant [27], a test of symmetry and equality may be constructed as follows

$$f(x, \theta_1, \theta_2, \ldots, \theta_{N-1}) = \begin{bmatrix} f_1(x, \theta_1) \\ f_2(x, \theta_2) \\ \vdots \\ f_{N-1}(x, \theta_{N-1}) \end{bmatrix} \tag{47}$$

where the $f_i(x, \theta)$ terms will be recognized as the share of the i^{th} monetary asset. If θ_i is the same in all equations, then

$$f(x, \theta_1, \theta_2, \ldots, \theta_{N-1}) = f(x, \theta) , \tag{48}$$

and hence the restriction

$$\theta_1 = \theta_2 = \ldots = \theta_{N-1} \tag{49}$$

represents the null hypothesis of equality and symmetry.

In the present study, because we use the INSUR method of estimation, in which the estimated variance-covariance matrix is held fixed in both constrained and unconstrained estimates, the Souza-Gallant [36] test statistic has been used. Further, with the variance-covariance matrix held fixed in both the restricted and the unrestricted estimations, one can write the test statistic as

$$- 2 \ln\lambda = 2 \ln L_u - 2 \ln L_r$$

$$= \Sigma_{t=1}^{n} \hat{e}_t' \hat{\Sigma}^{-1} e_t - \Sigma_{t=1}^{n} \tilde{e}_t' \hat{\Sigma}^{-1} \tilde{e}_t , \quad (50)$$

in which u and r refer to the unrestricted and restricted cases, respectively, and $\hat{e}$ and $\tilde{e}$ are the residuals vectors for the null hypothesis and the alternative hypothesis, respectively. This test statistic is distributed asymptotically as chi-square with degrees of freedom equal to the number of independent parameter restrictions (see Souza-Gallant [36]).

Data

The data that are used in the current study were provided by official sources at the Federal Reserve Board. Most of the available data (domestic data) were monthly and, therefore, have been converted to average quarterly data for our use [1969 I - 1979 IV]; foreign data and GNP were quarterly. The interest rate on money narrowly defined (M1) is the implicit interest rate that was constructed by B. Klein [32] and updated by Offenbacher [34]. Klein's data were annual and hence are interpolated (after updating) into monthly by Offenbacher using the method proposed by Chow and Lin [13] for the construction of time series by related series.(10)

A number of specific points regarding the construction of the data should be made. First, a series on the U.S. population that was provided by FRB was used to derive a per capita calculation. Second, R_t, the benchmark which is also used as the discount rate required in the construction of the rental rate in equation (3), was chosen to be the highest interest rate series derived from all available interest rates.(11) Third, for estimating purposes, it has been argued that when a Fourier flexible form is used the scaling of the data is important (in Gallant [27], [28], and

El Badawy, Gallant, and Souza [18]). A Fourier series is a periodic function in each of its arguments and an indirect utility function is not. A Fourier series approximation of the true indirect utility function can be made as accurate as desired on a region which is completely within the cube $\times_{i=1}^{N} [0, 2\pi]$. This can be done by rescaling the inc me normalized prices to fall between 0 and 2 π such that the rescaled prices satisfy

$$0 < x_i^{\ell} < x_i^{u} < 2\pi \tag{51}$$

In this study, however, the data are scaled so that [(12)]

$$\max \{x_{it}: t=1,2,\ldots,n\} = 6, \quad i=1,2,3 . \tag{52}$$

VI. Empirical Results

In what follows we will be pursuing a dual objective. On the one hand we wish to provide a convincing demonstration of the usefulness of the Fourier functional form on a standard set of data and on the other we wish to contribute to the debate over actual "monetary substitutability" in the United States; the latter is obviously of concern in the conduct of American monetary policy. The comparisons will be between old M1 (equal to currency plus demand deposits) and various measures of time and savings deposits at commercial and at savings banks. The comparisons will be to the Offenbacher [34], Barnett [2], and Ewis-Fisher [20] papers with the understanding that a considerable earlier literature exists (and is documented in these studies) and has not succeeded in settling the question of the degree of substitution.

In our theoretical work in earlier sections of this paper we emphasized three aspects of our approach while providing the hope for some improvement. Firstly, and most importantly, the Fourier flexible form is a newly proposed form which provides a consistent method of estimating the AES. Secondly, by providing estimates of the standard errors of the AES we are able to make judgments as to the statistical significance of our measure of substitutability. Thirdly, we will present estimates of the AES and its asymptotic standard errors over the entire sample period. This will enable us to address the question of whether or not the AES is constant over time; if it is not (and it generally is not), then the frequent point estimates of substitutability one sees in the literature are certainly misleading. Indeed, as we shall see, some dramatic things have been happening to the AES in recent years, at least as revealed by our approach.

VI.I Money and Near Money

In order to gain a quick impression of the nature of our results reference could be made to Figures (1) and (2), below; these exhibit the behavior of the AES between M1 and savings and time deposits in commercial banks (in Figure (1)) or in savings and loan banks (Figure (2)). As claimed, the AES is not constant in these cases and, indeed changes quite sharply, especially at the end of the 43 quarter series.[(13)] Complete tabular detail for these results would be at the expense of other interesting cases, and so we forbear; Table (1), though, presents the estimates of the AES for the two tests for three calendar quarters of the sample for 1979 (for commercial banks) and for 1978 (for S&Ls). Note that the numbers in parentheses are asymptotic standard errors. Also note that both these tests pass the test for equality and symmetry.[(14)]

For the relation between M1 and savings deposits in commercial banks we note that starting in late 1975 (Quarter #28) an essentially indeterminate relation turns to one of mild and then significantly strong substitutability (in 1979); the figure shows how sharp this change is, and the table confirms the statistical significance. M1 and time deposits in commercial banks show an opposite pattern (in σ_{13}), ending up with a fairly strong (and significant) complementarity in 1979. For σ_{23} --measuring the AES between savings and time deposits at commercial banks--we find, as expected, significant substitutability throughout the period with a sharp increase in 1979. One can conjecture that the institutional changes in the United States--involving automatic transfers, NOW accounts, and a changed style of monetary policy under Federal Reserve Chairman Paul Volcker--have brought dramatic changes in the definition of, for example, money.

For the second part of Table (1) and for Figure (2) we note that M1 and savings deposits at S&Ls suddenly turn from a complementarity to a substitutability relation; again, this is probably on account of increased ease of transferability of these funds (or on account of the volatility of interest rates). M1 and time deposits in S&Ls remain substitutes throughout, although not without some minor fluctuation. For the AES between time and savings deposits at S&Ls we find that as with commercial banks, these two types of accounts are significantly close substitutes; indeed, this substitution is sharply increasing at the end of the period.

Looking across the two sets of results we note that while M1 and SDCB are substitutes (on average), M1 and SDSL are complements; similarly, M1 and TDCB are complements while M1 and TDSL are substitutes. In the Ewis-Fisher paper referred to above, the possibility of "institutional loyalty" among small savers was postulated to help explain a similar sort of result; in any event when money is included in the aggregate, summation across all of these liquid assets (to produce an old M3) would not produce a satisfactory aggregate,

THE BEHAVIOR OF AES

M1 SDCB TDCB

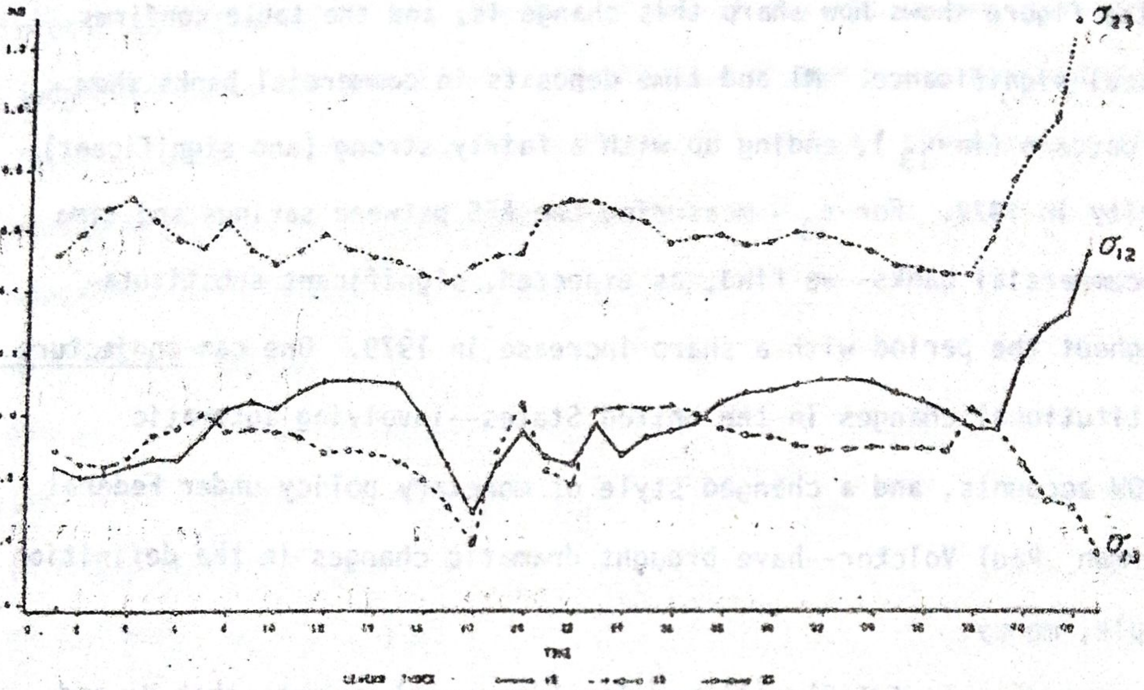

FIGURE 1. CURVE RELATING AES TO TIME
1969–1979

THE BEHAVIOR OF AES

M1 SDSL STDSL

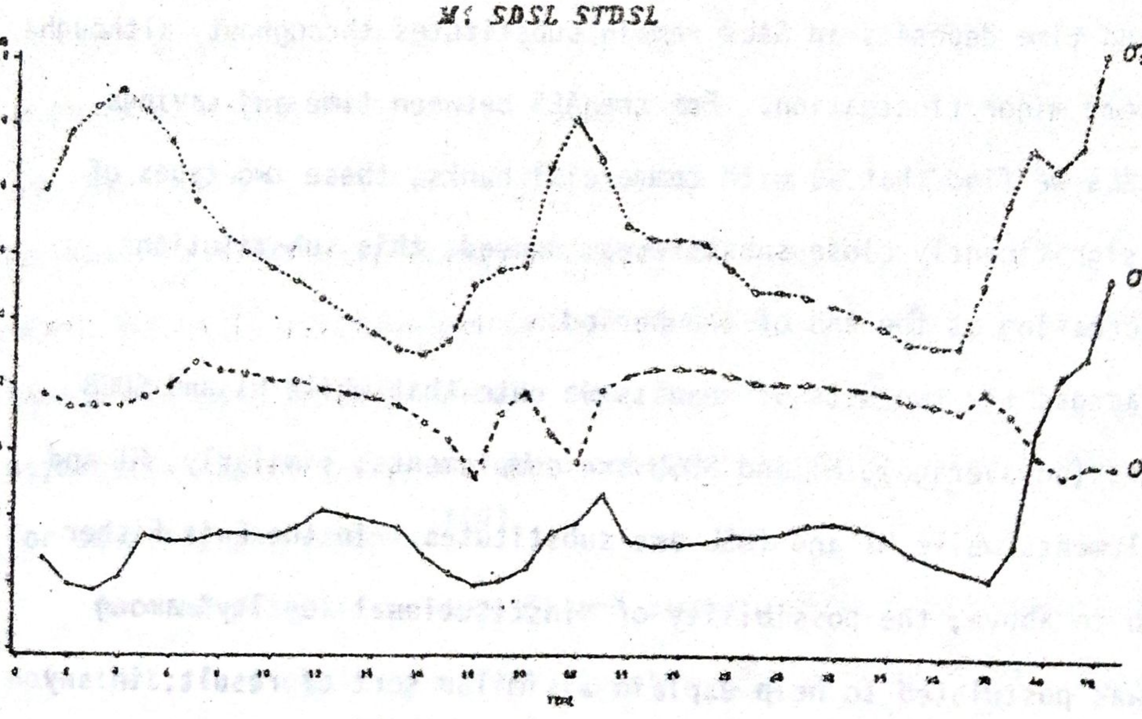

FIGURE 2. CURVE RELATING AES TO TIME
1969–1979

Table 1: Estimates of the AES for Money and Near Monies
US Data, 1969-1979*

	σ_{12}	σ_{13}	σ_{23}
a. M1-SDCB-TDCB			
1979(2)	.2690	-.2751	.8524
	(.1147)	(.0958)	(.1393)
1979(3)	.3135	-.2940	.9491
	(.1287)	(.1103)	(.1549)
1979(4)	.5078	-.4350	1.2648
	(.1878)	(.1593)	(.2057)
b. M1-SDSL-SDSL			
1978(1)	-.3460	.1302	.3064
	(.1744)	(.0590)	(.1840)
1978(2)	-.3676	.1112	.3068
	(.1690)	(.0648)	(.1838)
1978(3)	-.3905	.1714	.4919
	(.1782)	(.0529)	(.1283)

*numbers in parentheses are asymptotic standard errors.

a priori. Note, especially, that SDCB and TDCB are close substitutes--as are SDSC and TDSL--a pattern which is consistent with the institutional loyalty hypothesis. These results also confirm Barnett's findings.

VI. II. Substitutability among Savings and Time Deposits

Following Barnett, a natural extension of the foregoing is to inquire into the relationships among "like assets" both within and across institutions. Since we have constrained ourselves to a three asset framework, a natural way to proceed is to find some other asset, less "like," in order to test the robustness of the likeness. In Figure (3) we show a comparison between SDCB and SDSL, designed to show "across institutions" substitution; thus σ_{12} shows substitution from S&Ls to commercial banks and the comparison with a third asset--savings bonds--in σ_{13} and σ_{23}--show a remarkably similar (and generally complementary) pattern. We note, however, that there is a sudden and sharp switch to complementarity between SDCB and SDSL at the end of the period. The results for the large dips in σ_{13} and σ_{23} are given in Table (2). These show how close these estimates are and how little this affects σ_{12}. Evidently "institutional loyalty" is not as strong as the results of Section VI.I suggested, although, to be sure, within-institution substitutability is greater than across-institution substitutability (for savings accounts).

THE BEHAVIOR OF AES

SDCB SDSL SB

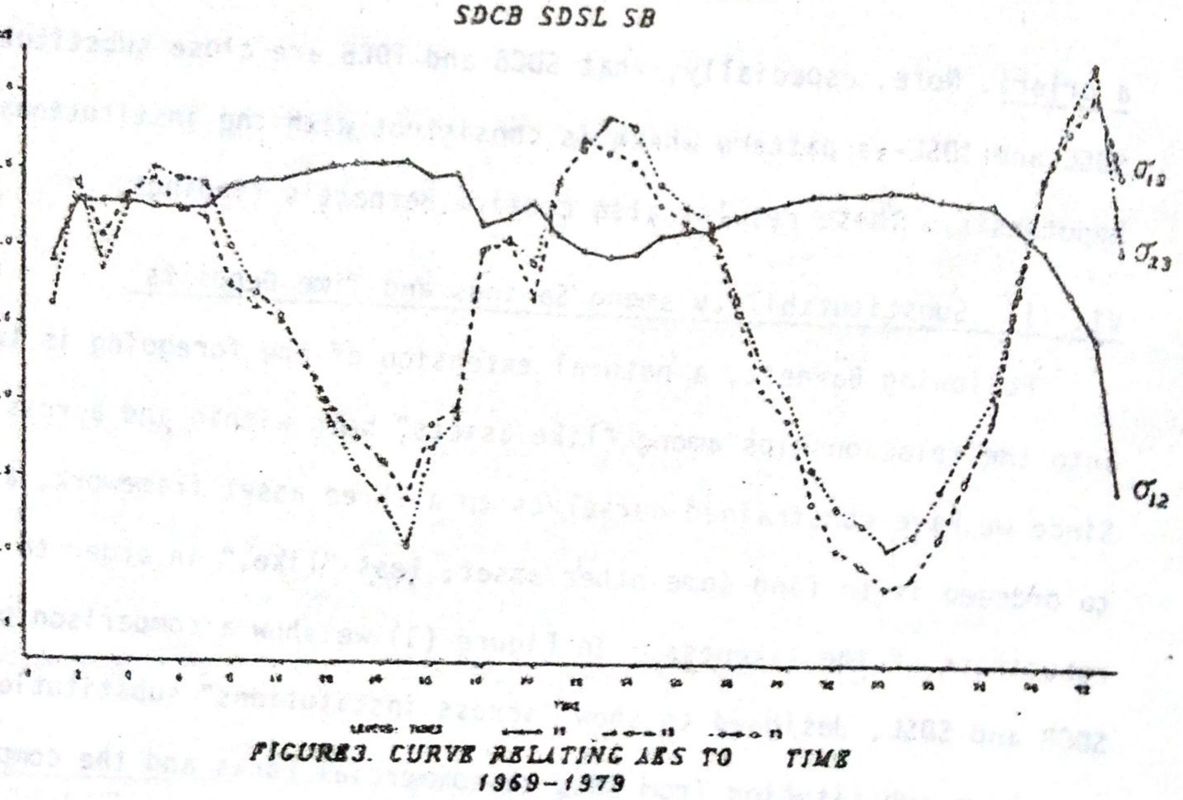

FIGURE3. CURVE RELATING AES TO TIME
1969-1979

Table (2) The elasticities of substitution between SDCB, SDSL, SB*

	σ_{12}	σ_{13}	σ_{23}
1972(1)	.5218	-1.0433	-1.1431
	(.2699)	(.4807)	(.7027)
1972(2)	.5415	-1.2268	-1.4291
	(.2671)	(.5539)	(.8021)
1972(3)	.5417	-1.3751	-1.6297
	(.2774)	(.61866)	(.8765)
1972(4)	.5576	-1.6176	-1.9400
	(.3092)	(.7609)	(1.0640)

*Asymptotic standard errors in parentheses.

A second "like asset" comparison--with the same rationale--is between S&Ls' (STDSL) and commercial banks' (STDCB) time deposits; the control asset in this case is short term Treasury securities (STTS). Figure (4) shows that for time deposits, again, the "like assets" are substitutes and, even, increasingly so, at the end of the period. Again there is a close--but not as close as Figure (3)--relation between each of the "like" assets and the control variable; we conclude again that "like assets" are substitutes across institutions. Again we note that across-institution substitution is less than within-institution as suggested by our "institutional loyalty" hypothesis.

Overall, these two tests have established that across financial institutions savings deposits are substitutes and, separately, that time deposits are substitutes across institutions. Since they were seen to be close substitutes within the institution in Section VI.1, this seems to dispose of the matter; institutional loyalty, so far as these tests are concerned, seems to be in the value of the AES; it does not here produce the exceptions noted in the Ewis-Fisher [20] test. Again Barnett's conclusions

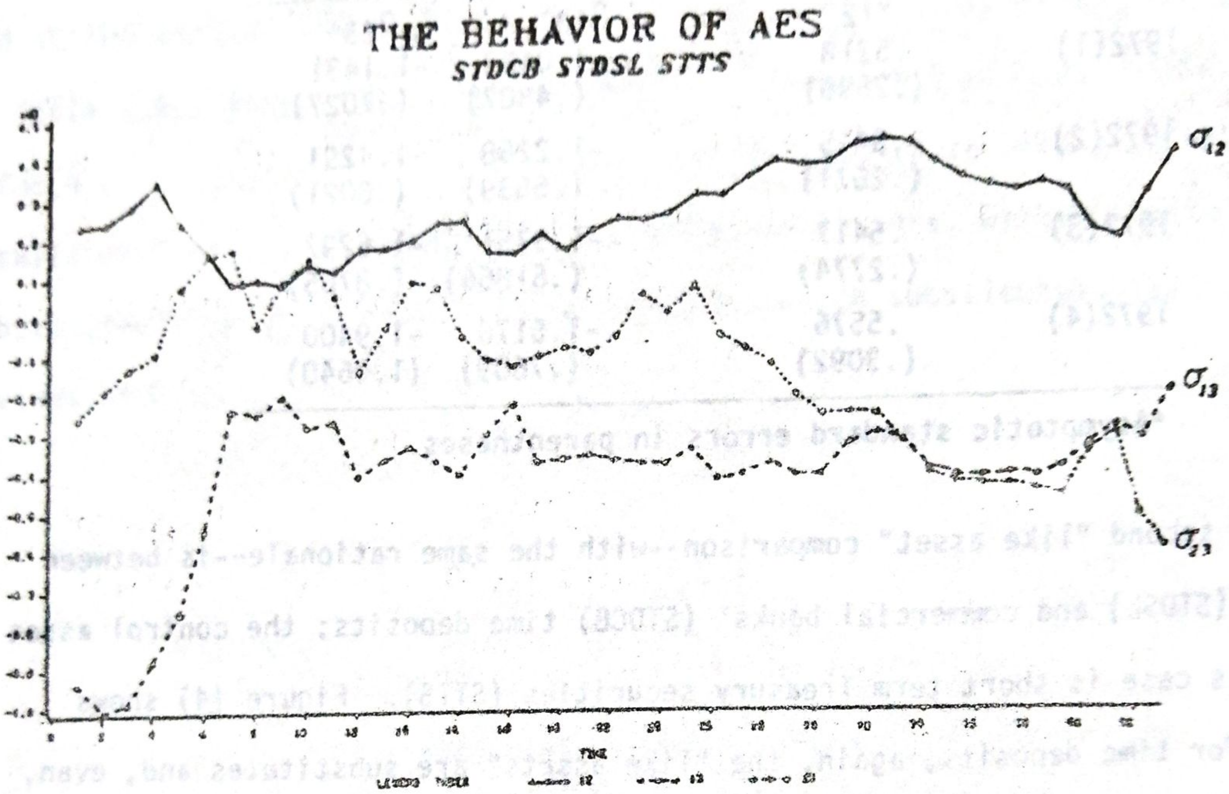

FIGURE 4. CURVE RELATING AES TO TIME
1969–1979

concerning the relative magnitude of the AES among similar assets across institutions is confirmed. We note, again, that symmetry and equality restrictions were accepted in both tests.(15)

VI.III: Disaggregation of the Money Stock

The last empirical question tackled in this paper concerns an issue raised by Offenbacher [34] concerning the within-M1 substitution. Offenbacher could not confirm that currency and demand deposits were close substitutes; indeed, he found that

$$\sigma_{DT} > \sigma_{CD} > \sigma_{CT} \quad (53)$$

and that σ_{CT} actually showed complementarity. Time deposits operate as the control asset here, of course, and in Figure (5) and Table (5) we report the results; the latter are representative ones plucked from the entire set (at the more dramatic turning points).

Table (5) the elasticity of substitution between DD, CUE, TDCB

	σ_{12}	σ_{13}	σ_{23}
1970(2)	.1499	.3730	.1945
	(.0535)	(.2192)	(.0656)
1972(2)	.1129	-.5312	.1692
	(.0360)	(.3047)	(.0500)
1973(2)	.1417	.4498	.2017
	(.0551)	(.2201)	(.0658)
1975(2)	.0284	-.9332	.1235
	(.0435)	(.2433)	(.0586)

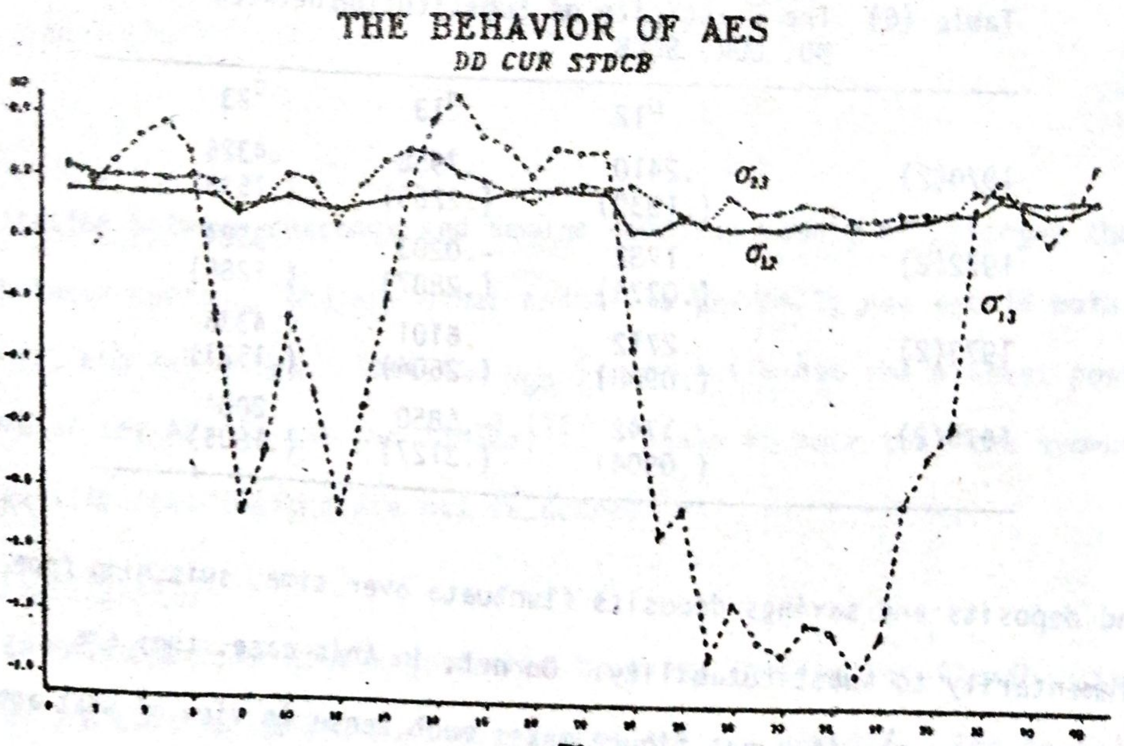

FIGURE5. CURVE RELATING AES TO TIME
1969–1979

The relationship between currency and bank demand deposits is one of substitution, in general--and is significantly so--but it is not as strong a one as we have found (for example) among the savings assets. Similarly, currency and time deposits show a stable substitutability relation over time. On the other hand, demand deposits and time deposits at commercial banks are usually complements, but are sometimes even substitutes--as, for example, in the second quarter of 1973. This is certainly disconcerting for the use of M2 as a "control" variable in monetary policy; we will comment on M1 in a moment, but for now we underscore the low substitution.

For savings deposits at commercial banks we repeat the test, as described in Figure (6) and Table (6). Demand deposits and currency and currency and savings deposits are again (mildly) significant substitutes in this test (we compare the same dates in Tables (5) and (6)), and again,

Table (6) The Elasticity of Substitution Between DD, CUR, SDCB

	σ_{12}	σ_{13}	σ_{23}
1970(2)	.2410 (.1039)	.7958 (.2265)	.4326 (.1538)
1972(2)	.1782 (.0774)	-.0203 (.2887)	.3384 (.1286)
1973(2)	.2712 (.0968)	.6101 (.2604)	.4338 (.1527)
1975(2)	.1742 (.0904)	-.5850 (.3127)	.2004 (.1805)

demand deposits and savings deposits fluctuate over time, switching from complementarity to substitutability. On net, in this case, they are substitutes, though, if a net figure makes much sense in view of what appears in Figure (6). In any case in <u>both</u> tests a condition laid down by Offenbacher--that if M1 is to be a successful aggregate then the

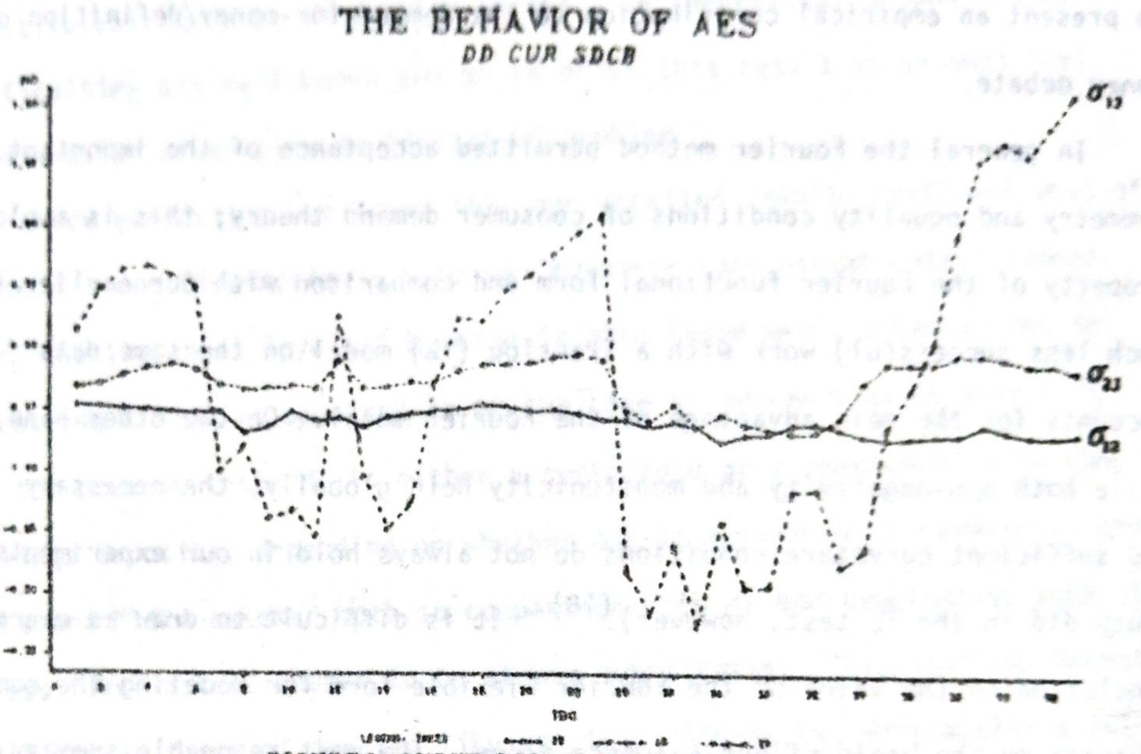

FIGURE 6. CURVE RELATING AES TO TIME
1969–1979

substitution between currency and demand deposits should be stronger than that between currency and any other asset--is generally not met in both Table (5) and Table (6). That is, σ_{CD} is generally not the highest positive measure of the AES in the two tables.(16) Again we note that the symmetry and equality restrictions are not rejected.(17)

VII. Conclusions

In this paper we have argued that by using the Fourier flexible functional form we are able to obtain consistent estimates of the AES. The technique employed also permits us to calculate the AES over time and, of equal importance, to obtain estimates of the standard errors for each of the estimates of the AES. This is particularly useful, as it turns out, in view of the apparent instability of the U.S. demand for money and has enabled us to present an empirical contribution to the demand-for-money/definition of money debate.

In general the Fourier method permitted acceptance of the important symmetry and equality conditions of consumer demand theory; this is a global property of the Fourier functional form and comparison with our earlier (and much less successful) work with a Translog (TL) model on the same data accounts for the main advantage of the Fourier model. On the other hand, while both non-negativity and monotonicity held globally, the necessary and sufficient curvature conditions do not always hold in our experiments (they did in the TL test, however).(18) It is difficult to draw an overall conclusion on the value of the Fourier Flexible form for modeling the consumer's behavior on the basis of our evidence alone. The most resonable summary, however, would suggest that this form is quite viable, offering estimates of AES with considerable precision.

Turning to our explicit empirical results the most important finding, we feel, is that the AES estimates are decidedly non-constant over time, even changing from (e.g.) complementarity to substitutability on occasion. There are, indeed, strong cyclical patterns noticable and there appears to be a tendency for the monetary innovations of the late 1970s to contribute to the instability of the AES.

An important implication is that the popular monetary aggregates constructed as simple sum of money components should be redefined as weighted sum aggregates of those components. The weights should not be constant over time as long as the substitutability relationships are not constant. Any aggregation that is based (either implicitly or explicitly) on a constant AES or an AES calculated at a single point may well produce shaky support for American monetary policy since it does not internalize the substitutability over the entire period. On theoretical grounds these difficulties are well known and so we offer this result as an empirical confirmation of an already identified problem.

More explicitly, we found that our detailed results confirmed most of Barnett's and Offenbacher's work on "monetary substitutability;" indeed, our "institutional loyalty" hypothesis also fared well, although not in as strong a form as appeared to be the case in our earlier TL work.[19] In the present tests M1 is either a substitute or a complement with time and savings deposits, depending on whether one is referring to commercial banks or S&Ls and "like assets" both in comparison with M1 and separately, with other "less like" assets appear to be strong substitutes. This confirms Barnett's findings in favor of "nested like assets." Similarly, Offenbacher's result concerning the effect of the disaggregation of M1 is confirmed in that substitutability between currency and deposits is surprisingly low and lower than the substitutability between these components of M1 and at least one other asset (either savings deposits or time deposits). This raises a serious question about the suitability of even M1 for monetary control and certainly suggests that further work is needed on this important issue. Finally we have noted again that across-institution elasticities of substitution are generally lower (for the same categories of assets) than within-institution elasticities, as if customers have a certain amount of "institutional loyalty" in the aggregate.

VII. Appendix

It is argued that the truncated versions of the Fourier form (Gallant [27]) are tractable for testing hypothesis and in yielding accurate and consistent estimate of the elasticities of substitution. The problem, then, is to choose specific value for K; the choice depends on whether the problem is hypothesis testing or estimation (see [18], [27], and [28, section 5]).

In the present study, however, we choose A=6 and J=1. The Fourier form for these values is simplified as

$$g_K(x) = u_0 + [b_1\ b_2\ b_2] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [x_1\ x_2\ x_3]\ C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{aligned}
&+ u_{01} + 2[u_{11} \operatorname{Cos}(x_1) - v_{11} \sin(x_1)] \\
&+ u_{02} + 2[u_{12} \operatorname{Cos}(x_2) - v_{12} \sin(x_2)] \\
&+ u_{03} + 2[u_{13} \operatorname{Cos}(x_3) - v_{13} \sin(x_3)] \\
&+ u_{04} + 2[u_{14} \operatorname{Cos}(x_1+x_2) - v_{14} \sin(x_1 + x_2)] \\
&+ u_{05} + 2[u_{15} \operatorname{Cos}(x_1+x_3) - v_{15} \sin(x_1+x_3)] \\
&+ u_{06} + 2[u_{16} \operatorname{Cos}(x_2+x_3) - v_{16} \sin(x_2+x_3)]
\end{aligned} \tag{54}$$

$$\text{the sequence of } K_\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{55}$$

and then, $C = -\Sigma_\alpha^A\ u_{0\alpha}\ K_\alpha K_\alpha'$ is

$$C = -u_{01}\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix} - u_{02}\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} - u_{03}\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$-u_{04}\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix} - u_{05}\begin{bmatrix}1&0&1\\0&0&0\\1&0&1\end{bmatrix} - u_{06}\begin{bmatrix}0&0&0\\0&1&1\\0&1&1\end{bmatrix} \qquad (56)$$

The gradient is

$$(\partial/\partial x)g_k(x) = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} + C\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$-2\,[u_{11}\,\mathrm{Cos}(x_1) - v_{11}\,\mathrm{Cos}(x_1)]\begin{bmatrix}1\\0\\0\end{bmatrix} \qquad (57)$$

$$-2\,[u_{12}\mathrm{Cos}(x_2) - v_{12}\mathrm{Cos}(x_2)]\begin{bmatrix}0\\1\\0\end{bmatrix}$$

.

$$-2\,[u_{16}\mathrm{Cos}(x_1+x_3) - v_{16}\mathrm{Cos}(x_2+x_3)]\begin{bmatrix}0\\1\\1\end{bmatrix}$$

*We are grateful to A. Ronald Gallant for his help in fitting the Fourier demand system and for his comments on a earlier draft of this paper. All remaining errors are, of course, our responsibility.

(1) There is a rapid growing literature following the same lines of Barnett [2]. Because of space limitation the reader is referred to [3], [4], [5], [6], and [7].

(2) The existence of a "representative consumer" requires two sets of assumptions: (1) Gorman's conditions of Linear Engel curves that are parallel across consumers and (2) rationality of individual consumers. While the present study tests for condition (2) given (1) as the maintained hypothesis, it would be also interesting to test for the reverse. Discussion of this point is in [3, p. 220-21].

(3) See, for example, Christensen et. al. [14], Gallant [27], and Wales [38].

(4) But see below, footnote (19) in the empirical section.

(5) Standard errors for the estimated elasticities of substitution are fairly easy to obtain using the translog flexible form (for example), when we impose the linear homogeneous assumption (as in Offenbacher's model [34]). In this case

$$\sigma_{ij} = 1 - \beta_{ij}/S_iS_j ,$$

and hence its asymptotic standard error is

$$SE\ (\sigma_{ij}) = \{(1/S_iS_j)^2\ Var\ (\beta_{ij})\}^{\frac{1}{2}}$$

These standard errors are based on the assumption that the cost shares are nonstochastic. It is known that such a specification contradicts the assumptions used in estimating the model, but it has been argued (Binswanger [9], Humphrey and Moroney [30]) that these relations hold asymptotically. Evaluating the statistics at the mean cost share values and treating them as estimates of the central tendency of the distribution might serve to reduce the error in the approximating standard errors. This point is made by Dennis and Smith [15, p. 804], although Offenbacher does not use this rationale.

(6) For a fairly comprehensive household maximization problem the reader is referred to Barnett's work in [1], [2], and [3, Chap. 7] and to the Ewis-Fisher work in [14]. The latter is based on Barnett's theoretical work. However, they differ in using assumptions and/or theorems to retreat to a single period optimization problem with financial assets only, in the objective function. Barnett acquired a current period utility function by assuming intertemporal weak separability; the latter assumption is needed to get a current period utility function depending only upon current period quantities and hence the theory behind superlative or Divisia quantity aggregates is applicable. Ewis-Fisher [14] -- in contrast -- use Hadar's collapsibility theorem to acquire a current period utility function. This is not appropirate when index number theory is to be used to

construct a monetary aggregate; this is not the purpose of the their study.

(7) The simple construction of such a form for the rental rate was derived first by B. Klein as [$P_{it} = R_t - r_{it}$] and has been used by many researchers (e.g. Offenbacher [34]). Construction of an extension is by Donovan [17] although the theoretical derivation in this case is by W. Barnett [1]. This form was

$$P_{it} = \bar{P}_t \quad \frac{R_t - r_{it}}{1+R_t}$$

where $\bar{P}_t$ is a price index. A similar form is used by Ewis-Fisher [30]. All these forms are connected to each other since they are independent of $\bar{P}_t$ or $(1+R_t)$.

(8) In testing for the serial independence of each u_i--in an early stage--the Durbin-Watson test reveals that the hypothesis of serial independence among the ordinary least squares residuals in each u_i was rejected sharply. A possible explanation for these rejections may be that the elements of these individual equation residual vector are generated by a first-order autoregressive scheme, as is assumed above. Later estimates of the nonlinear equations system reveal high autocorrelation coefficients.

(9) The algorithm and computer program used in the estimation of the model are developed by Gallant [26]. Using the INSUR method to estimate (θ_i, $\hat{R}$) simultaneously enables us to consider the autoregressive specification in all iterative processes as well as Σ. In this process, one starts with an estimated value of $\hat{R}$ and minimizes the sum of squares by the choice of $\hat{\theta}$. The next step is to minimize the sum of squares by the choice of $\hat{R}$ for this $\hat{\theta}$. This process continues until estimators $\hat{\theta}$ and $\hat{R}$ are obtained that do not significantly differ from those obtained on the previous step.

(10) For more discussion of the construction of such implicit interest rates the reader is referred to Offenbacher [34]. The implicit interest rate that is used in the present study is the full competitive interest rate. The flavor of the debate of using such implicit interest rate may be found in [22] and [23].

(11) More detail about the data can be found in W. Barnett [3, Chap. 7].

(12) To do so, we simply use the following formula

$$x^*_{it} = [6/\max(x_{it})]\, x_{it} \qquad i=1,2,\ldots,3 \quad t=1,2,\ldots,n$$

where x^*_{it} is the rescaled normalized price.

(13) The data was for 44 quarters starting in 1969 and running to 1979 IV. Because of the treatment of the autoregressive specification, one observation is lost.

(14) The computed value of the test statistic for equality and symmetry for the results reported in Figure (1) is

$$L = 83.68 - 51.72 = 31.96$$

which is not significant at a level of 1%. For the Figure (2) the test statistic is

$$L = 82.19 - 44.4 = 37.79$$

which is significant at a lavel of 1%, but not at .001.

(15) For the first test, the computed value of the statistic for equality and symmetry is

$$L = 83.75 - 48.8 = 34.9$$

which is not significant at a level of .005. For the second test the statistic is

$$L = 85.17 - 55.21 = 29.96$$

which is not significant at a level of .025.

(16) Unfortunately, Offenbacher estimates the substitutability relationships according to incorrect derivation of AES as

$$\sigma_{ij} = 1 - \frac{B_{ij}}{S_i S_j} , \quad i \neq j$$

while the correct derivation is

$$\sigma_{ij} = 1 + \frac{B_{ij}}{S_i S_j} , \quad i \neq j$$

Having his estimates of σ_{ij} and B_{ij} one is able to get the correct AES and therefore his main conclusions--based upon correct derivations--should be written as

$$\sigma_{CT} > \sigma_{CD} > \sigma_{DT} ,$$

and all assets are noticed to be substitutes. Fortunately, our findings tend to confirm his corrected conclusion.

(17) The equality and symmetry restrictions for both tests cannot be rejected. For the first test, the computed value of the test statistic is

$$L = 80.66 - 52.69 = 27.97$$

which is not significant at a level of .025. For the second test the statistic is

$$L = 65.43 - 37.54 = 27.89$$

which is not significant at a level of .025.

(18) It might be more appropriate to test directly for the curvature condition and hence impose it. It is not clear, however, how to impose the weaker of the two convexity restrictions. Hopefully, further work will produce Fortran Code which will allow one to impose the restriction of quasi-convexity.

(19) In the earlier Ewis-Fisher [20] paper a variable which was successfully included was for "foreign assets" (in the demand for money). We made a number of experiments with foreign assets but were unable to obtain convergence. Two factors seem involved, as near as we can tell: (1) there is a high degree of auto-correlation in the tested equations and (2) the share of the foreign assets in total asset holdings is just too low for a precise estimate to be effected. In both papers the foreign assets variable was U. S. nondirect private claims on foreigners and the corresponding interest rate was a four country weighted average of a three-month interest rate (from the FRB Multi-Country Model Data Base).

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