The Classroom Assignment Problem (CAP): For Universities with Separate Male/Female Classes

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Contents

Figures Index iii
Tables Index iii
1. Introduction 1
2. Previous Work 1
3. Problem Definition 2
4. General Observations 3
5. Definitions 4
6. Notation 6
7. The Mathematical Model 6
    7.1 Conflict Resolving Constraints 6
    7.2 Room Assignment Constraints 7
    7.3 Class Assignment Constraints 7
    7.4 The Objective Function 7
8. Example 9
9. Problem Size 13
10. Implementation 13
11. The Timetabling Problem 15
12. Conclusions 15
Bibliography 17
APPENDIX 1

Figures Index

Figure 1: There a e 12, 2-hours, time slots in a day with 13 time penods........ 8

Figure 2: Classes in conflict with class i may not be assigned to slot 10. Classes in conflict with class *i'* may be assigned to slot 10........ 8

Figure 3: The best utilization of rooms by the 3 classes of the above example........ 11

Figure 4: Instructor Preference table........ 16

Tables Index

Table 1: Penalties assigned to a class of 40 students when assigned to any of the 4 different room sizes........ 5

Table 2: 3 rooms are available: a, b, and c. 3 classes are to be assigned: 1, 2, and 3. The *seating capacities* of the rooms, and the *number of students* in each class are shown........ 11

Table 3: Courses offered by the department........ 13

Table 4: The courses are broken into classes........ 13

Table 5: The set of rooms (R_i) available for class i (i=1,2,3,4,5), and the set of classes (I_r) that may be assigned to room r (r=1,2)........ 14

Table 6: Instructors' preferences for the 4 time slots........ 14

Table 7: Room assignment penalties........ 15

Table 8-a: Penalties for assigning classes to room 1-male and room 1-female in the four time slots........ 15

Table 8-b: Penalties for assigning classes to room 2-male and room 2-female in the four time slots........ 15

1. Introduction

Colleges and secondary schools over the world are faced with large classroom assignment problems every semester[1]. Most institutions solve the problem manually, and face a never ending chorus of complaints from instructors. A few places employ computer assisted heuristics.

The University of Qatar employs more than 10 experienced people every semester, approximately five weeks to develop the initial room assignments, this is in addition to one or more representative from each faculty. The process is tiring, time consuming and many assignments have to be changed more than once to incorporate last minute updates. Yet, the result is far from being optimal. Rooms are underutilized, a class conflict becomes a fact of life, and most instructors seem to be unhappy with their schedules.

Many techniques and approaches were presented in the literature. Some decomposed the problem in two sub problems: the *Timetabling Problem*, and the *Room Assignment Problem* [1]. The timetabling problem is concerned with the assignment of classes to time periods. The room assignment problem deals with the assignment of a timetable to rooms.

2. Previous Work

The literature includes approaches to solve the classroom assignment problem or special versions of it. Techniques used were applied to either one or both of the above mentioned problems. Some used heuristic methods, some used mathematical programming and some used both. Others introduced automated multi-stage solution methodology and devised interactive routines that rely on the insight of the scheduler [2, 3, 4, 5, 6, 7, 8, 9].

de Werra [10] presented a survey of timetabling procedures and techniques. *Barham* and *Westwood* [4] used a heuristic based methodology to reach solutions. They proposed a model with an objective to minimize the number of periods needed to attain feasibility. *Tripathy* [7] devised a similar model but used a Lagrangian relaxation approach in conjunction with a network algorithm that uses the out-of-kilter method for solving the sub problems. *Glassey* and *Mizrach* [5] devised a decision support system that utilizes flexible interactive methodology. *Mulvey* [8] devised a model to be used interactively by a decision maker who considers the constraints and objectives not directly built into the model. He developed a network based optimizing approach to the classroom/time model which rapidly approximates

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1 In this paper we are concerned with educational institutions that apply the credit hour system. For those with fixed schedule from year to year, the problem is much easier because the changes are minimal.

the solutions. His model combines the insight of the scheduler with combinatorial and searching ability of a computer via a transshipment optimization network model. His model however, assumes that all classes are of equal sizes and each can be assigned to any of the available equal length periods subject to the constraints of the model. Thus no overlapping can occur. This is rarely the case, All universities investigated, offer classes of lengths that range frome 1 to 5 hours a week.

Gosselin and *Truchon* [9] presented a two stage fully automated procedure for allocating classrooms between different courses and requests. He used a penalty function that is adapted in this paper. When a room of appropriate size is selected a penalty of 1 is used, when a larger room is used a penalty of 2 is used. If an even larger room is selected then a coefficient of 4 is used and so on. The penalties assigned to each of the four rooms shown in Table 1 below for a class of 40 students, are shown.

Room	No. of Seats	Penalty
1	50	1
2	100	2
3	150	4
4	200	8

Table 1

Penalties assigned to a class of 40 students when assigned to any of the 4 different room sizes.

Ferland and *Roy* [1] presented a mathematical programming approach. Their model consists of two sub problems that are solved sequentially. In the first sub problem, a timetable is specified such that class conflicts are minimized and the instructors' preferences for certain teaching hours are respected as much as possible. Then, given a timetable, classes are assigned to rooms according to their specific requirements such that room utilization is maximized. If no solution appears to be valid for the second problem, then the timetabling problem (the first problem) is solved again with constraints on certain peak hours to allow the spread of room usage over the time periods of a day.

3. Problem Definition

It is required that courses be assigned to time periods and rooms such that instructors' preferences and room utilization are optimized subject to course conflicting constraints and room availability constraints. The following are some examples of the problem constraints:

1- Each course must be assigned to a number of time periods in a week that equals the credit hours honored for that course (or weekly work load).

2- Courses of the same level (courses that are to be attended by one student) may not meet in the same time period or two overlapping time periods.

3- An instructor may not be assigned to a male class in time period *q*, if he is assigned to a female class in time period *q*-1. This is because female classes start 15 minutes after the male classes, to allow the instructor enough time to move between the male and female buildings.

4- An instructor is assigned to any time period only once.

5- Many courses meet more than once weekly. Such courses may not meet more than once in any day.

6- A room may not be assigned more than one class meeting in any given time period in any day.

4. General Observations

It was noticed that courses offered from year to year in each semester in some of the universities in the Gulf Cooperative Council (GCC) countries rarely change[2]. That is, courses offered in the spring semester of 1993 are almost the same that were offered in the spring semester of 1992. Therefore, it is safe to assume that, once the problem is solved for a given academical year, then it can be copied for a number of years while keeping an eye on the magnitude of the accumulated change and adjust the solutions. Another run may be necessary every 3 to 5 years depending on the nature and type of changes occurring (i.e., the number of rooms available, the instructors, the number of students, courses being added to the program or deleted, etc.). Some changes are less frequently occurring than others. Many changes can be planned for in advance, thus reducing the effect on an already constructed timetable.

Another fact was also evident. Most faculties have their own set of rooms. Or a set of rooms that are available during certain hours in a week. Thus, it is possible to decompose the problem into smaller sub problems, one for each faculty (or department). It can be easily shown that such a decomposition, although does not guarantee optimality, it assures feasibility. This is of course, under the assumption that the main problem is feasible, see [11]. The problem decomposition may not be straight forward. This is because each faculty will have rooms of its own along with other rooms that are shared with other faculties. In this case class assignments may be made based on a priority list. If a room is shared between two faculties then one of the two will have a priority over the other in this room's usage. In this

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2 Based on a review of courses offered in the University of Qatar (over the past 7 years), the University of the United Arab Emirates (over the past 3 years), and the University of Kuwait (over the past 3 years).

case the problem is solved for the first faculty and the remaining unscheduled periods for this room may be used by the other faculty. This is easily done by removing the variables associated with the assignment of any course to this room in the periods occupied by the first faculty from the variable set of the problem of the second faculty (or assigning a value of zero to those variables). To simplify the model presentation it is assumed that such a case is not present and that problem decomposition is possible and straight forward. That is, each faculty has a predefined set of rooms that it can use. Another difficulty that may arise, is the case when an instructor teaches courses in more than one faculty. This can also be handled in a manner similar to the room sharing case. A higher priority will be given to courses taught in the faculty the instructor belongs to. Later, when solving the assignment problem of the other faculty, the time periods that he was assigned to in the first faculty will be blocked for assignment in the second faculty. This of course, will have no major effect on the preferences of the instructor. His preferences will still be respected as much as possible in each of the two problems.

In this paper a new restriction is introduced. An instructor may be assigned to male or female classes. Some universities (like the University of Qatar) have different buildings and rooms for male and female classes. Actually, the male classes are in a different location than the female classes. This requires extra time for the instructor to move between them. In the University of Qatar, male classes start 15 minutes earlier than female classes. That is, the first male class meets at 7:15 am, where the first female class meets at 7:30 am. This creates a problem, because the class periods do not match. There are rooms and resources for male classes and different rooms and resources for female classes, but the instructors, in most cases, are the same.

5. Definitions

Time period: A time period is the length of the shortest class meeting in any day. In most cases a class period starts every hour, with a length of 55 minutes, and a 5 minutes break.

Class: A part of the course that consists of a single meeting in a day (it may occupy 1 or 2 consecutive time periods).

Time slot: A time slot is the longest class meeting. A time slot consists of 1, 2 or more time periods. It is fixed for each problem solved. In this paper we will assume that a time slot is of length 2. See Figure 1 below.

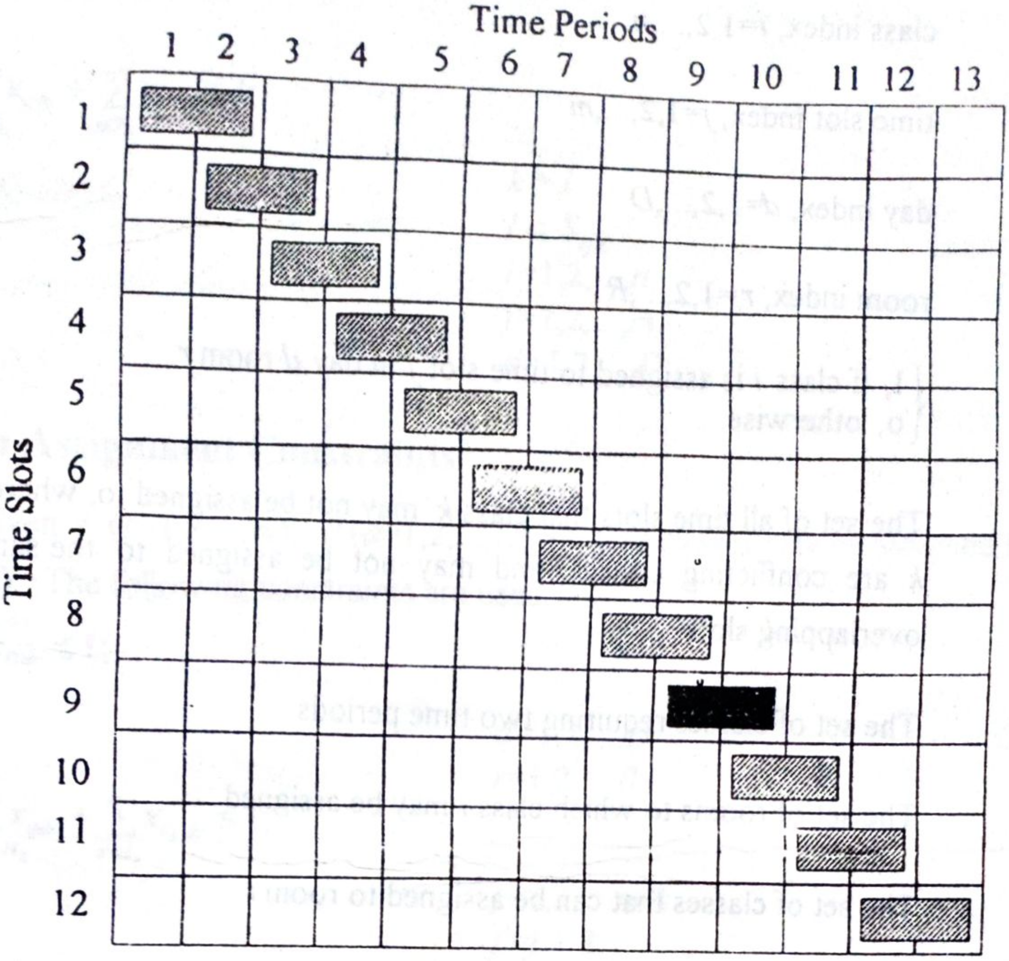

Figure 1

There are 12, 2-hours, time slots in a day with 13 time periods.

If a two hour class i is assigned to slot 9, then slot 10 will not be available for classes in conflict with class i (or to the same room). But if a one hour class i' is assigned to slot 9, then slot 10 will be available to classes in conflict with class i'. See Figure 2 below.

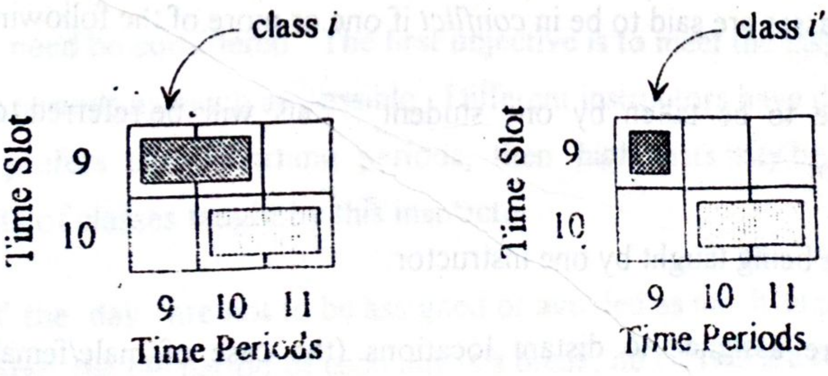

Figure 2

Classes in conflict with class i may not be assigned to slot 10. Classes in conflict with class ... is assigned to slot ...

6. Notation

i = class index, i=1,2,...,n

j = time slot index, j=1,2,...,m

d = day index, d=1,2,...,D

r = room index, r=1,2,...,R

x_{ijdr} = $\begin{cases} 1, & \text{if class } i \text{ is assigned to time slot } j \text{ in day } d \text{ room } r \\ 0, & \text{otherwise} \end{cases}$

S_{ijk} = The set of all time slots that class k may not be assigned to, where classes i and k are conflicting classes, and may not be assigned to the same slot or two overlapping slots.

H_2 = The set of classes requiring two time periods

R_i = The set of rooms to which class i may be assigned.

I_r = The set of classes that can be assigned to room r.

t_{ij} = The cost of assigning class i ($i \in R_i$) to slot j (j=1,2,...,m) in any day any room.

f_{ir} = The cost of assigning class i (i=1,2,...,n) to room r ($r \in R_i$).

7. The Mathematical Model

The mathematical model is defined in terms of the following constraints and objective function.

7.1 Conflict Resolving Constraints

Two or more classes are said to be in *conflict* if one or more of the following is true:

a. They are to be taken by one student. This will be referred to as the *course level* constraint.

b. They are being taught by one instructor.

c. They are assigned to distant locations (the case of male/female locations in the University of Qatar). Enough time must be allowed for the instructor and/or the students to move between the two distant locations.

Conflicting classes may not be assigned to the same time periods. The following constraints are used:

$$\sum_{r \in R_i} x_{ijdr} + \sum_{s \in R_k} x_{klds} \leq 1; \tag{1}$$

$$k > i$$
$$l \in S_{ijk}$$
$$i = 1,2,...,n$$
$$j = 1,2,...,m$$
$$d = 1,2,...,D$$

7.2 Room Assignment Constraints

For any given j $(j=1,2,...,m)$, d $(d=1,2,...,D)$, no two classes may be assigned to any room r $(r=1,2,...,R)$. The following constraints are used:

$$\sum_{i \in I_r} x_{i1dr} \leq 1; \tag{2}$$

$$d = 1,2,...,D$$
$$r = 1,2,...,R$$

$$\sum_{i \in I_r \cap H_2} x_{ijdr} + \sum_{i' \in I_r} x_{i'j'dr} \leq 1; \tag{3}$$

$$j = 2,3,...,m-1$$
$$j' = j + 1$$
$$d = 1,2,...,D$$
$$r = 1,2,...,R$$

7.3 Class Assignment Constraints

Each class must be assigned exactly once.

$$\sum_{j=1}^{m} \sum_{d=1}^{D} \sum_{r \in R_i} x_{ijdr} = 1; \tag{4}$$

$$i = 1,2,...,n$$

7.4 The Objective Function

Two objectives need be considered. The first objective is to meet the instructors' preferences for special time periods as much as possible. Different instructors have different preferences. If an instructor prefers early morning periods, then high costs may be associated with late period assignments of classes taught by this instructor.

Certain hours of the day are not to be assigned or avoided as much as possible. (i.e., in the University of Qatar the 6th period of each day is a break, no classes are to be assigned to this period, except in cases where a feasible solution would not be obtained otherwise). Heavy penalties can be associated with such periods. It may also be desirable to leave a set of periods without assignment, to allow for special meetings (i.e., department board meetings).

A scale of 10 may be used, where a value of 1 means that the time period is most preferred, and a value of 10 means it is least preferred and is to be avoided if possible.

A penalty value t_{ijdr} is associated with the assignment of class i (i=1,2,...,n) to time slot j (j=1,2,...,m) in day d (d=1,2,...,D) and room r ($r \in R_i$).

The second objective is to maximize the room utilization. A room is said to be fully utilized when the number of students of a class assigned to it, is equal to the seating capacity of that room. Consider the example presented in Table 2 below.

Room	Seating Capacity
a	30
b	60
c	90

Class	Number of Students
1	40
2	20
3	80

Table 2

3 rooms are available: a, b, and c. 3 classes are to be assigned: 1, 2, and 3. The *seating capacities* of the rooms, and the *number of students* in each class are shown.

The best utilization is obvious. Assuming there are no conflicts between these three classes, only one time period is needed. See Figure 3 for illustration.

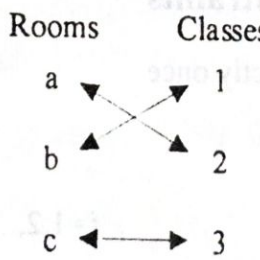

Figure 3

The best utilization of rooms by the 3 classes of the above example.

Note that any other assignment will mean that at least one room will be empty while another room is busy during the day. Consider the case where class 1 is assigned to room b, and class 2 to room c, in the first time period. Class 3 has to wait until class 2 is done, because room c is the only room that it can be assigned to. A cost can be associated with class assignments to each of the rooms available. When class i is assigned to the smallest room in R_i, a cost of 1 is incurred. A cost of 2 is incurred for the assignment of class i to the second smallest room in R_i, 4 to the next size, and so on.

Thus, a penalty of f_{ijdr} is incurred when class i (i=1,2,...,n) is assigned to room r ($r \in R_i$) in any time slot j (j=1,2,...,m) in day d (d=1,2,...,D).

An objective function coefficient, c_{ijdr} is computed as the product of the two penalty functions: t_{ijdr} and f_{ijdr}.

The objective function of the model is given below.

$$\text{Minimize} \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{d=1}^{D}\sum_{r \in R_i} c_{ijdr} x_{ijdr} \qquad (5)$$

The Classroom assignment problem (CAP) is given below:

$$\text{Minimize} \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{d=1}^{D}\sum_{r \in R_i} c_{ijdr} x_{ijdr}$$

subject to:

$$\sum_{r \in R_i} x_{ijdr} + \sum_{s \in R_k} x_{klds} \le 1; \qquad k > i,\ l \in S_{ijk},\ i=1,2,...,n,\ j=1,2,...,m,\ d=1,2,...,D$$

$$\sum_{i \in I_r} x_{i1dr} \le 1; \qquad d=1,2,...,D,\ r=1,2,...,R$$

$$\sum_{i \in I_r \cap H_2} x_{ijdr} + \sum_{i' \in I_r} x_{i'j'dr} \le 1; \qquad j=2,3,...,m-1,\ j'=j+1,\ d=1,2,...,D,\ r=1,2,...,R$$

$$\sum_{j=1}^{m}\sum_{d=1}^{D}\sum_{r \in R_i} x_{ijdr} = 1; \qquad i=1,2,...,n$$

$$x_{ijdr} = 0,1; \qquad i=1,2,...,n,\ j=1,2,...,m,\ d=1,2,...,D,\ r \in R_i$$

8. Example

Consider the case where courses of one department are to be assigned. Two rooms of capacity 50 and 100 are available in the male section of the university, and similar rooms in the female section. Table 3 below summarizes the problem data.

Course	M/F [3]	Student Level	Size Expected	Instructor	Credit Hours
1	M	1	70	A	3
2	M/F	2	40	B	2
3	F	2	60	C	1

Table 3

Courses offered by the department.

Courses that require 3 hours in a week, are broken in two classes, a two hour class and a one hour class. Courses that require 2 hours a week, are represented by classes that occupy two connected time periods. Courses that require only 1 hour a week, are represented by classes that occupy one time period. Classes are to be arranged so that the smallest classes are first on the list, followed by the larger classes. Table 4 below expands and reorders data given in Table 3 above.

Course	Class	M/F	Student Level	Expected Size	Instructor	Length of the class
1	1	M	1	70	a	1
3	2	F	2	60	a	1
2	3	M	2	40	b	2
2	4	F	2	40	b	2
1	5	M	1	70	a	2

Table 4

The courses are broken into classes.

The sets of the problem are defined in Table 5 below.

[3] M, F indicates whether the course is being offered to the *male* or *female* students, respectively.

Male Classes		Female Classes	
i	R_i	i	R_i
1	2	2	2
3	1,2	4	1,2
5	2		

r	I_r	r	I_r
1	3	1	4
2	1,3,5	2	2,4

Table 5

The set of rooms (R_i) available for class i (i=1,2,3,4,5), and the set of classes (I_r) that may be assigned to room r (r=1,2).

For simplicity purposes, it is assumed that the instructor's preferences do not change from day to day. That is, the cost of assigning class i (i=1,2,...,5) to time slot j (j=1,2,...,4) is the same for all days d (d=1,2,...,5) and all rooms r ($r \in R_i$). Table 6 below summarizes the preferences of the three instructors, where a lower value indicates a higher preference and a higher value indicates a lower preference.

	Time slot (j)			
class (i)	1	2	3	4
1	5	2	1	10 [4]
2	5	2	1	10
3	1	2	10	2
4	1	2	10	2
5	5	2	1	10

Table 6

Instructors' preferences for the 4 time slots.

Similarly, assume that the penalty of assigning class i (i=1,2,...,5) to room r ($r \in R_i$) is fixed in all time slots j (j=1,2,...,4) and days d (d=1,2,...,5). Table 7 below, summarizes the cost of assigning class 1 to room r (where $r \in R_1$). A penalty of 1 in the table, is associated with the assignment of class 1 to the smaller class in R_1, a penalty of 2 is used otherwise.

Although classes 1, 2, and 5 are taught by the same instructor, they do not have the same preferences. An instructor may be asked to select a general preference for all classes (courses), or a special preference for each class (course)

	Male Classes		Female Classes	
Class (i)	Room 1	Room 2	Room 1	Room 2
1	-	1	N/A[5]	-
2	N/A[6]	-	-	1
3	1	2	N/A	-
4	N/A	-	1	2
5	-	1	N/A	-

Table 7

Room assignment penalties

Finally, if we assume (without loss of generality) that the penalty of assigning classes to rooms, will not change from day to day, then the penalties may be given as shown in Tables 8-a and 8-b below.

	Male classes Time slots (j) 1	2	3	4	Female classes Time slots (j) 1	2	3	4
1	-	-	-	-				
2					-	-	-	-
3	1	2	10	2				
4					1	2	10	2
5	-	-	-	-				

Table 8-a

Penalties for assigning classes to room 1-male and room 1-female in the four time slots.

	Male classes Time slots (j) 1	2	3	4	Female classes Time slots (j) 1	2	3	4
1	5	2	1	10				
2					5	2	1	10
3	2	4	20	4				
4					2	4	20	4
5	5	2	1	10				

Table 8-b

Penalties for assigning classes to room 2-male and room 2-female in the four time slots.

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5 This room is available only to the female classes. Classes 1, 3, and 5 are male classes.
6 This room is available only to the male classes. Classes 2, and 4 are female classes.

The objective function and problem constraints, are shown in Appendix A.

9. Problem Size

The CAP model, theoretically, consists of $n.m.D.R$ variables. In the example presented there are 200 variables. But room 1 is not accepted for classes 1, 2, and 5 because of the limited capacity. Thus the variables associated with assigning classes 1, 2, and 5 to room 1 over d=1,2,...,5 may be dropped from the problem. Therefore, the total number of variables is reduced to 140 (200-60).

The number of variables may further be reduced by removing all variables associated with the assignment of any class to a period with an instructor preference of 10. Thus, classes 1, 2 and 5 may not be assigned to period 4 in any day, and classes 3 and 4 may not be assigned to period 3 in any of the 5 lecture days. The number of variables is thus reduced to 115 variables.

The number of constraints is also dependent on the nature of the problem. It has been evident from the cases observed (in the University of Qatar) that the total size of the problem is actually less than the theoretical size by 30 to 60%.

10. Implementation

The CAP model was applied to the classroom assignment problem of classes offered by the faculty of Administrative Sciences and Economics in the University of Qatar in the Fall semester of 1993. During the first half of each semester, classes offered for the next semester are determined by each of the four scientific departments in the faculty. Special forms are then prepared by the scientific department to present the classes that are being offered. These forms include information like the number of groups for each course that is being offered, the instructor's name, the number of credit hours (weekly class hours), student level allowed to take the course, type of student group (male/female), maximum number allowed, and pre-requisites. These forms are then collected and approved by the dean of the faculty. The forms are then turned to a special scheduling committee. The scheduling committee contacts the instructors to obtain their special preferences. This is done via a form that the instructor fills. The form is actually a blank timetable, showing empty cells representing the different time slots. Each instructor inserts a mark "X" in the time slot that he wishes to avoid as much as possible, see Figure 4.

	1	2	3	4	5	B	6	7	8	9	10	11	12
Sat	X	X	X	X	X	X	X	X	X	X	X	X	X
Sun							X						
Mon							X						
Tue							X						X
Wed							X						X

Figure 4

Instructor Preference table.

Based on the problem constraints discussed earlier, the committee translates the classes offered to a timetable taking in consideration the instructor preferences as much as possible. This timetable is then handed over to another set of experienced personnel that try to assign these classes to appropriate rooms. The whole process is done manually, with a big opportunity for mistakes, and to a never ending instructor complaints. In many cases it becomes impossible to find rooms that fit the proposed timetable. If this is the case, then the timetable must be altered. This causes conflicts between classes that must be taken by one student, and in most cases the instructor is assigned to a time slot that is not convenient to him/her. In many situations the rooms are either over utilized (a set of rooms is full in all time slots every day) or under utilized (small classes occupy very large capacity rooms).

The proposed model will not only solve these problems, but will also attempt to solve the problem in one run. That is, assigning classes to time slots as well assigning them to appropriate rooms. Thus, a much easier process, a more satisfied instructor, a better utilized room and an improved overall schedule that increases the selection range for the student and improve the overall productivity of the university.

It was found that each instructor wants to have one day with no classes to use it for office hours. In addition, most instructors prefer morning over afternoon classes. Also, most instructors would not want to be assigned to the first time slot. This creates a shortage for morning time slots, especially 2nd, 3rd, 4th and 5th time slots. To reduce this problem, each instructor was requested to select at least one afternoon day. Based on this, and in an attempt to further reduce the size of the problem, the variables representing unwanted slots are removed from the problem. This may of course yield infeasible solutions. If an infeasible solution is generated, the removed variables may be plugged back into the model as needed. For example, if the set of feasible solutions is empty, then the least unwanted time slots is returned to the variable set of the problem. This requires that the personnel in charge of defining the model be in close contact with the instructor to determine which time slots to include.

If an instructor, prefers earlier classes, a larger penalty value is assigned to the late time slots. A proposed penalty value can be the index of the time slot. That is, time slot 1 would carry a penalty of 1, slot 2 a penalty of 2, and so on. If an instructor has a preference for afternoon classes, then higher penalties are given to the earlier time slots.

No class is to be assigned to the break period (the period between the fifth and sixth time slots) unless absolutely necessary, (e.g., feasibility may not be obtained otherwise). The break period is usually given a very high penalty that is higher than the maximum time slot penalty (twice the maximum time slot penalties would be appropriate). There are cases however, were the assignment of classes to the break period is not allowed at all. In such cases the variables representing the assignment of classes to the break period are removed from the problem.

Not all rooms are available for assignment to the faculty of Administrative Sciences and Economics. On the other hand, some classes require very large rooms. Thus, the room set is defined such that these rooms are actually available, fully or partially. If a room is partially available, i.e., in some time slots in some days, because it is being used by another faculty, then the variables corresponding to these assignments are also removed from the variable set of the problem.

The solution of this problem and improvement to the model will be the subject of a follow up paper that is under preparation by the author. However, the problem size although very big, it would be within acceptable sizes that can be handled with the support of a binary optimization code.

11. The Timetabling Problem

The timetabling problem considers the assignment of classes to time slots, irrespective of the room assignments. Simply said, we may drop the room index of the variables, thus reducing the size of the problem significantly. Many, as discussed earllier, have proposed solutions to the problem in two phases. The first phase produces the timetable, and the second phase assigns the timetable to the available rooms. It was found from the universities surveyed, that certain classes are assigned to certain rooms every time they are offered. This is due to the special requirements of these classes, the location of the rooms with respect to the department teaching the class, and/or due to special preferences by the instructors. In such cases, the assignment of classes to rooms is straight forward. The assignment of rooms may then be done manually, or via one of the techniques presented in the literature.

12. Conclusions

The Classroom Assignment Problem CAP, is a very large problem which is hard to solve. It was shown that the actual problem may be reduced in size by more than 50% of the actual theoretical size. The resulting model consists of a much smaller variable and constraint sets. The special case that is considered in this paper deals with the presence of two different sets of classes: male classes, and female classes. What makes the problem harder is the fact that the time slots of the male classes do not match those of the female classes. This was meant to allow enough time for the instructor to move between the male and female rooms. Special constraints were constructed to deal with this situation.

Also presented was a simple technique for calculating the penalty functions for the different assignments, which takes in consideration both, the instructor preferences and the maximum room utilization objectives simultaneously.

The use of this model would minimize the complaints of the instructors, provide a better utilization of the available rooms, and reduce the repeated effort of preparing such assignments manually (or even via computer assisted procedures).

Extensions to this paper include a detailed investigation of the constraint matrix of the problem in an attempt to further decompose the problem into smaller sets that are easier to solve. In addition, a man-machine interactive procedure may be developed, where the output of the above smaller sets of problems may be evaluated to determine lower and upper bounds for the room assignments as well as alternative penalty functions.

Bibliography

1- Ferland, Jacques A. and Roy, Serge, "Timetabling Problem for University as Assignment of Activities to Resources". Computers & Operations Research, Vol. 12 No. 2, 1985. (207-218)

2- Barr, R., Glover, D., and Klingman, D., "An Improved Version of the Out-of-kilter Method and a Comparative Study of Computer Codes". Mathematical Programming, Vol. 7, 1974. (60-86)

3- Bloomfield, S. D., and McScharry, M. M., "Preferential Course Scheduling". Interfaces, Vol. 9, 1979. (24-31)

4- Barham, A., and Westwood, J., "A Simple Heuristic to Facilitate Course Timetabling". Operations Research Society, Vol. 29, 1978. (1055-1060)

5- Glassey, C. Roger, and Mizrach, Michael, "A Decision Support System for Assigning Classes to Rooms". Interfaces, Vol. 16, 1986. (92-100)

6- Knauer, B. A., "Solution of a Timetable Problem". Computational Operations Research, Vol. 1, 1974. (363-375)

7- Tripathy, A., "A Lagrangian Rleaxation Approach to Course Timetabling". Journal of Operations Research Society, Vol. 31, 1980. (599-603)

8- Mulvey, John M., "A Classroom / Time Assignment Model". European Journal of Operational Research, Vol. 9, 1982. (64-70)

9- Gosselin, Karl, and Truchon, Michael, "Allocation of Classrooms by Linear Programming". Journal of Operational Research Society, Vol. 37, No. 6, 1986. (561-569)

10- de Werra, D., "An Introduction to Timetabling". European Journal of Operational Research, Vol. 19, No. 2, 1985. (151-162)

11- Carter Michael W. and Tovey, Craig A., "When Is the Classroom Assignment Hard?". The Journal of Operations Research Society of America, Vol. 40, Supplement 1, 1992. (S28-S39)

4- Arkin, Esther M. and Silverberg, Ellen B., "Scheduling Jobs With Fixed Start and End Times". Discrete Applied Mathematics, Vol. 18, 1987. (1-8)

12- Tovey, Craig A., "A Simplified NP-Complete Satisfiability Problem". Discrete Applied Mathematics, Vol. 8, 1984. (85-89)

APPENDIX

The mathematical model of the porblem given in the example is given below.

Minimize
$$5x_{1112} + 5x_{1122} + 5x_{1132} + 5x_{1142} + 5x_{1152} + 2x_{1212} + 2x_{1222} + 2x_{1232} + 2x_{1242} +$$
$$2x_{1252} + x_{1312} + x_{1322} + x_{1332} + x_{1342} + x_{1352} + 10x_{1412} + 10x_{1422} + 10x_{1432} +$$
$$10x_{1442} + 10x_{452} + 5x_{2112} + 5x_{2122} + 5x_{2132} + 5x_{2142} + 5x_{2152} + 2x_{2212} +$$
$$2x_{2222} + 2x_{2232} + 2x_{2242} + 2x_{252} + x_{2312} + x_{2322} + x_{2332} + x_{2342} + x_{2352} +$$
$$10x_{2412} + 10x_{2422} + 10x_{2432} + 10x_{2442} + 10x_{2452} + x_{3111} + 2x_{3112} + x_{3121} +$$
$$2x_{3122} + x_{3131} + 2x_{3132} + x_{3141} + 2x_{3142} + x_{3151} + 2x_{3152} + 2x_{3211} + 4x_{3212} +$$
$$2x_{3221} + 4x_{3222} + 2x_{3231} + 4x_{3232} + 2x_{3241} + 4x_{3242} + 2x_{3251} + 4x_{3252} +$$
$$10x_{3311} + 20x_{3312} + 20x_{3321} + 20x_{3322} + 10x_{3331} + 20x_{3332} + 10x_{3341} +$$
$$20x_{3342} + 10x_{3351} + 20x_{3352} + 2x_{3411} + 4x_{3412} + 2x_{3421} + 4x_{3422} + 2x_{3431} +$$
$$4x_{3432} + 2x_{3441} + 4x_{3442} + 2x_{3451} + 4x_{3452} + x_{4111} + 2x_{4112} + x_{4121} + 2x_{4122} +$$
$$x_{4131} + 2x_{4132} + x_{4141} + 2x_{4142} + x_{4151} + 2x_{4152} + 2x_{4211} + 4x_{4212} + 2x_{4221} +$$
$$4x_{4222} + 2x_{4231} + 4x_{4232} + 2x_{4241} + 4x_{4242} + 2x_{4251} + 4x_{4252} + 10x_{4311} +$$
$$20x_{4312} + 10x_{4321} + 20x_{4322} + 10x_{4331} + 20x_{4332} + 10x_{4341} + 20x_{4342} +$$
$$10x_{4351} + 20x_{4352} + 2x_{4411} + 4x_{4412} + 2x_{4421} + 4x_{4422} + 2x_{4431} + 4x_{4432} +$$
$$2x_{4441} + 4x_{4442} + 2x_{4451} + 4x_{4452} + 5x_{5112} + 5x_{5122} + 5x_{5132} + 5x_{5142} + 5x_{5152} +$$
$$2x_{5212} + 2x_{5222} + 2x_{5232} + 2x_{5242} + 2x_{5252} + x_{5312} + x_{5322} + x_{5332} + x_{5342} +$$
$$x_{5352} + 10x_{412} + 10x_{5422} + 10x_{5432} + 10x_{5442} + 10x_{5452}$$

Subject to the following constraints:

$x_{1112} + x_{4111} + x_{4112} \leq 1$
$x_{1122} + x_{4121} + x_{4122} \leq 1$
$x_{1132} + x_{4131} + x_{4132} \leq 1$
$x_{1142} + x_{4141} + x_{4142} \leq 1$
$x_{1152} + x_{4151} + x_{4152} \leq 1$
$x_{1212} + x_{4211} + x_{4212} \leq 1$
$x_{1222} + x_{4221} + x_{4222} \leq 1$
$x_{1232} + x_{4231} + x_{4232} \leq 1$
$x_{1242} + x_{4241} + x_{4242} \leq 1$
$x_{1252} + x_{4251} + x_{4252} \leq 1$
$x_{1312} + x_{4311} + x_{4312} \leq 1$
$x_{1322} + x_{4321} + x_{4322} \leq 1$
$x_{1332} + x_{4331} + x_{4332} \leq 1$
$x_{1342} + x_{4341} + x_{4342} \leq 1$
$x_{1352} + x_{4351} + x_{4352} \leq 1$
$x_{1412} + x_{4411} + x_{4412} \leq 1$
$x_{1422} + x_{4421} + x_{4422} \leq 1$
$x_{1432} + x_{4431} + x_{4432} \leq 1$
$x_{1442} + x_{4441} + x_{4442} \leq 1$
$x_{1452} + x_{4451} + x_{4452} \leq 1$

$x_{1112} + x_{2112} \leq 1$
$x_{1122} + x_{2122} \leq 1$
$x_{1132} + x_{2132} \leq 1$
$x_{1142} + x_{2142} \leq 1$

$x_{1152} + x_{2152} \leq 1$
$x_{1212} + x_{2212} \leq 1$
$x_{1222} + x_{2222} \leq 1$
$x_{1232} + x_{2232} \leq 1$
$x_{1242} + x_{2242} \leq 1$
$x_{1252} + x_{2252} \leq 1$
$x_{1312} + x_{2312} \leq 1$
$x_{1322} + x_{2322} \leq 1$
$x_{1332} + x_{2332} \leq 1$
$x_{1342} + x_{2342} \leq 1$
$x_{1352} + x_{2352} \leq 1$
$x_{1412} + x_{2412} \leq 1$
$x_{1422} + x_{2422} \leq 1$
$x_{1432} + x_{2432} \leq 1$
$x_{1442} + x_{2442} \leq 1$
$x_{1452} + x_{2452} \leq 1$

$x_{1112} + x_{5112} \leq 1$
$x_{1122} + x_{5122} \leq 1$
$x_{1132} + x_{5132} \leq 1$
$x_{1142} + x_{5142} \leq 1$
$x_{1152} + x_{5152} \leq 1$
$x_{1212} + x_{5212} \leq 1$
$x_{1222} + x_{5222} \leq 1$
$x_{1232} + x_{5232} \leq 1$

$x_{1242} + x_{5242} \leq 1$
$x_{1252} + x_{5252} \leq 1$
$x_{1312} + x_{5312} \leq 1$
$x_{1322} + x_{5322} \leq 1$
$x_{1332} + x_{5332} \leq 1$
$x_{1342} + x_{5342} \leq 1$
$x_{1352} + x_{5352} \leq 1$
$x_{1412} + x_{5412} \leq 1$
$x_{1422} + x_{5422} \leq 1$
$x_{1432} + x_{5432} \leq 1$
$x_{1442} + x_{5442} \leq 1$
$x_{1452} + x_{5452} \leq 1$

$x_{1112} + x_{2212} \leq 1$
$x_{1122} + x_{2222} \leq 1$
$x_{1132} + x_{2232} \leq 1$
$x_{1142} + x_{2242} \leq 1$
$x_{1152} + x_{2252} \leq 1$
$x_{1212} + x_{2312} \leq 1$
$x_{1222} + x_{2322} \leq 1$
$x_{1232} + x_{2332} \leq 1$
$x_{1242} + x_{2342} \leq 1$
$x_{1252} + x_{2352} \leq 1$
$x_{1312} + x_{2412} \leq 1$
$x_{1322} + x_{2422} \leq 1$
$x_{1332} + x_{2432} \leq 1$
$x_{1342} + x_{2442} \leq 1$
$x_{1352} + x_{2452} \leq 1$

$x_{1112} + x_{5212} \leq 1$
$x_{1122} + x_{5222} \leq 1$
$x_{1132} + x_{5232} \leq 1$
$x_{1142} + x_{5242} \leq 1$
$x_{1152} + x_{5252} \leq 1$
$x_{1212} + x_{5312} \leq 1$
$x_{1222} + x_{5322} \leq 1$
$x_{1232} + x_{5332} \leq 1$
$x_{1242} + x_{5342} \leq 1$
$x_{1252} + x_{5352} \leq 1$
$x_{1312} + x_{5412} \leq 1$
$x_{1322} + x_{5422} \leq 1$
$x_{1332} + x_{5432} \leq 1$
$x_{1342} + x_{5442} \leq 1$
$x_{1352} + x_{5452} \leq 1$

$x_{2112} + x_{5112} \leq 1$
$x_{2122} + x_{5122} \leq 1$
$x_{2132} + x_{5132} \leq 1$
$x_{2142} + x_{5142} \leq 1$
$x_{2152} + x_{5152} \leq 1$
$x_{2212} + x_{5212} \leq 1$
$x_{2222} + x_{5222} \leq 1$
$x_{2232} + x_{5232} \leq 1$
$x_{2242} + x_{5242} \leq 1$
$x_{2252} + x_{5252} \leq 1$
$x_{2312} + x_{5312} \leq 1$
$x_{2322} + x_{5322} \leq 1$
$x_{2332} + x_{5332} \leq 1$
$x_{2342} + x_{5342} \leq 1$
$x_{2352} + x_{5352} \leq 1$
$x_{2412} + x_{5412} \leq 1$
$x_{2422} + x_{5422} \leq 1$
$x_{2432} + x_{5432} \leq 1$
$x_{2442} + x_{5442} \leq 1$
$x_{2452} + x_{5452} \leq 1$

$x_{3111} + x_{3112} + x_{4111} + x_{4112} \leq 1$
$x_{3121} + x_{3122} + x_{4121} + x_{4122} \leq 1$
$x_{3131} + x_{3132} + x_{4131} + x_{4132} \leq 1$
$x_{3141} + x_{3142} + x_{4141} + x_{4142} \leq 1$
$x_{3151} + x_{3152} + x_{4151} + x_{4152} \leq 1$
$x_{3211} + x_{3212} + x_{4211} + x_{4212} \leq 1$
$x_{3221} + x_{3222} + x_{4221} + x_{4222} \leq 1$
$x_{3231} + x_{3232} + x_{4231} + x_{4232} \leq 1$
$x_{3241} + x_{3242} + x_{4241} + x_{4242} \leq 1$
$x_{3251} + x_{3252} + x_{4251} + x_{4252} \leq 1$
$x_{3311} + x_{3312} + x_{4311} + x_{4312} \leq 1$
$x_{3321} + x_{3322} + x_{4321} + x_{4322} \leq 1$
$x_{3331} + x_{3332} + x_{4331} + x_{4332} \leq 1$
$x_{3341} + x_{3342} + x_{4341} + x_{4342} \leq 1$
$x_{3351} + x_{3352} + x_{43\,1} + x_{4352} \leq 1$
$x_{3111} + x_{3112} + x_{4211} + x_{4212} \leq 1$
$x_{3121} + x_{3122} + x_{4221} + x_{4222} \leq 1$
$x_{3131} + x_{3132} + \;_{4231} + x_{4232} \leq 1$
$x_{3141} + x_{3142} + x_{4241} + x_{4242} \leq 1$
$x_{3151} + x_{3152} \; x_{4251} + x_{4252} \leq 1$
$x_{3211} + x_{3212} + x_{4311} + x_{4312} \leq 1$
$x_{3221} + x_{322\,} + x_{4321} + x_{4322} \leq 1$
$x_{3231} + x_{32\;2} + x_{4331} + x_{4332} \leq 1$
$x_{3241} + x_{3\;42} + x_{4341} + x_{4342} \leq 1$
$x_{3251} + x_{\,252} + x_{4351} + x_{4352} \leq 1$

$x_{2112} + x_{\;112} + x_{5112} \leq 1$
$x_{2122} + \;_{3122} + x_{5122} \leq 1$

$$x_{2132} + x_{3132} + x_{5132} \leq 1$$
$$x_{2142} + x_{3142} + x_{5142} \leq 1$$
$$x_{2152} + x_{3152} + x_{5152} \leq 1$$

$$x_{1112} + x_{4112} \leq 1$$
$$x_{1122} + x_{4122} \leq 1$$
$$x_{1132} + x_{4132} \leq 1$$
$$x_{1142} + x_{4142} \leq 1$$
$$x_{1152} + x_{4152} \leq 1$$

$$x_{1112} + x_{1212} + x_{1312} + x_{1412} + x_{1122} + x_{1222} + x_{1322} + x_{1422} + x_{1132} + x_{1232} + x_{1332} + x_{1432} + x_{1142} + x_{1242} + x_{1342} + x_{1442} + x_{1152} + x_{1252} + x_{1352} + x_{1452} = 1$$

$$x_{2112} + x_{2212} + x_{2312} + x_{2412} + x_{2122} + x_{2222} + x_{2322} + x_{2422} + x_{2132} + x_{2232} + x_{2332} + x_{2432} + x_{2142} + x_{2242} + x_{2342} + x_{2442} + x_{2152} + x_{2252} + x_{2352} + x_{2452} = 1$$

$$x_{3111} + x_{3112} + x_{3211} + x_{3212} + x_{3311} + x_{3312} + x_{3411} + x_{3412} + x_{3121} + x_{3122} + x_{3221} + x_{3222} + x_{3321} + x_{3322} + x_{3421} + x_{3422} + x_{3131} + x_{3132} + x_{3231} + x_{3232} + x_{3331} + x_{3332} + x_{3431} + x_{3432} + x_{3141} + x_{3142} + x_{3241} + x_{3242} + x_{3341} + x_{3342} + x_{3441} + x_{3442} + x_{3151} + x_{3152} + x_{3251} + x_{3252} + x_{3351} + x_{3352} + x_{3451} + x_{3452} = 1$$

$$x_{4111} + x_{4112} + x_{4211} + x_{4212} + x_{4311} + x_{4312} + x_{4411} + x_{4412} + x_{4121} + x_{4122} + x_{4221} + x_{4222} + x_{4321} + x_{4322} + x_{4421} + x_{4422} + x_{4131} + x_{4132} + x_{4231} + x_{4232} + x_{4331} + x_{4332} + x_{4431} + x_{4432} + x_{4141} + x_{4142} + x_{4241} + x_{4242} + x_{4341} + x_{4342} + x_{4441} + x_{4442} + x_{4151} + x_{4152} + x_{4251} + x_{4252} + x_{4351} + x_{4352} + x_{4451} + x_{4452} = 1$$

$$x_{5112} + x_{5212} + x_{5312} + x_{5412} + x_{5122} + x_{5222} + x_{5322} + x_{5422} + x_{5132} + x_{5232} + x_{5332} + x_{5432} + x_{5142} + x_{5242} + x_{5342} + x_{5442} + x_{5152} + x_{5252} + x_{5352} + x_{5452} = 1$$

$$x_{ijdr} = 0, 1;$$

$i=1,2,\ldots,n$
$j=1,2,\ldots,m$
$d=1,2,\ldots,D$
$r \in R_i$