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ANALYSIS OF FRAMES RESTING ON ELASTIC FOUNDATIONS AND SUBJECTED TO UNSYMMYTRICAL LOADS

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ABSTRACT

This investigation includes the determination of the elastic critical summation of loading of different varieties of portal and box frames resting on elastic foundations and subjected to unsymmetrical loading. The critical buckling loads of these frames are obtained for practical values of the stiffness of the elastic springs. In addition, the effect of the stiffness of the elastic foundations on the elastic stability of these frames is thoroughly examined by changing the stiffness of the elastic springs. Different families of curves are obtained to represent the relationship between the critical buckling loads and the parameters affecting it. For unsymmetrical loading P1 & P2, The elastic critical sum of loading is found to be nearly constant and is irrespective of the load ratio $\frac{P_1}{P_2}$ even for the most severe case when $P_1 = zero$ and the whole load P_2 acts on the other column. In general, there was a complete analogy between frames subjected to unsymmetrical loads and resting on elastic springs and those resting on rigid supports for both portal and box frames.

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KEY WORDS

Structural Analysis, Elastic Stability, Frames, Elastic Foundations.

NOMENCLATURE

The following symbols are used in this paper:

P	External vertical acting load on a frame
E	Young's modulus of elasticity
M	Bending moment
k	Stiffness of column
$\mathbf{k_b}$	Stiffness of beam
$\mathbf{k_t}$	Stiffness of elastic translational spring
$\mathbf{k_r}$	Stiffness of elastic rotational string
Δ	Vertical displacement of a frame due to applied load only
Δ′	Vertical displacement of a frame due to secondary vertical forces V produced in columns due to side sway effect
$\mathbf{\Delta_1}$	Total vertical displacement at joint 1
$\mathbf{\Delta_2}$	Total vertical displacement at joint 2
θ	Angel of rotation
L	Length of beam or span of frame
h	Height of columns or height of story
H	Secondary force causing shear on columns produced due to side sway of frame
V	Secondary vertical forces produced in columns due to side sway effect
$\mathbf{P_{cr}}$	Elastic critical load
$\mathbf{P_E}$	[$\pi^2 EI/h^2$] Euler's load of column member
ρ	[P / P_E] Ratio of axial load to Euler's load
ρ_{cr}	[P_{cr} / P_E] Critical load parameter
s	μh [1- μh cot(μh)] / [2 tan (μh /2) – μh] Stiffness factor for fixed end member when side sway is prevented
c	[μh – sin(μh)] / [sin(μh) – μh cos(μh)] Carry-over factor for fixed end member when side sway is prevented
***s*”**	[s (1-c^2)] Stiffness factor for pinned end member when side sway is prevented
n	[μh cot(μh)] Stiffness factor for fixed end member which is in a state of no-shear sway
o	[μh cosec(μh)] Carry-over factor for fixed end member which is in a state of no-shear sway
***n*”**	[μh tan(μh)] Stiffness factor for pinned end member which is in a state of no-shear sway
m	[2 tan(μh /2) / μh] Magnification factor for moments produced at ends of fixed member which is in a state of pure-shear sway due to axial force effect

INTRODUCTION

Studying the elastic stability of frames resting on elastic foundations is very important because of its wide, useful and important applications such as; stability of structural frameworks resting directly on soil, which is considered as an elastic foundation, stability of buried and partially buried structures resting directly or surrounded by soil, and stability of floating bridges which are box frames with large bearing area resting on water, which is also considered as an elastic foundation, besides several types of special civilian and military structures. In this study, the analysis is performed on frames resting on vertical and horizontal elastic translational springs with and without elastic rotational springs and subjected to symmetrical loading. The elastic foundations are represented by elastic translational and rotational springs, which offer axial and rotational stiffness to resist translation and rotation. The elastic critical buckling loads are presented as a function of the stiffness of the translational elastic springs. They depend also on the stiffness of the rotational springs, the bending stiffness ratio of beams to columns.

The elastic stability of structural frameworks has been previously studied by many authors. The followings are brief highlights on some of these studies:

Livesley, R. K., and Chandler, D. B., [2] in 1956, derived the stability functions for structural frameworks and tabulated it for both axial compression and axial tension. *Merchant, W., and Salem, A. H. [3] in 1960,* studied the use of the stability functions in the analysis of rigid frames. *Zweig, A., and Kahn, A., [6] in 1968,* presented the buckling analysis of one story portal frames in which either columns with unequal stiffness are subjected to equal loads or columns with identical stiffness are subjected to unequal loads. *Salem, A. H., [7] in 1969,* studied the buckling analysis of one-story single-bay and multi-bay frames, studied before by *Zweig and Kahn,* using a quicker and direct method. It is based on the use of ready-prepared operations of rotations and sway of axially compressed struts and the resolution of the general state of sway of an isolated strut (or a structure as a whole) to the components of no-shear sway and pure-shear sway. The axial compression is the same in all cases. *Beshara, and Attabi, [8] in 1987,* studied the frame-foundation interaction and the effect of foundation stiffness on the critical buckling load of frames for the first time. *Hanna, M. T., et al., [10] in 1999,* studied the effect of variation of loads ratios on column on the elastic buckling strength of frameworks. *El-Desouky, O., et al., [11] in 2003,* studied the elastic stability of structural frameworks resting on elastic foundations.

METHOD OF ANALYSIS

General Assumptions

Besides the main assumptions of direct method of analysis, the following assumptions are applied to all cases of the present study:

a. No axial deformations in columns are considered and all axial displacements are due to compression of elastic springs.
b. Horizontal displacement of supports is prevented i.e. horizontal spring is very rigid ($k_{th} = \infty$).

If the horizontal displacement is permitted, the horizontal spacing between L.H.S. and R.H.S. supports (L) is kept constant by the use of a lower non-deformable member.

Direct Method

The applications carried out through the present study for the determination of the elastic critical loads of structural frameworks are all analyzed using The Direct Method of Analysis.

The direct method of stability analysis is based mainly on two main items; first, on the ready prepared operations of rotation and sway of axially compressed members, second, on the pre-study of the possible buckling modes of the frame and choosing the one which is expected to get the elastic critical load of a frame.

EQUATIONS OF THE ELASTIC CRITICAL LOAD OF FRAMES

Symmetric Portal Frame Subjected to Unsymmetrical Loading in an Unsymmetrical Sway Buckling Mode

Development of the elastic critical load equation

Consider the rectangular portal frame shown in Fig. 1.a resting on elastic vertical and horizontal translational springs and elastic rotational springs. The frame is subjected to two equal vertical loads at its top corners. When the vertical loads reach their critical value, any small sidesway can take place without requiring any external shearing forces. Thus the frame as a whole will be in a state of no-shear sway and the columns will also be in states of no-shear sway. Due to the small amount of sidesway of the frame, two equal and opposite secondary vertical forces V will be developed at the bases.

$$P_2 = k_t \Delta_2 \qquad\qquad P_1 = k_t \Delta_1$$

$$V = k_t \frac{\Delta'}{2} \qquad\qquad \Delta = \Delta_2 - \Delta_1 = \frac{P_2 - P_1}{k_t}$$

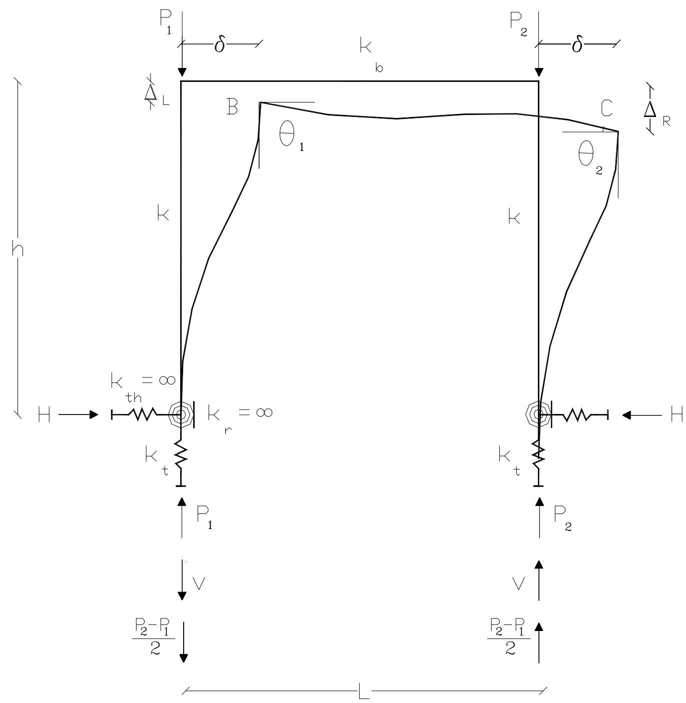

Fig. 1.a. Distorted configuration of the frame

$$4k_b(\theta_2 - \frac{\Delta_t}{L}) \qquad 2k_b(\theta_2 - \frac{\Delta_t}{L})$$

$$2k_b(\theta_1 - \frac{\Delta_t}{L}) \qquad 4k_b(\theta_1 - \frac{\Delta_t}{L})$$

$$n_2 k\theta_2 - \frac{m_2}{2}Hh \qquad C \qquad\qquad B \qquad n_1 k\theta_1 + \frac{m_1}{2}Hh$$

Fig. 1.b. Operations of sway and rotations

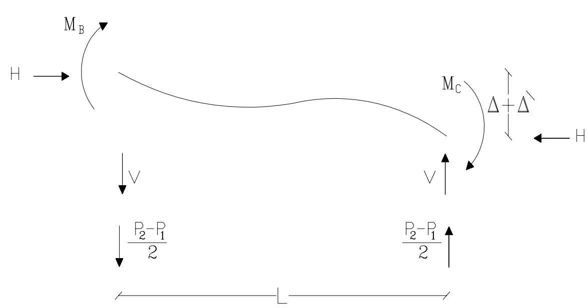

Fig. 1.c. Equilibrium of beam

$$\Delta_t = \Delta + \Delta'$$

$$\Delta_R = \Delta_2 + \frac{\Delta'}{2} \qquad \Delta_L = \Delta_1 - \frac{\Delta'}{2}$$

The operations of sway and rotations are built up for every member of the frame separately as shown in Fig. 1.b corresponding to the distorted configuration of the frame under the critical values of the vertical load. From these operations, the equation of the sway buckling load is obtained:

When $\boldsymbol{\frac{k_r}{k} = \infty}$;

$$\begin{matrix} \theta_1 & \theta_2 & \frac{Hh}{k} & \frac{\Delta_t}{L} \end{matrix}$$

$$\begin{vmatrix} (n_1 + 4\frac{k_b}{k}) & 2\frac{k_b}{k} & \frac{m_1}{2} & -6\frac{k_b}{k} \\ 2\frac{k_b}{k} & (n_2 + 4\frac{k_b}{k}) & -\frac{m_2}{2} & -6\frac{k_b}{k} \\ \frac{m_1}{2} & -\frac{m_2}{2} & -\left[\frac{m_1}{2s_1(1+c_1)} + \frac{m_2}{2s_2(1+c_2)}\right] & zero \\ -6\frac{k_b}{k} & -6\frac{k_b}{k} & zero & \frac{1}{2}(\frac{k_t}{k}L^2 + 24\frac{k_b}{k}) \end{vmatrix} = zero \qquad (1)$$

When $\frac{k_t}{k}L^2 = \infty$;

$$-(n_1 + 4\frac{k_b}{k})(n_2 + 4\frac{k_b}{k})\left[\frac{m_1}{2s_1(1+c_1)} + \frac{m_2}{2s_2(1+c_2)}\right] + (2\frac{k_b}{k})^2\left[\frac{m_1}{2s_1(1+c_1)} + \frac{m_2}{2s_2(1+c_2)}\right]$$

$$-2(\frac{m_1}{2})(\frac{m_2}{2})(2\frac{k_b}{k}) - \frac{m_2^{\,2}}{4}(n_1 + 4\frac{k_b}{k}) - \frac{m_1^{\,2}}{4}(n_2 + 4\frac{k_b}{k}) = zero$$

Which is the equation of critical buckling load of fixed base frame.

For symmetrical loading, $P_1 = P_2, H = zero, \theta_1 = \theta_2 = \theta, \Delta_t = \Delta'$

$$\begin{matrix} \theta & \frac{V}{k_t L} \end{matrix}$$

$$\begin{vmatrix} (n + 6\frac{k_b}{k}) & -12\frac{k_b}{k} \\ -12\frac{k_b}{k} & (\frac{k_t}{k}L^2 + 24\frac{k_b}{k}) \end{vmatrix} = zero \qquad (2)$$

Which is the equation of critical buckling load for the case of symmetrical loading.

When $\frac{k_r}{k} = zero$;

$$\begin{array}{cccc} \theta_1 & \theta_2 & \frac{Hh}{k} & \frac{\Delta_t}{L} \end{array}$$

$$\begin{vmatrix} (-\frac{\pi^2\rho_1}{n_1}+4\frac{k_b}{k}) & 2\frac{k_b}{k} & \frac{1}{n} & -6\frac{k_b}{k} \\ 2\frac{k_b}{k} & (-\frac{\pi^2\rho_2}{n_2}+4\frac{k_b}{k}) & -\frac{1}{n} & -6\frac{k_b}{k} \\ \frac{1}{n} & -\frac{1}{n} & -\left[\frac{1}{s_1'' n_1}+\frac{1}{s_2'' n_2}\right] & zero \\ -6\frac{k_b}{k} & -6\frac{k_b}{k} & zero & \frac{1}{2}(\frac{k_t}{k}L^2+24\frac{k_b}{k}) \end{vmatrix} = zero \qquad (3)$$

When $\frac{k_t}{k}L^2 = \infty$;

$$-(-\frac{\pi^2\rho_1}{n_1}+4\frac{k_b}{k})(-\frac{\pi^2\rho_2}{n_2}+4\frac{k_b}{k})\left[\frac{1}{s_1'' n_1}+\frac{1}{s_2'' n_2}\right]+(2\frac{k_b}{k})^2\left[\frac{1}{s_1'' n_1}+\frac{1}{s_2'' n_2}\right]$$
$$-2(\frac{1}{n_1})(\frac{1}{n_2})(2\frac{k_b}{k})-(\frac{1}{n_2})^2(-\frac{\pi^2\rho_1}{n_1}+4\frac{k_b}{k})-(\frac{1}{n_1})^2(-\frac{\pi^2\rho_2}{n_2}+4\frac{k_b}{k}) = zero$$

Which is the equation of critical buckling load of hinged base frame.

For symmetrical loading, $P_1 = P_2, H = zero, \theta_1 = \theta_2 = \theta, \Delta_t = \Delta'$

$$\begin{array}{cc} \theta & \frac{V}{k_t L} \end{array}$$

$$\begin{vmatrix} (-\frac{\pi^2\rho}{n}+6\frac{k_b}{k}) & -12\frac{k_b}{k} \\ -12\frac{k_b}{k} & (\frac{k_t}{k}L^2+24\frac{k_b}{k}) \end{vmatrix} = zero \qquad (4)$$

Which is the equation of critical buckling load for the case of symmetrical loading.

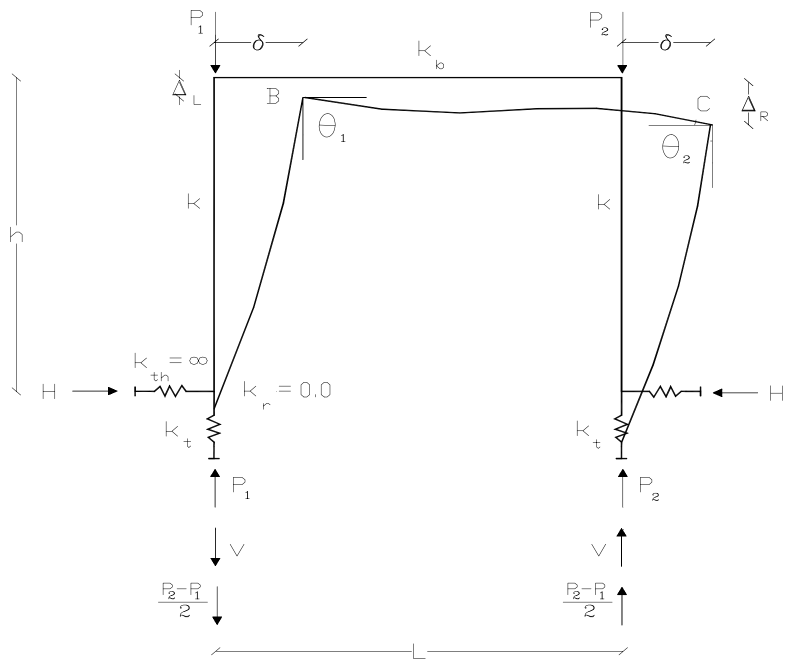

Fig. 2. Distorted configuration of the frame

These equations are solved by the method of trial and error. At each trial a remainder is obtained which may not be equal to zero. Several trials are made, using a more appropriate value for ρ_1 each time. Care is always taken that the values obtained of ρ_1 and ρ_2 are the least ones.

Results

The results are presented where the elastic critical sum of buckling loads divided by twice the Euler's load $\frac{(P_{cr_1} + P_{cr_2})}{2P_E}$ is plotted against the ratio of the two loads $\frac{P_1}{P_2}$ for different values of the stiffness ratio of beam to column $\frac{k_b}{k}$ and the stiffness ratio $\frac{k_t}{k} L^2$. The elastic critical sum of loading is found to be nearly constant and is irrespective of the load ratio $\frac{P_1}{P_2}$ even for the most severe case when $P_1 = zero$ and the whole load P_2 acts on the other column.

These results are represented graphically in Figs. 3, 4, 5, 6, 7, and 8.

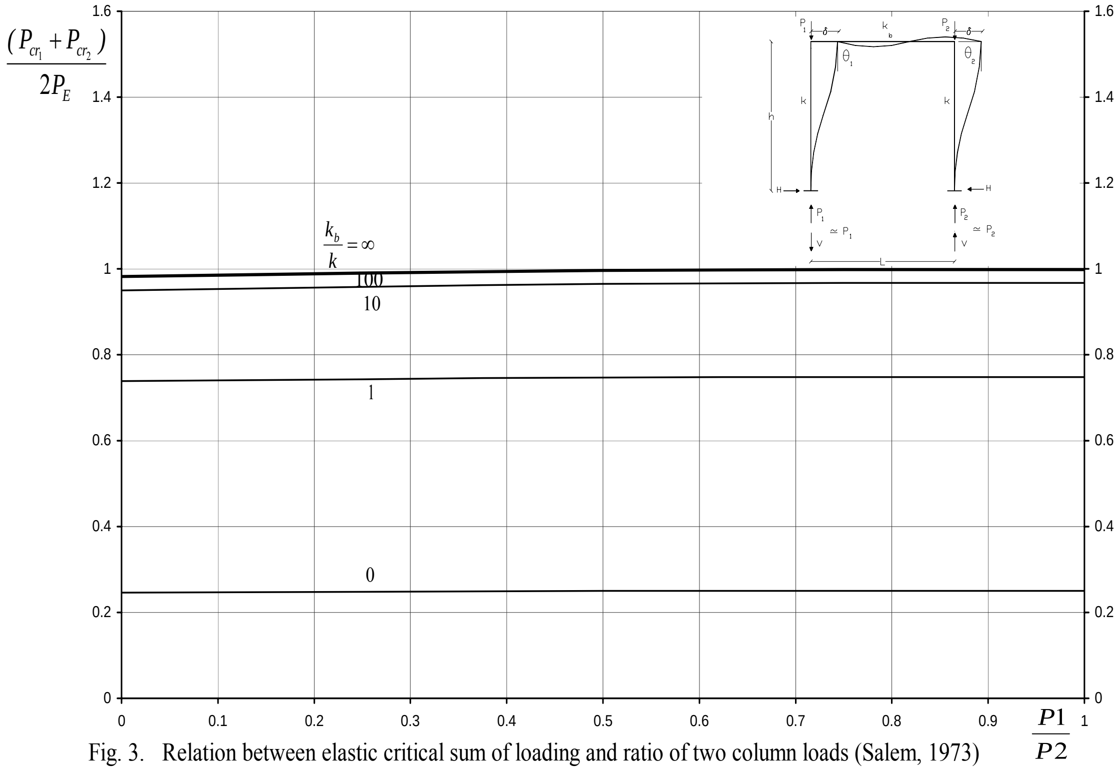

Fig. 3. Relation between elastic critical sum of loading and ratio of two column loads (Salem, 1973)

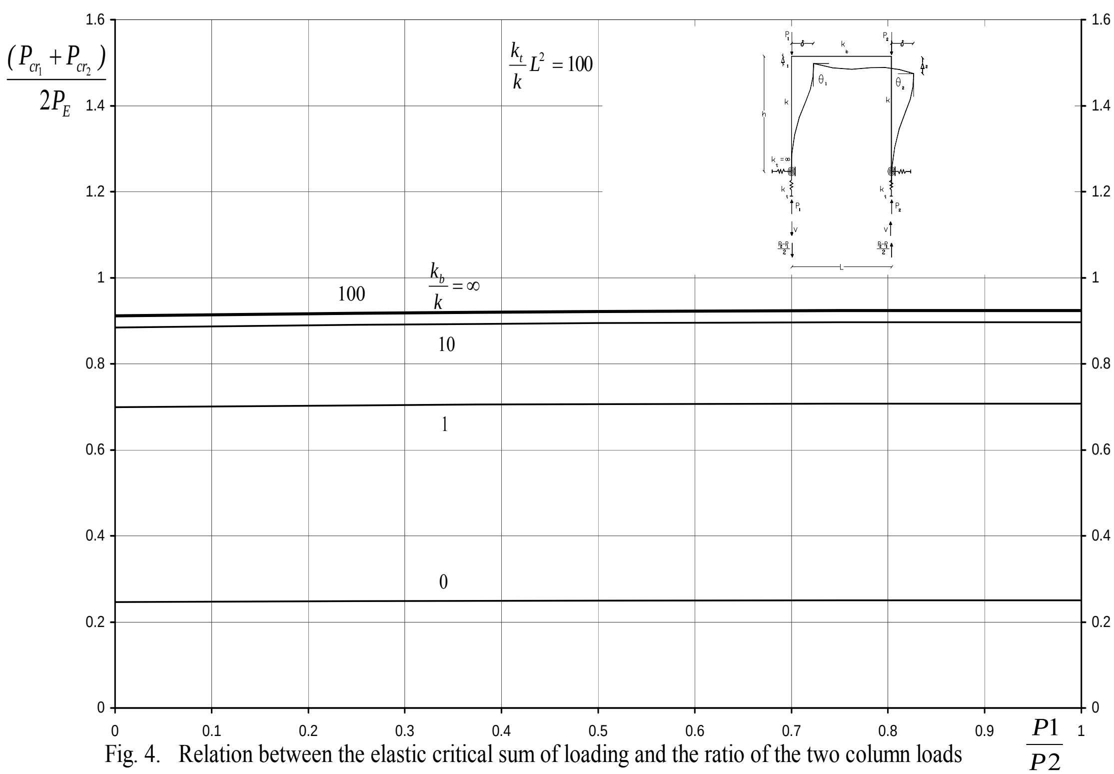

Fig. 4. Relation between the elastic critical sum of loading and the ratio of the two column loads

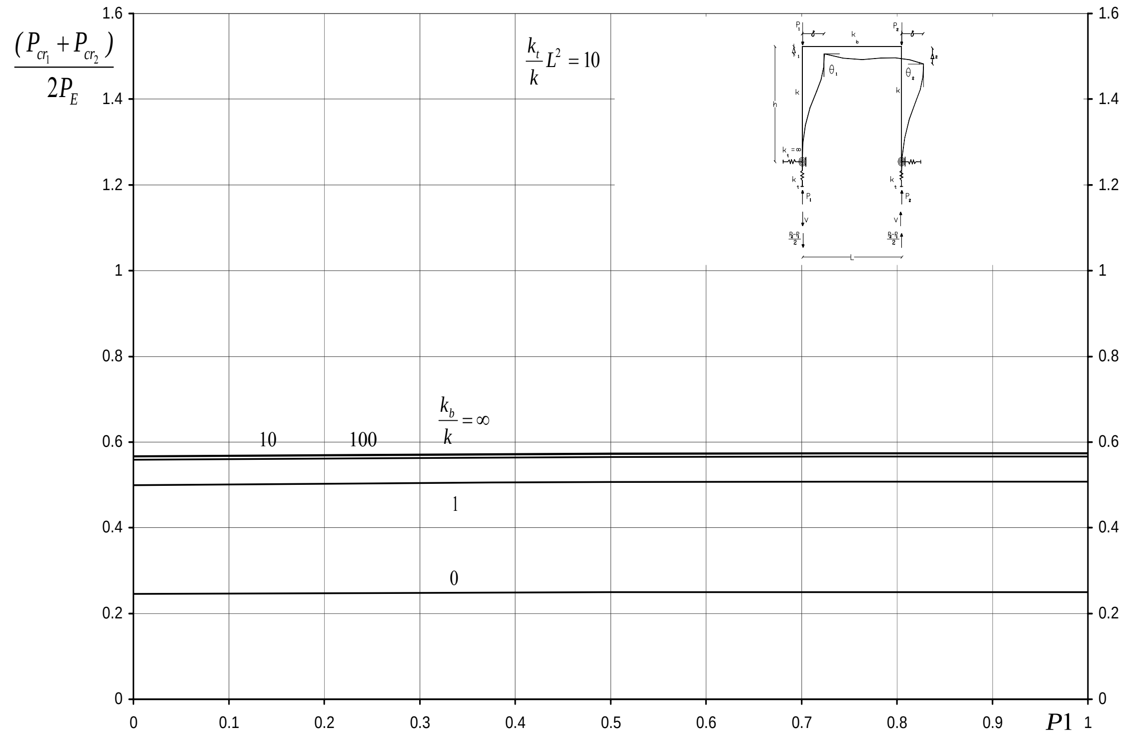

Fig. 5. Relation between the elastic critical sum of loading and the ratio of the two column loads

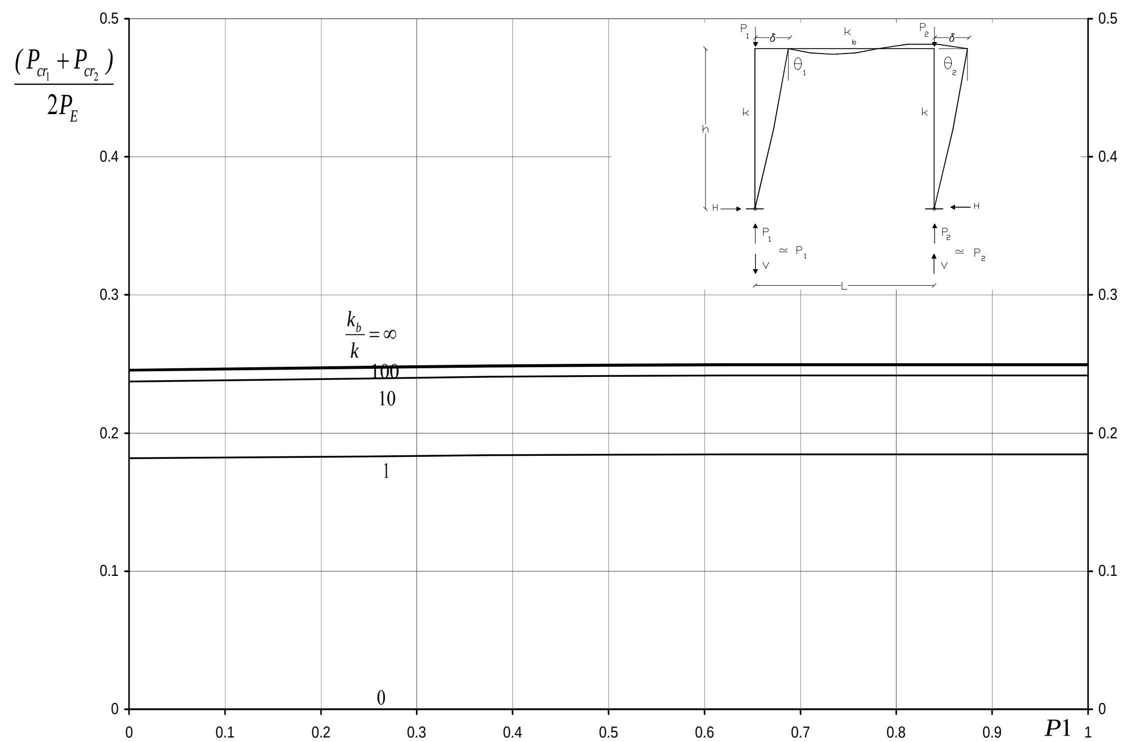

Fig. 6. Relation between elastic critical sum of loading and ratio of two column loads (Salem, 1958)

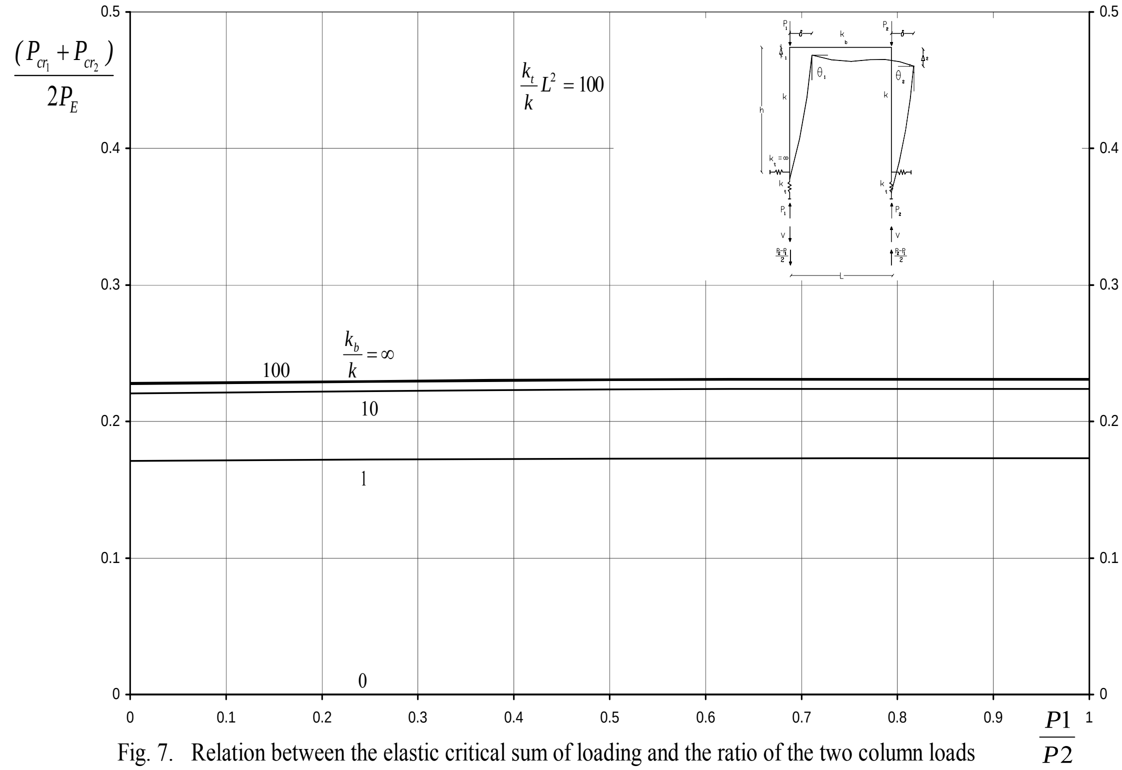

Fig. 7. Relation between the elastic critical sum of loading and the ratio of the two column loads

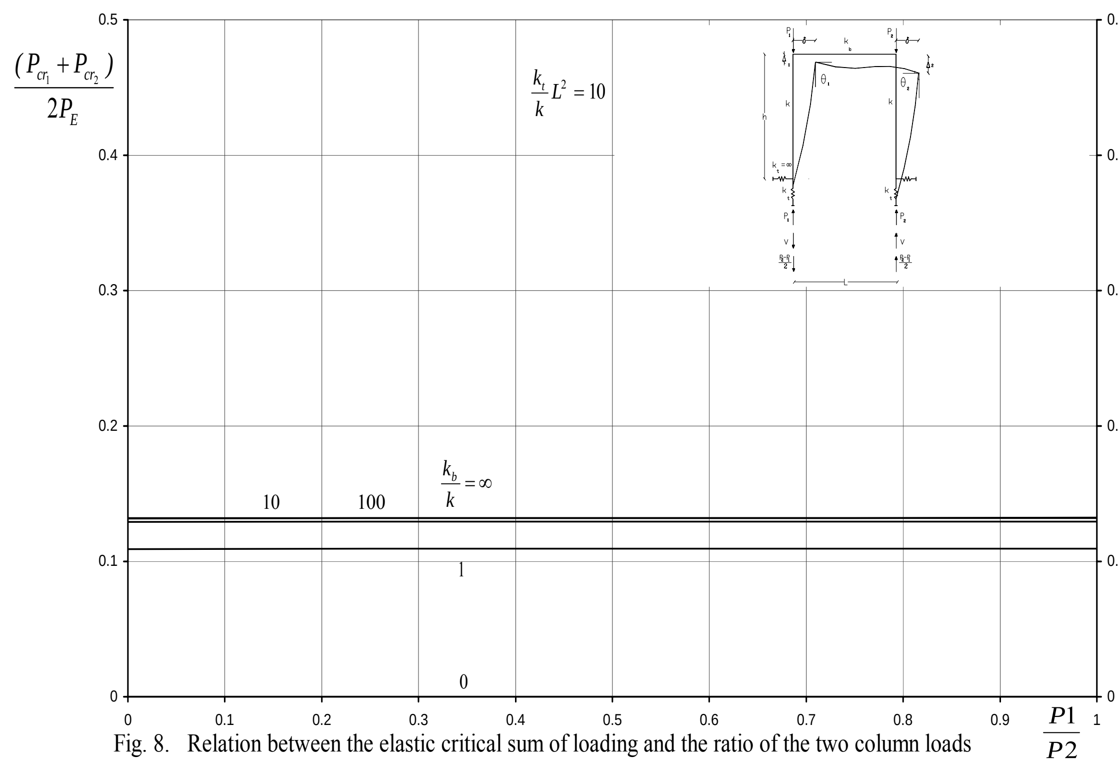

Fig. 8. Relation between the elastic critical sum of loading and the ratio of the two column loads

Symmetric Box Frame Subjected to Unsymmetrical Loading in an Unsymmetrical Sway Buckling Mode

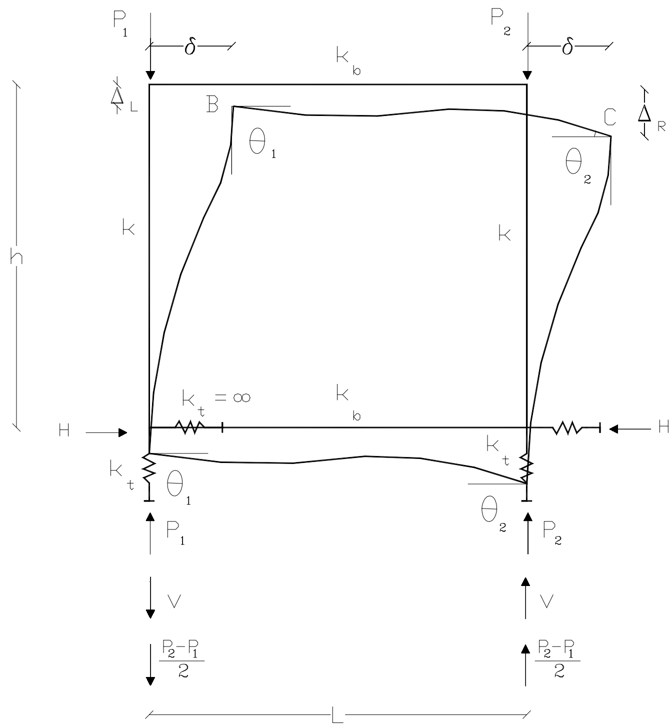

Fig. 9. Distorted configuration of the frame

Development of the elastic critical load equation

Consider the rectangular box frame shown in Fig. 9 and using the same displacements as for the previous case where H_o is the internal shear acting on columns, the equation of the critical buckling load can be developed as follows:

$$\begin{array}{cccc} \theta_1 & \theta_2 & \dfrac{H_o h}{k} & \dfrac{\Delta_t}{L} \end{array}$$

$$\begin{vmatrix} (n_1 - o_1 + 4\dfrac{k_b}{k}) & 2\dfrac{k_b}{k} & \dfrac{m_1}{2} & -6\dfrac{k_b}{k} \\ 2\dfrac{k_b}{k} & (n_2 - o_2 + 4\dfrac{k_b}{k}) & -\dfrac{m_2}{2} & -6\dfrac{k_b}{k} \\ \dfrac{m_1}{2} & -\dfrac{m_2}{2} & -\dfrac{1}{2}\left[\dfrac{m_1}{2s_1(1+c_1)} + \dfrac{m_2}{2s_2(1+c_2)}\right] & zero \\ -6\dfrac{k_b}{k} & -6\dfrac{k_b}{k} & zero & \dfrac{1}{4}(\dfrac{k_t}{k}L^2 + 48\dfrac{k_b}{k}) \end{vmatrix} = zero$$

(5)

When $\frac{k_t}{k}L^2 = \infty$;

$$-\frac{1}{2}(n_1 - o_1 + 4\frac{k_b}{k})(n_2 - o_2 + 4\frac{k_b}{k})\left[\frac{m_1}{2s_1(1+c_1)} + \frac{m_2}{2s_2(1+c_2)}\right] - 2(\frac{m_1}{2})(\frac{m_2}{2})(2\frac{k_b}{k})$$

$$+(2\frac{k_b}{k})^2\left[\frac{m_1}{2s_1(1+c_1)} + \frac{m_2}{2s_2(1+c_2)}\right] - \frac{m_2^{\,2}}{4}(n_1 - o_1 + 4\frac{k_b}{k}) - \frac{m_1^{\,2}}{4}(n_2 - o_2 + 4\frac{k_b}{k}) = zero$$

Which is the equation of critical buckling load of box frame resting on rigid supports and subjected to unsymmetrical loading P_1, P_2.

For symmetrical loading, $P_1 = P_2, H_o = zero, \theta_1 = \theta_2, \Delta_t = \Delta'$

$$\begin{matrix} \theta & \frac{V}{k_t L} \end{matrix}$$

$$\begin{vmatrix} (n - o + 6\frac{k_b}{k}) & -12\frac{k_b}{k} \\ -12\frac{k_b}{k} & \frac{1}{2}(\frac{k_t}{k}L^2 + 48\frac{k_b}{k}) \end{vmatrix} = zero \qquad (6)$$

Which is the equation of critical buckling load for symmetrical loading.

Results

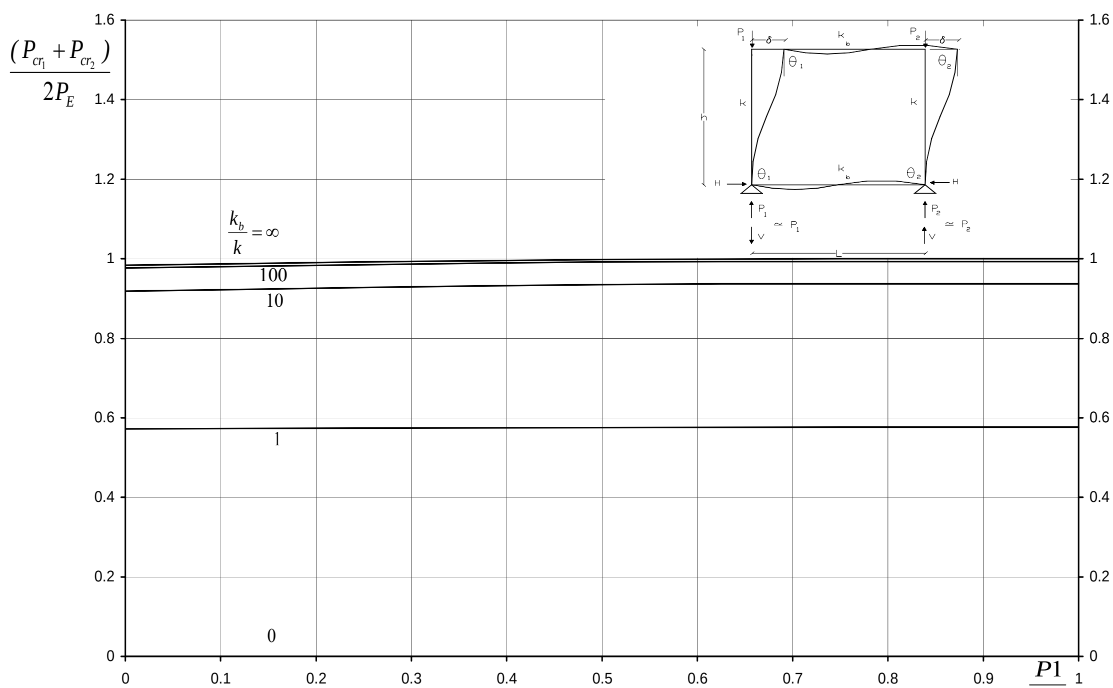

Fig. 10. Relation between the elastic critical sum of loading and the ratio of the two column loads

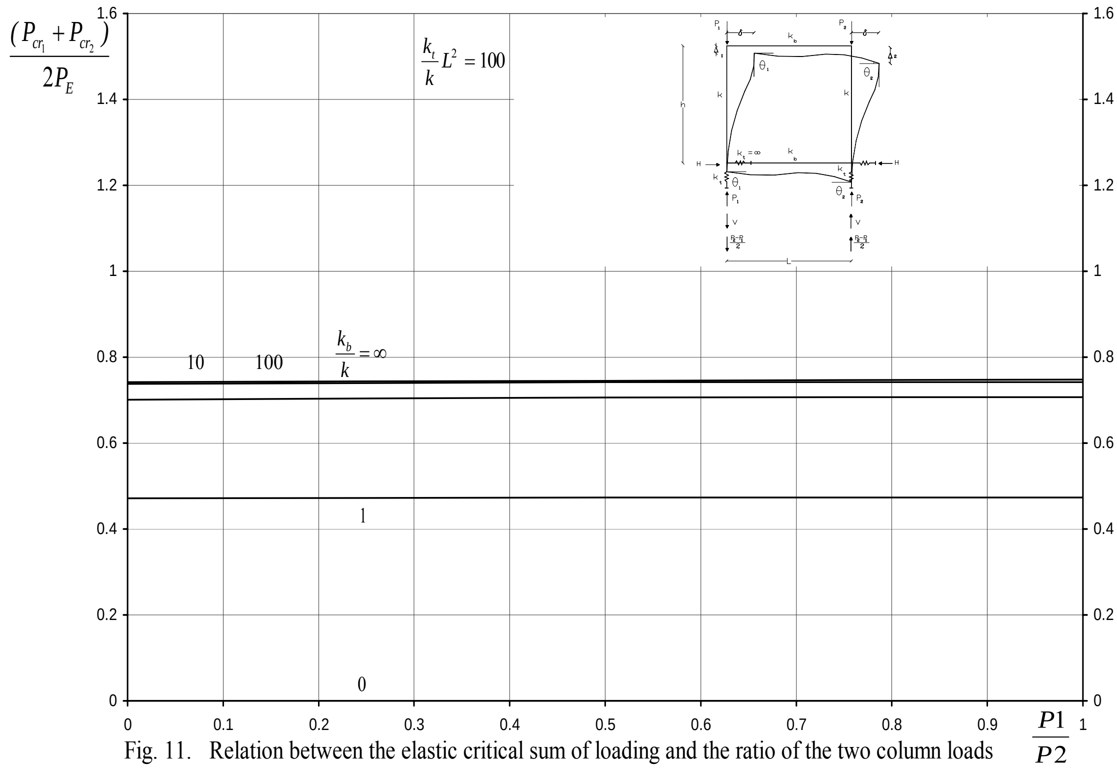

Fig. 11. Relation between the elastic critical sum of loading and the ratio of the two column loads

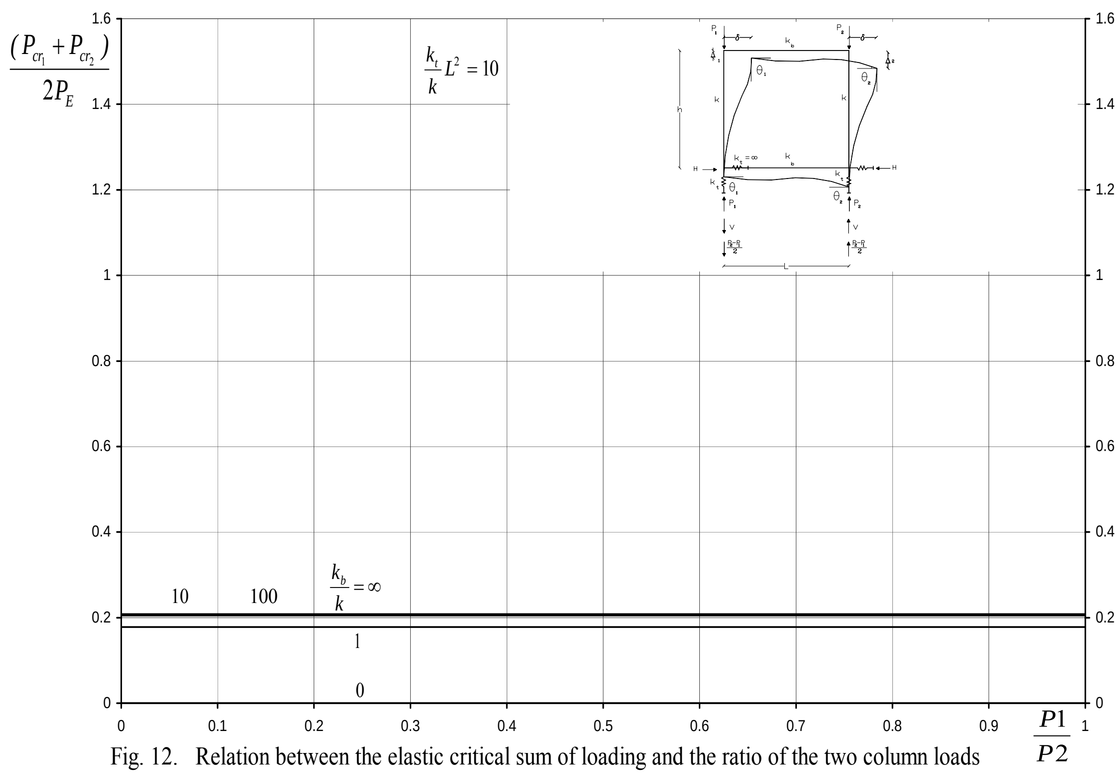

Fig. 12. Relation between the elastic critical sum of loading and the ratio of the two column loads

Symmetric Box Frame with Variable Beam Stiffness Subjected to Unsymmetrical Loading in an Unsymmetrical Sway Buckling Mode

Consider the rectangular box frame shown in Fig. 13 and using the same displacements as for the previous case, the equation of the critical buckling load can be developed, equation (7).

For symmetrical loading, $P_1 = P_2, H_o = zero, \theta_1 = \theta_2, \theta_3 = \theta_4, \Delta_t = \Delta'$

$$\begin{matrix} \theta_1 & \theta_2 & \dfrac{V}{k_t L} \end{matrix}$$

$$\begin{vmatrix} (n+6\dfrac{k_{b_1}}{k}) & -o & -12\dfrac{k_{b_1}}{k} \\ -o & (n+6\dfrac{k_{b_2}}{k}) & -12\dfrac{k_{b_2}}{k} \\ -12\dfrac{k_{b_1}}{k} & -12\dfrac{k_{b_2}}{k} & \left[\dfrac{k_t}{k}L^2 + 24(\dfrac{k_{b_1}}{k}+\dfrac{k_{b_2}}{k})\right] \end{vmatrix} = zero \qquad (8)$$

Which is the equation of critical buckling load of box frame with variable beam stiffness subjected to symmetrical loading.

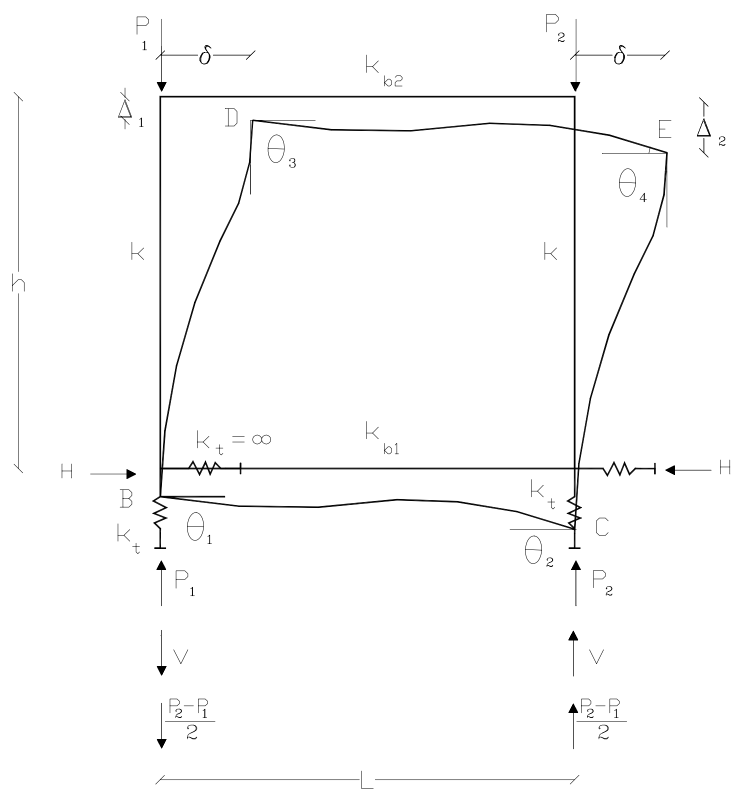

Fig. 13. Distorted configuration of the frame

$$
\begin{matrix}
\theta_1 & \theta_2 & \theta_3 & \theta_4 & \dfrac{H_o h}{k} & \dfrac{\Delta_t}{L}
\end{matrix}
$$

$$
\begin{vmatrix}
(n_1+4\frac{k_{b_1}}{k}) & 2\frac{k_{b_1}}{k} & -o_1 & zero & \frac{m_1}{2} & -6\frac{k_{b_1}}{k} \\
2\frac{k_{b_1}}{k} & (n_2+4\frac{k_{b_1}}{k}) & zero & -o_2 & -\frac{m_2}{2} & -6\frac{k_{b_1}}{k} \\
-o_1 & zero & (n_1+4\frac{k_{b_2}}{k}) & 2\frac{k_{b_2}}{k} & \frac{m_1}{2} & -6\frac{k_{b_2}}{k} \\
zero & -o_2 & 2\frac{k_{b_2}}{k} & (n_2+4\frac{k_{b_2}}{k}) & -\frac{m_2}{2} & -6\frac{k_{b_2}}{k} \\
\frac{m_1}{2} & -\frac{m_2}{2} & \frac{m_1}{2} & -\frac{m_2}{2} & -\left[\frac{m_1}{2s_1(1+c_1)}+\frac{m_2}{2s_2(1+c_2)}\right] & zero \\
-6\frac{k_{b_1}}{k} & -6\frac{k_{b_1}}{k} & -6\frac{k_{b_2}}{k} & -6\frac{k_{b_2}}{k} & zero & \frac{1}{2}\left[\frac{k_t}{k}L^2+24(\frac{k_{b_1}}{k}+\frac{k_{b_2}}{k})\right]
\end{vmatrix} = zer
$$

(7)

DISCUSION OF RESULTS

The results are presented to demonstrate the effect of the stiffness of the elastic translational and rotational spring on the critical buckling load of frame.

The following results may be stated:

a. For unsymmetrical loading P1 & P2, The elastic critical sum of loading is found to be nearly constant and is irrespective of the load ratio $\frac{P_1}{P_2}$ even for the most severe case when $P_1 = zero$ and the whole load P_2 acts on the other column, i.e. there is a complete analogy with the case of frames resting on rigid supports.

b. For $\frac{k_r}{k} = \infty$ and the ratio $\frac{k_t}{k} L^2 \geq 100$, the values of critical buckling load are very close to those for fixed base frame and when the ratio $\frac{k_t}{k} L^2$ approaches infinity, the values of critical buckling load approach those for fixed base frame.

c. For $\frac{k_r}{k} = zero$ and the ratio $\frac{k_t}{k} L^2 \geq 100$, the values of critical buckling load are very close to those for hinged base frame and when the ratio $\frac{k_t}{k} L^2$ approaches infinity, the values of critical buckling load approach those for hinged base frame.

d. The very small differences between elastic critical sums of loading slightly increase as the ratio $\frac{k_t}{k} L^2$ increase.

e. For $\frac{k_b}{k} \geq 10$, the critical buckling loads are very close. This was observed for all values of $\frac{k_t}{k} L^2$.

f. In general, there is a complete analogy between portal frames resting on elastic springs and those resting on rigid supports.

CONCLUSIONS

- In general, there is a complete analogy between frames subjected to unsymmetrical loads and resting on elastic springs and those resting on rigid supports for both portal and box frames.

- For practical values of the ratios $\frac{k_b}{k}$ and $\frac{k_t}{k}L^2$ (in case of average values of modulus of sub-grade reaction for different values of sandy soils), it was found that the ratio $\frac{k_t}{k}L^2$ has bigger values which lead to higher values of the buckling loads.

- For unsymmetrical loading P1 & P2, The elastic critical sum of loading is found to be nearly constant and is irrespective of the load ratio $\frac{P_1}{P_2}$ even for the most severe case when $P_1 = zero$ and the whole load P_2 acts on the other column.

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