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ANALYSIS OF ELASTIC-PLASTIC DEFORMATIONS OF STRAIGHT WINGS

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ABSTRACT

The paper presents a computer analysis for the determination of the wing twist. It is suitable for the calculation of the elastic and plastic angle of twist of straight wings. The wing twisting is assumed due to the action of aerodynamic and mass forces. The computer program is applied to L-29 advanced jet trainer for the calculation of wing twist.

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INTRODUCTION

Heavy stick is a common trouble that appears after some flight hours. The cause of this defect is related to the plastic deformation of the wings. After performing high load factor manoeuvres or after heavy landings, the airplane wings are expected to be plastically deformed. These deformations, commonly appearing in the form of wing twist, result in assymmetrical wing loading. The effect of this assymmetrical load on the stick force is investigated in a previous paper of the first auther. Mean while, in this work, a theoretical analysis of the plastic twist is presented.
In fact, it is impossible to investigate every loading condition which an airplane might encounter during service. Therefore, the following analysis is performed for specified loading conditions which may exceed the usual permissible cases of flight prescribed for the airplane.

THEORETICAL ANALYSIS

1- WING LOADS

The acting loads are the resultant of the distributed mass forces and the distributed aerodynamic loads. The dynamic pressure is given by the airworthiness requirements according to the prescribed loading case. The corresponding distributed aerodynamic load is calculated as

$$q_y = (C_y \; b) \; q$$

where

$(C_y \; b)$: spawise lift distribution

q : dynamic pressure

The distributed mass forces due to the weight and inertia of the wing is given by :

$$q_{G_w} = K \; b^2$$

where $K = \dfrac{n \; f \; G_w}{S}$

n load factor

f safety factor

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G_w weight of the wing
S wing area

2- LIFT DISTRIBUTION

For calculating the lift distribution, it was chosen Multhopp,s lifting-line theory. It represents a characteristic solution of prandtl,s equation. It is widely employed for calculating the lift distribution because it has been elaborated to a high degree of applicability.
The equation of prandtl is :

$$\Gamma = \frac{1}{2}\, b\, V_\infty\, a_\infty\, (\alpha_a - \alpha_i)$$

in its dimensionless form, it is written :

$$\gamma = \frac{1}{2}\, c\, \bar{b} \left\{ \alpha_a - \frac{1}{2\pi} \int_0^\pi \frac{d\gamma}{d\theta_1} \frac{d\theta_1}{(\cos\vartheta_1 - \cos\theta)} \right\}$$

where:-

$$\gamma(\theta) = \frac{2}{m+1} \sum_{n=1}^{m} \gamma_n(\theta) \sum_{K=1}^{n} \sin K\theta_n \;\; \sin K\theta$$

provides Multhopp,s approximate formula for the distribution of circulation.
The use of equation (3) for solving the γ - distribution leads to the problem to determine the separate values of γ_n satisfying the equation of Prandtl.
Consequently, the induced angle of attack is given by:

$$\alpha_i = \sum_{n=1}^{m} \gamma_n \sum_{K=1}^{m} \frac{K}{m+1} \sin K\theta_n \frac{\sin K\theta}{\sin\theta}$$

Or in Multhopp,s rearrangement

$$(\alpha_i)_\nu = \gamma_\nu\, b_{\nu\nu} - \sum_{n=1}^{m} \gamma_n\, b_{n\nu}$$

where

$$b_{\nu\nu} = \frac{m+1}{4 \sin \theta_\nu}$$

$$b_{\nu n} = \frac{1-(-1)^{n-\nu}}{2\,(m+1)} \qquad \frac{\sin \theta_n}{(\cos \theta_n - \cos \theta_\nu)^2}$$

In case of a general wing configuration, it is of advantage to superpose the resulting effect from individualcontributions (planform, twist, flaps, ailerons) by using the minimum number of solutions.

The wing twist is represented by a spanwise variation of sectional angles of attack. The real process of aerodynamic incidence with respect to the flight velocity is given by

$$\alpha_a = \alpha_w + \alpha_{w_0} - \alpha_t\,(z)$$

where the wing zero incidence α_{w_0} is given by the relation

$$\alpha_{w_0} = \frac{2}{S} \int_0^{1/2} b\, \alpha_t\, dz$$

and α_t is the local angle between the section root line and the zero lift line.

Flaps and ailerons are treated as special cases of abrupt twist. Within the extend of flaps, the angle cf flap deflection δ_{fl} produces the local lift coefficient of the value

$$C_y = \frac{\partial C_y}{\partial \delta}\, \delta_{fl} = \frac{\partial C_y}{\partial \alpha}\, \frac{\partial \alpha}{\partial \delta}\, \delta_{fl}$$

$$= \frac{\partial C_y}{\partial \alpha}\, \alpha_{eq}$$

The effect of flap deflection is substituted by the equivalent amount of the aerodynamic incidence

$$\alpha_{eq} = \frac{\partial \alpha}{\partial \delta}\, \delta_{fl}$$

Prandtl,s equation transfers simply to

$$\Gamma = \frac{1}{2} V_\infty b a_\infty \left(\frac{\partial\alpha}{\partial\delta} \delta_{fl} - \alpha_i \right)$$

where δ_{fl} has a constant value within the extent of flaps and equals zero in the remaining parts of the wing. The process of the derivative $\partial\alpha/\partial\delta$ is determined from the Data sheets (Ref.3). The effect of ailrrons is involved in prandtl,s equation in a similar manner.

The fixed tabs have significant effects on the wing loading. These can be easily studied, considering only the tabs influence on the distribution of local angles of attack. The change in the section chord, due to the tab existance, is clearly very small. The relatively small depth of the tab enables to neglect this effect.

The Guass elimination method was used for the solution of the linear set of equations reprsenting the distrubution of circulation. For better accuracy, the Seidel method was also employed. The results obtained by the first method will be the input values for the Seidel iterative method. The needed accuracy can be obtained according to the number of iterations made during the calculations.

3- STRAIGHT WING TWIST:

For the straight wing, it was proved that it is possible to separate the bending and twist by using the cancept of shear centre. Therefore, the external aerodynamic forces acting on the wing structure will be transfered to the elastic flexural axis. These forces are transfered by shearing forces and twisting moments. The shearing forces produce only bending of the wing, while the twisting moments produce pure twist of the structure.

The effective parts of the wing structure for the twist analysis are the skin and webs of the spars. These parts, at each section, are devided into n elements of the sheet material. The centroid, area moment of inertia and the areas enclosed by individual cells are calculated using the sheet elements for each section.

The problem of detemination of the position of the shear centre of m-cell structure is m times statically indeterminate. The position of the shear centre is obtained from the condition of moment equilibrium of internal force and the external forces defined by a shear force acting in the assumed shear centre. The redundant internal forces are calculated from the conditions of zero twist of individual cells of the cross-section. The angle of twist at any section is determined from the relation

$$\varphi = \int_{tip}^{z} \vartheta \, dz$$

where ϑ : rate of twist at any section

By this relation, two values of the angle of twist are determined. The first is the total angle of twist corresponding to the elastic and plastic deformations. The second is the plastic angle of twist corresponding to the permanent plastic deformation after the removal of the acting load. The rate of twist at each section due to the aerodynamic and mass forces is calculated as:

$$\vartheta = \frac{1}{2U_i} \oint_i \frac{\tau_t}{G\,t} \, dS$$

τ_t	is the resultant shear flow in the i^{th} cell of the wing section
V_i	Area enclosed by cell
G	Shear modulus
t	thickness of thinwall
	arc length

APPLICATION AND RESULTS

The L-29 jet trainer airplane was chosen as a model for the calculation of the wing angle of twist. The airplane is intended for a highly economic operation when training pilots in initial flying of jet aircraft and for advanced training of pilots in the pilotage, blind flying air firing from aircraft weapons and droping.

The leveling procedures were carried out for a group of airplane still in service. The aim of leveling is to check the geometrical shape of airplane, the correct mutual position of the main joined parts and the permanent deformations of different parts of airframe. The results obtained are tabulated in table 1. The required ailerons deflections, and the corresponding stick forces to cancel the effect of these deformations are given in figure 1 as a function of the speed.

Airplanes may be subjected to a wide variety of loading conditions in flight. The pilot may perform a large number of intentical manoeuvres. Each manoeuvre may be performed at various speeds and with various rates of displacement of the controls.

In fact, it is impossible to investigate every loading condition which an airplane might encounter. Then it is necessary to select a few conditions such that one of these conditions will be critical for every structural member of the airplane.

The method is used to calculate the permanent angle of twist φ at extreme conditions of loadings [n = 9]. The result [φ =.0745 deg.] agrees with that measured for a group of airplanes. The differences between the calculated and the measured values are mainly due to the fact that the exact load factors causing these measured deformations and the frequency of their occurence during the airplane service are not exactly known.

CONCLUSION

The loading of the airplane during manoeuvres, even in case of load factor smaller than the permissible one, yeilds to a plastic deformation of the wing. The magnitude of the wing plastic influences mainly the wing loading and the control characteristics of the airplane. The plastic twist of the wing accumulates up to a certain limit. This limit influences directly the serviceability of the airplane leveling after heavy manoeuvres.

It is therefore required to determine the wing service life from the point of view of plastic twisting. To fulfill this requirement, it is necessary to have the load spectrum of the airplane for some prescribed flying hours. This is the expected continuation of the problem in further work.

Airplane number	leveling difference at the wing tip.		corresponding angle of plastic twist	
	right	left	right	left
A	0	+ 2	0	0,1219
B	0	+ 1,5	0	0,0943
C	0	- 1	0	- 0,06095
D	+ 0,5	- 1	0,0305	- 0,06095
E	0	- 1	0	- 0,06095
F	+ 2	+ 1	0,1219	0,06095
G	- 1	+ 2	- 0,06095	0,1219

Tab.1,Measured plastic angle of twist at the wing tip

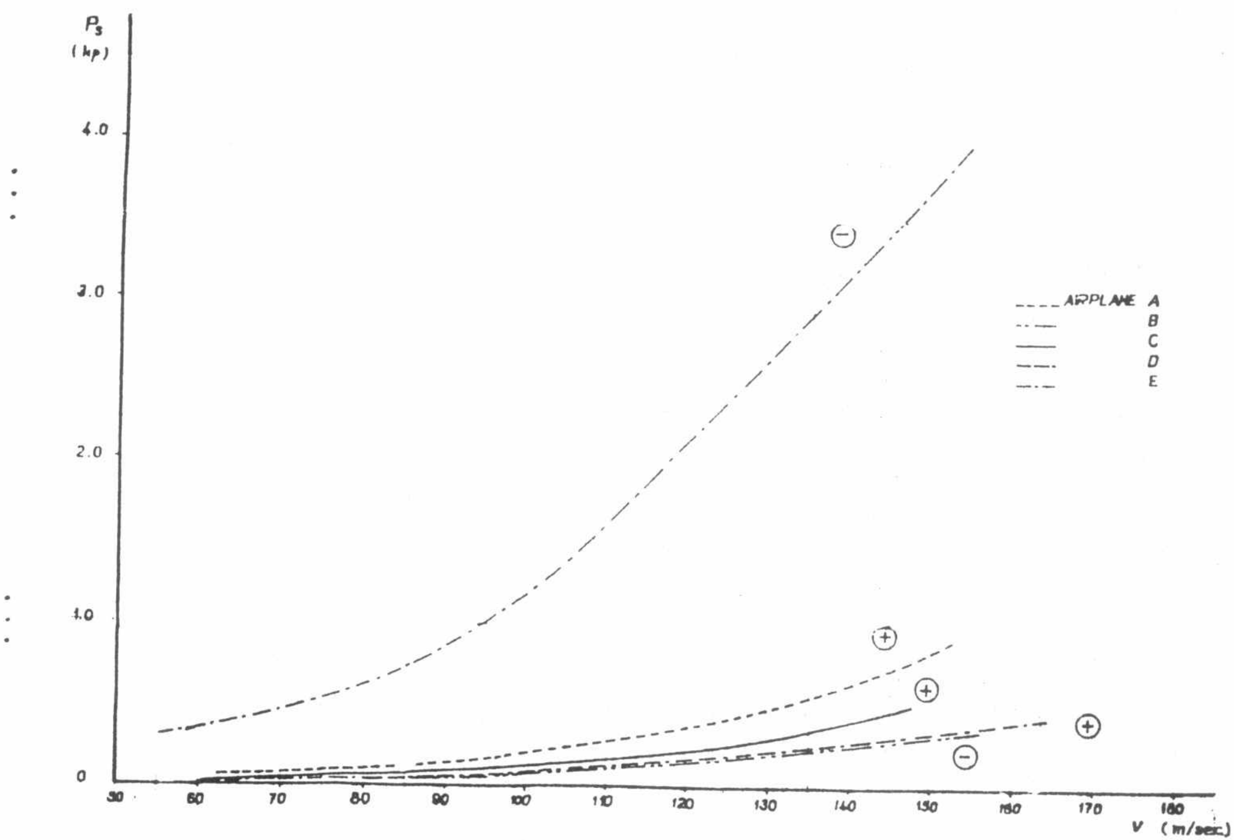

Fig.1, Stick force due to plastic deformation

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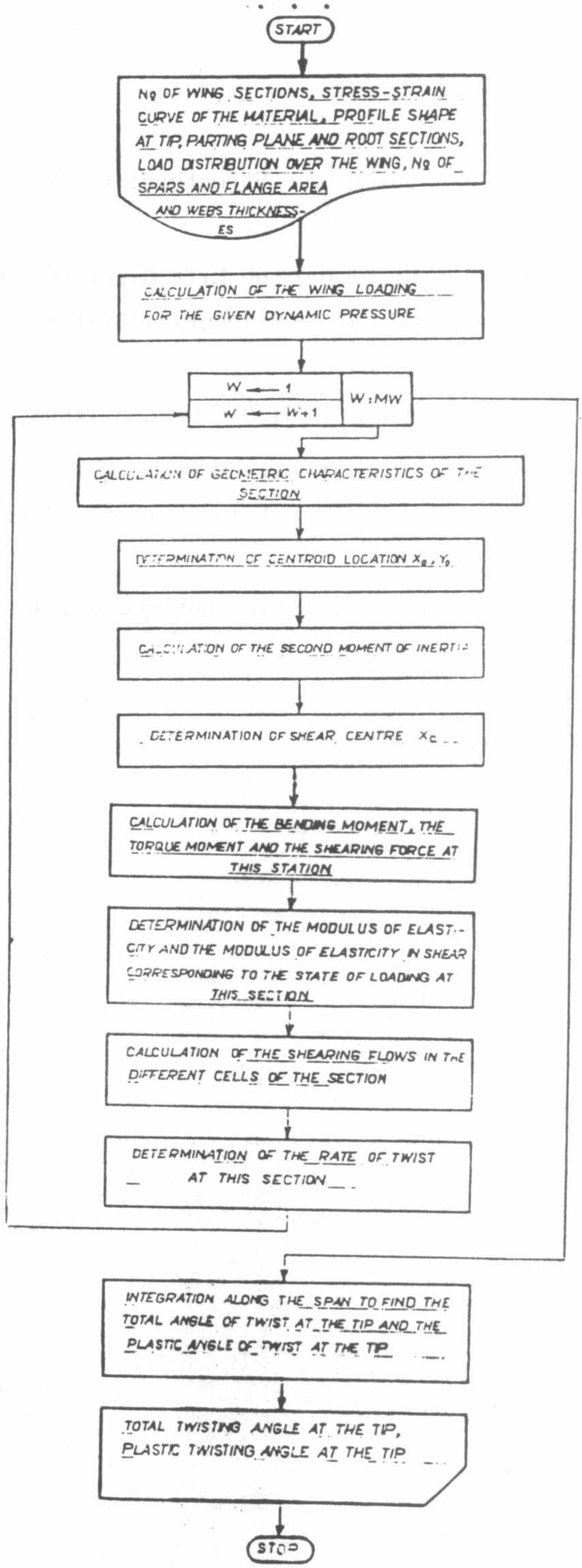
START
Nº OF WING SECTIONS, STRESS-STRAIN CURVE OF THE MATERIAL, PROFILE SHAPE AT TIP, PARTING PLANE AND ROOT SECTIONS, LOAD DISTRIBUTION OVER THE WING, Nº OF SPARS AND FLANGE AREA AND WEBS THICKNESSES
CALCULATION OF THE WING LOADING FOR THE GIVEN DYNAMIC PRESSURE
W ← 1
W ← W+1
W : MW
CALCULATION OF GEOMETRIC CHARACTERISTICS OF THE SECTION
DETERMINATION OF CENTROID LOCATION X_o, Y_o
CALCULATION OF THE SECOND MOMENT OF INERTIA
DETERMINATION OF SHEAR CENTRE X_c
CALCULATION OF THE BENDING MOMENT, THE TORQUE MOMENT AND THE SHEARING FORCE AT THIS STATION
DETERMINATION OF THE MODULUS OF ELASTICITY AND THE MODULUS OF ELASTICITY IN SHEAR CORRESPONDING TO THE STATE OF LOADING AT THIS SECTION
CALCULATION OF THE SHEARING FLOWS IN THE DIFFERENT CELLS OF THE SECTION
DETERMINATION OF THE RATE OF TWIST AT THIS SECTION
INTEGRATION ALONG THE SPAN TO FIND THE TOTAL ANGLE OF TWIST AT THE TIP AND THE PLASTIC ANGLE OF TWIST AT THE TIP
TOTAL TWISTING ANGLE AT THE TIP, PLASTIC TWISTING ANGLE AT THE TIP
STOP

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