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AXLE DIFFERENTIAL LOADING SPECTRA ASSESSMENT AND RATING OF ITS GEARING LOAD CAPACITY

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ABSTRACT

This contribution deals with the methodology of a lifetime calculation for gearings of automobile differentials in bending and a contact. Basis of a calculation is an operating loading spectrum whose determination for a differential is very difficult. In this contribution a procedure for a determination of a hypothetic loading spectrum by the mathematical formula with an exploitation of an experiment is published. A calculation of a contact stress and a bending stress in the root is also made by using modified techniques in accordance with appropriate standards. An assessment of gearing service life is based on a calculation of an intensity of damages using Haibach's theory of a cumulating of damages. But there's no point in to use this method for differential gearing. The new procedure is designed – this new procedure evaluates loading capacity of gearing by a utilization of an ultimate or reliable recommended safety factors which are subjected to the endurance limit.

KEY WORDS

Differential, loading spectrum, gearing, service life.

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INTRODUCTION

A premise for each effectual gearing loading capacity calculation in a contact and bending is knowledge of loading spectrum and using of a suitable method of its schematization when gearing is stochastically loaded. Knowledge of a suitable method for computing of a bending stress in the root and a Hertzian stress at the tooth flank is necessary as well. Parameters of Wöhler's curves of teeth material for bending and a contact and a suitable theory of a cumulative damage are needed to know too. This contribution uses this approach for gearing of differentials D_M and D_N with regard to that the core of the principle is the created schematized loading spectrum especially its speed part – it is generally a tough task.

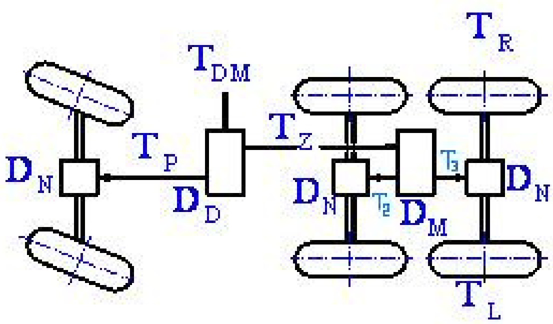
When truck is driving on the straight road the rotational speed of differentials gears are zero ones. On the contrary the wheelspin occurs when driving wheels or axles are moving along different track lengths (road turnings, transverse or lengthwise road irregularities), when tires have different tires rolling radii or immediate loading and when there is a different drive slip. This contribution solves task of a determination of a hypothetic load with using of an experiment for getting of a particular gear frequency for a differential.

Individual types of a differential are signed using a division ratio J_D, generally true:

$$J_D = \frac{T_1}{T_2} \geq 1\,; \quad T_1 \geq T_2\,; \quad T_U = T_1 + T_2 \tag{1}$$

where T_1, T_2 – output torque moments, T_U – torque moment on the planet carrier.

The example displayed on Fig. 1 is the schema of a fully driven truck 6 x 6.



D_D – torque divider $J_D = \frac{T_Z}{T_P} \geq 1$

D_M – inter-axle differential $J_D = \frac{T_2}{T_3} = 1$

D_N – axle differential $J_D = \frac{T_R}{T_L} = 1$

Fig. 1. Schema of the 6 x 6 truck traction.

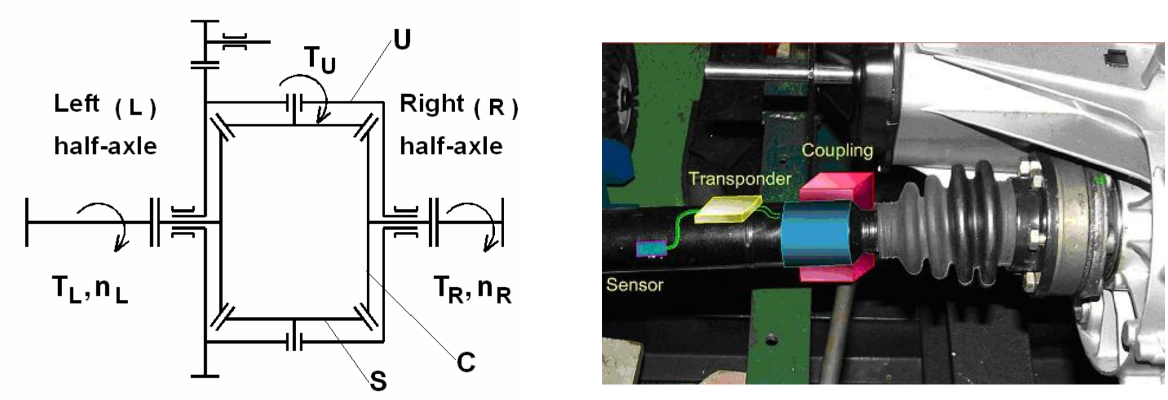
DETERMINATION OF A LOADING SPECTRUM FOR A DIFFERENTIAL

Schematized loading spectrum including a hypothetic loading spectrum usually express as a function between the load (in this case torque moment or bending or contact stress) on the appropriate level and the number of load cycles in an absolute or a relative values. The first step for a determination of the hypothetic loading spectrum will be finding of the sum total of load cycles for an estimated lifetime of gears of a truck differential.

Determination of a Sum Total of Load Cycles of Gears of a Truck Differential Using an Experiment

The experiment consisted in a synchronous measuring of torque moments and rotational speeds on the output half axles of the bevel axle differential of measuring personal car. The schema of this differential is on Fig. 2a, and the picture of the half axle with the installed measuring system is on Fig. 2b.

These measuring were worked out on selected road segments formed by a highway (290 km), out of town the first class route (32 km) and the second class road (29 km), and city roads (90 km). Recording sample from city roads is on Fig.3, next evaluation is on the Fig. 4.



a) Scheme of the differential with half axles. b) Measuring half axle of the car.

Fig. 2. Measuring of the torque moment (*T*) and rotation speed (*n*) on half axles [1].

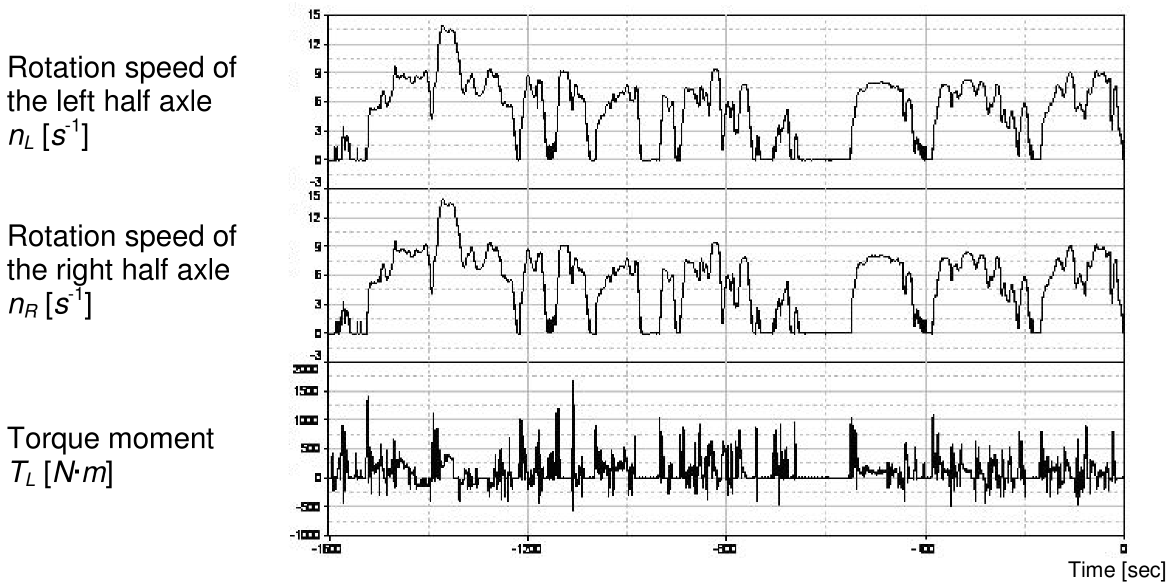


Fig. 3. Recording sample from city roads [1].

These measuring were worked out on selected road segments formed by a highway (290 km), out of town the first class route (32 km) and the second class road (29 km), and city roads (90 km). Recording sample from city roads is on Fig. 3 next evaluation is on Fig. 4.

Rotation speeds difference
$\Delta n = n_L - n_R \ [s^{-1}]$

Absolute value of the rotation speeds difference
$\Delta n_a = |n_L - n_R| \ [s^{-1}]$

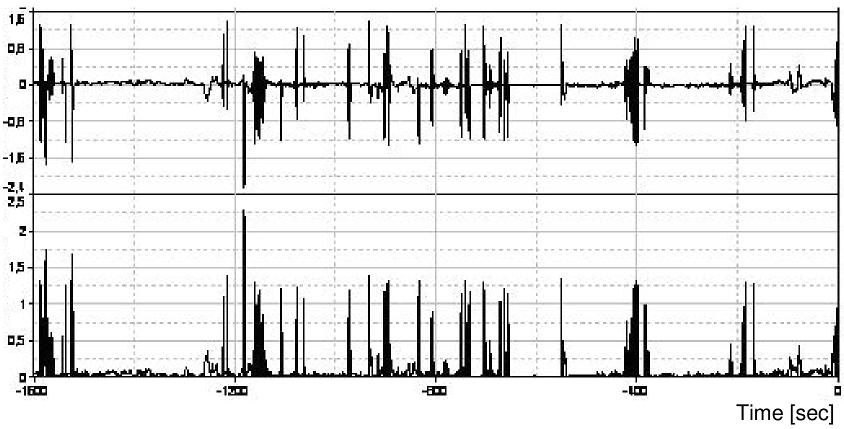


Fig. 4. Evaluation of the rotation speeds difference from the record on Fig. 3.

It is necessary to determine a frequency of revolutions of a differential for an assessment of differential gearings lifetime as a function of a path not of a time. Calculation of a frequency of a sun gear revolutions Δn_C for a given path 1 km can carry out:

$$\Delta n_{CK} = \frac{\Delta n_a \cdot t}{L} \ [rev \cdot km^{-1}] \qquad (2)$$

Where Δn_a is the absolute value of the rotation speeds difference between the left and the right half axle $\Delta n_a = | n_L - n_R |$, *t is the* driving time in seconds and L is the distance move in kilometers.

Computed the average frequency of revolutions of the sun gear Δn_C for selected road segments are in the Table 1. The frequency of revolutions of the satellite gear is:

$$\Delta n_S = \Delta n_C \cdot \frac{z_C}{z_S} \ [rev \cdot km^{-1}] \qquad (3)$$

where z_C is the number of teeth of the sun gear, and z_S is the number of teeth of the satellite gear.

Table 1. Average number of revolutions of the sun gear for 1 km [2].

Road segment	$\Delta n_{C,i}$	Share k_i	k_i – share of the road segment from overall one $k_i = \frac{\Delta n_{C,i}}{\sum \Delta n_{C,i}}$, $\sum_{i=1}^{n} k_i = 1$
Highway	3.088	k_1	
City road	9.059	k_2	
1. class road	5.275	k_3	
2. class road	5.841	k_4	

Total number of load cycles during demanded service life N_C (a spectrum total length) is then:

$$N_C = L_D \cdot \Delta n_{C,S} \quad [cyc.] \tag{4}$$

where L_D is the demanded service life in km, $\Delta n_{C,S}$ – average number of revolutions (cycles) for 1 km for a demanded service life.

Average number of revolutions $\Delta n_{C,S}$ is determined on the base of chosen or required compilation of road segments e.g. with a help of the Table 1 as:

$$\Delta n_{C,S} = \frac{\sum_{i=1}^{n} k_i \cdot \Delta n_{Ci}}{n} \Delta n_C \cdot \frac{z_C}{z_S} \quad [rev \cdot km^{-1}] \tag{5}$$

where n is the number of road segments.

Estimation of number of load cycles for automobile differentials for three basic types of vehicles is mentioned in the Table 2.

Table 2. Estimation of number of load cycles for differentials [2].

Parameter	Vehicle		
	Personal	**Commercial**	**Truck***
Estimated service life, L_D [km]	$2.5 \cdot 10^5$	$5 \cdot 10^5$	$2 \cdot 10^6$
Average number of rev., $\Delta n_{C,S}$ [cyc.·km^{-1}]	3 ÷ 5 (measured)	5 ÷ 10 (measured)	10 ÷ 20 (estimated)
Spectrum total length, N_C [cyc.]	$7.5 \cdot 10^5 \div 1.3 \cdot 10^6$	$2.5 \cdot 10^6 \div 5 \cdot 10^6$	$2 \cdot 10^7 \div 4 \cdot 10^7$
Total number of cycles during service life on maximal level, N_p [cyc.]	7.5 ÷ 13	25 ÷ 50	200 ÷ 400

* data acceptance from the manufacturer

Hypothetical Loading Spectrum

If there is no spectrum obtained by an experiment then it is possible to replace it by a hypothetical spectrum using a mathematical substitution. Substitution starts from knowledge of maximal (T_{max}) and minimal (T_{min}) torque moment or stresses of a spectrum (it is possible to make an educated guess) and from a number of load cycles N_C (for example by the Table 2). Next step is to design a number of spectrum levels p with the same width and to determine an aggressiveness of a spectrum which is characterized by a degree (index) of aggressiveness s. It is necessary to know or to estimate a number of load cycles on the maximal level N_p in the end. Curves of cumulative loading spectra for various degree of aggressiveness s are seen on Fig. 5. The equation is used for a mathematical expression of a curve of a relative or a relative cumulative loading spectrum:

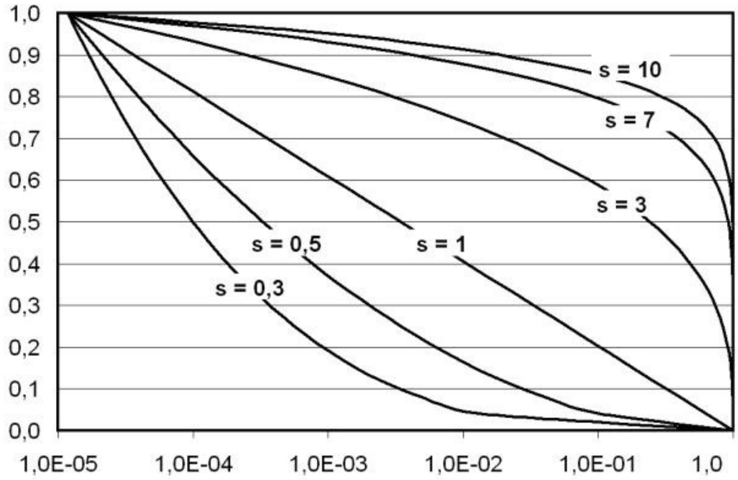


Fig. 5. Curves of relative cumulative loading spectra for a different degree of aggressiveness *s* [3].

$$\overline{N}_i^{a,k} = \left[N_c^p \cdot \left(\frac{N_p}{N_c} \right)^{(T_{si})^S} \right] \quad or \quad \overline{N}_i^{a,k} = \left[N_c^p \cdot \left(\frac{N_p}{N_c} \right)^{(\overline{T}_{si})^S} \right] \tag{6}$$

where T_{si} is the average torque moment on the *i.* level, $\overline{T}_{si}$ is the relative (dimensionless) torque moment on the *i.* level, p is the number of spectrum levels, i is the level number (i = 1…p), $\overline{N}_i^a$ is the relative frequency of load presence on *i.* level $\overline{N}_i^a = \frac{N_i^a}{N_c}$, N_i^a is the absolute frequency of load presence on *i.* level, $\overline{N}_i^k$ is the relative cumulative frequency of load presence on *i.* level $\overline{N}_i^k = \frac{N_i^k}{N_c}$, N_i^k is the cumulative (additive) frequency of load presence on *i.* and higher levels, for a cumulation from above (from the max. torque moment to the min. torque moment)

Average torque moment on *i.* level for a real load given by values T_{max} and T_{min} is derived from the constant spectrum width ΔT:

$$T_{si} = T_{\min} + \Delta T \cdot (i - 0.5) \ [N \cdot m] \ , \ \Delta T = \frac{T_{\max} - T_{\min}}{p} \ [N \cdot m] \tag{7}$$

when a relative load $\overline{T}_{si} \in (0,1)$ is thought, then the load on the *i.* level is [3]:

$$\overline{T}_{si} = \frac{i-1}{p-1} \ [-] \tag{8}$$

Average torque moment on the maximal (T_{sp}) and on the minimal (T_{s1}) level:

$$T_{sp} = T_{\max} - 0.5 \cdot \Delta T \ , \quad T_{s1} = T_{\min} + 0.5 \cdot \Delta T \tag{9}$$

Meaning of particular variables is evident from Fig. 6.

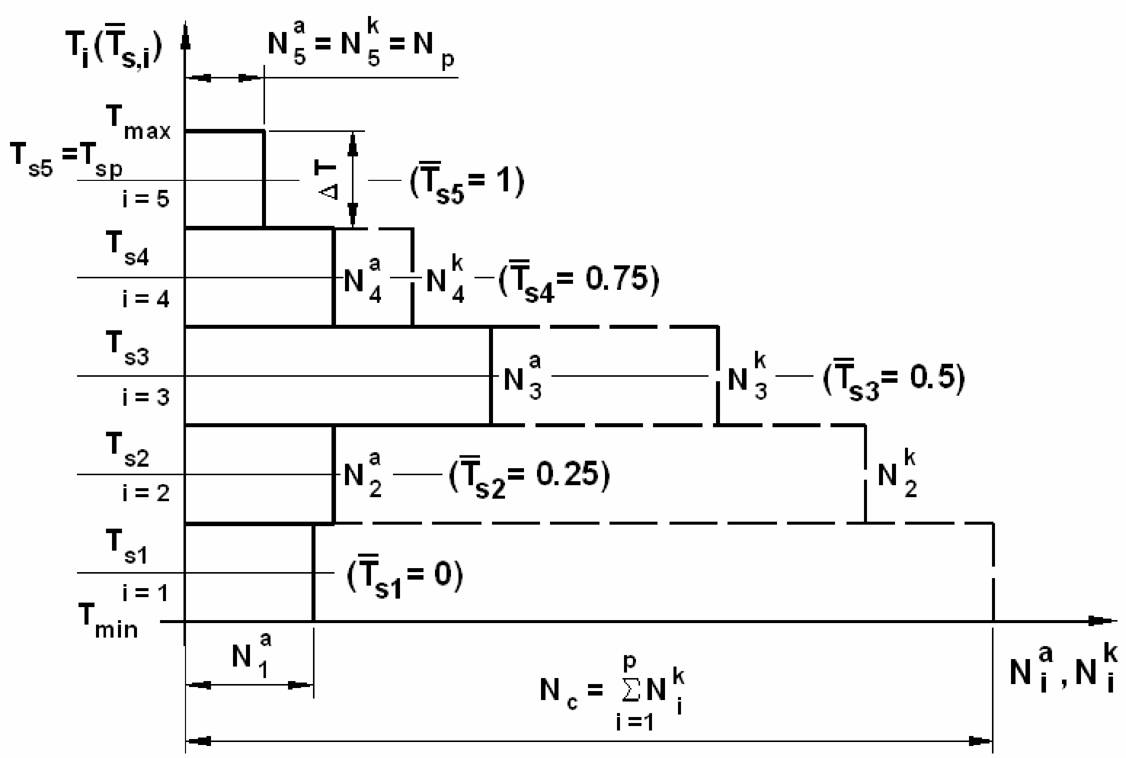


Fig. 6. Simplified schema of the loading spectrum for five levels.

Operating loading spectra measured on four road segments (Table 1) were modified into 23 loading levels using a two-parametric level schematization. The result as a graphic output is on Fig. 7 where it is the function between the torque moment $\overline{T}_{si}$ and the relative or relative cumulative frequency of torque presence $\overline{N}_i^{a,k}$. At the same time there is the calculated substitution of relative cumulative spectrum obtained by the equation 2.5 with the degree of aggressiveness s = 1.3.

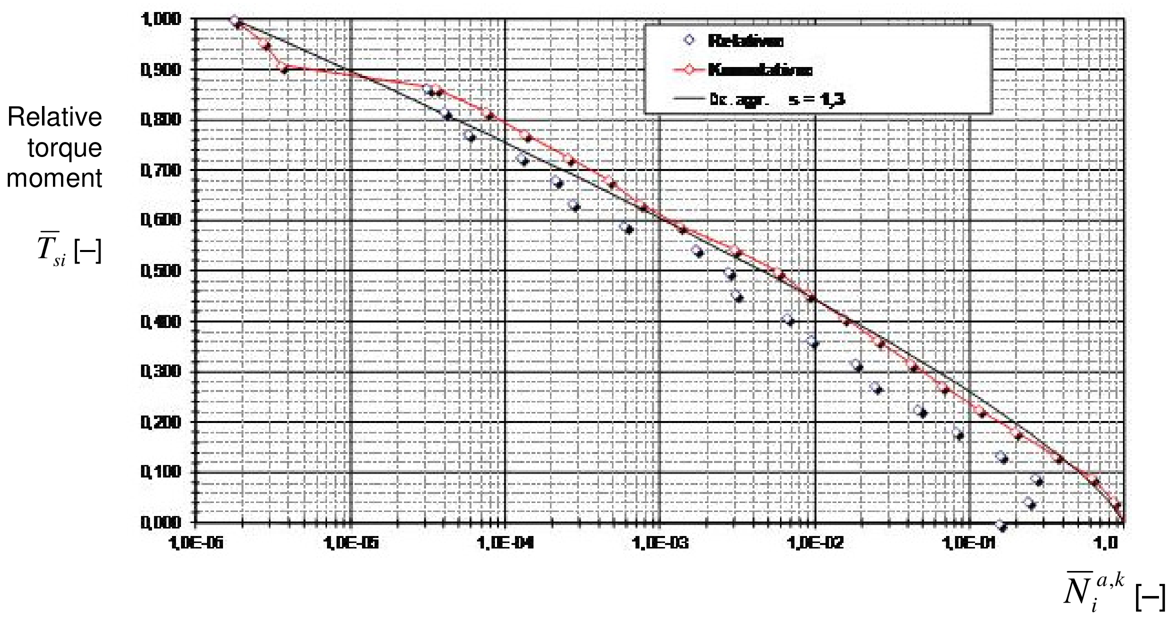


Fig. 7. Relative and relative cumulative frequency of torque presence [2].

Determination of the Degree of Aggressiveness

If there is no measured operating loading spectrum of differential it is possible to create a substitute hypothetic spectrum according to the equation 2.5 where a quality of the substitution is setting by good choice of the degree of aggressiveness *s*. This degree is choosing by an educated guess according to experiments which were made and published, for example curves from Fig. 5. We used for our experiment such a procedure that for each measured road segment *i* by the Table 1 we created an individual substitution for a spectrum by the equation 2.5 with the degree of aggressiveness s_i and the resultant degree of aggressiveness *s* we made out using the share k_i (Table1):

$$s = \frac{\sum_{i=1}^{n} k_i \cdot s_i}{n} \quad [-] \tag{10}$$

If the degree of aggressiveness *s* (in this case *s* = 1.3) is suited for the whole spectrum with the length N_c and for the resultant degree of aggressiveness *s* we can convince in the following way. We convert the equation 2.5 after double taking logarithm into:

$$s = \frac{\log\left[\dfrac{\log \dfrac{N_i}{N_c}}{\log \dfrac{N_p}{N_c}}\right]}{\log \overline{T}_{si}} \tag{11}$$

We calculate the appropriate value of the degree of aggressiveness *s* for each level of $\overline{T}_{si}$ (except levels $\overline{T}_{si} = 0$ and $\overline{T}_{si} = 1$). If calculated values of the degree of aggressiveness *s* on each level don't have big variances, it is possible to replace the measured loading spectrum by the average value of the index *s*.

SERVICE LIFE CALCULATION FOR DIFFERENTIAL GEARINGS ON THE BASIS OF THE LOADING SPECTRUM

We use a different approach for dimensioning of gears of automobile differentials than for other automobile gearings for other reasons:

- frequency of a maximal load for differential gears is low one and it is used to reduce by a tire adhesion
- number of loading cycles is derived from the rotation speeds difference of driven parts (axles, half-axles) and is very low one
- no internal dynamic forces are coming up because circumferential velocities are very low during differential gears meshing.

Calculation of Contact and Bending Stresses

Calculation is made by a modified procedure given by the standard ISO 6336 [4] which is practically identical as the standard DIN 3990-T41 [5]. We use equations stated in these standards for calculation of a contact stress σ_H and a bending stress σ_F. The calculation of both stresses is among other things based on a size of the circumferential force F_t which is acting at the reference or the pitch diameter of driving element of a differential:

$$F_t = \frac{2000 \cdot T}{d \cdot a} \delta \ [N] \tag{12}$$

where T is the torque moment on the driving element of a differential [$N{\cdot}m$], a is the number of satellites or a pairs of satellites, δ *is the* degree of an inequality of a forces distribution, it is given by [2], *and d is the* reference or the pitch diam. of driving element of a differential [*mm*] by Ref. [2].

Load factors in bending K_F and in a contact K_H by [4] and [5] perform in equations for a calculation of a contact stress σ_H and a bending stress σ_F. These coefficients are complying with an influence of internal and external dynamic forces, coefficient of an inequality of a forces distribution along the facewidth and an influence of a force fraction on individual teeth. For differentials we suppose that factors $K_F = K_H = 1$, namely when loading spectrum is used.

Calculation of an Expected Service Life of Gearing

For this calculation of an expected service life it came right to use the technique based on Haibach's theory of a cumulating of damages according to Fig. 8. On this figure, Wöhler's curves in a contact (*H*) and bending (*F*) for gears materials are schematically illustrated. Relevant material values which match to case hardened steels 16MnCr5 and 20MoCr4 (by Ref. [5]) are listed in the Table 3.

Table 3. Material characteristics for Wöhler's curve.

Parameter	Bending	Contact
HRC	-	58 – 64
σ_{lim} [*MPa*]	920 (2300)	1500
σ_{pst} [*MPa*]	1840	2400
N_{lim}	$3{\cdot}10^6$	$5{\cdot}10^7$
N_{pst}	$1{\cdot}10^3$	$1{\cdot}10^5$
q	11.5	13.2

Seriously expected service life $L_{c,s}$ [*km*] is determined by the equation:

$$L_{c,s} = \frac{L_c}{\max(D_{CF}, D_{CH})} \ [km] \tag{13}$$

where D_{CF}, D_{CH} is an accumulate intensity of damages in bending or in a contact.

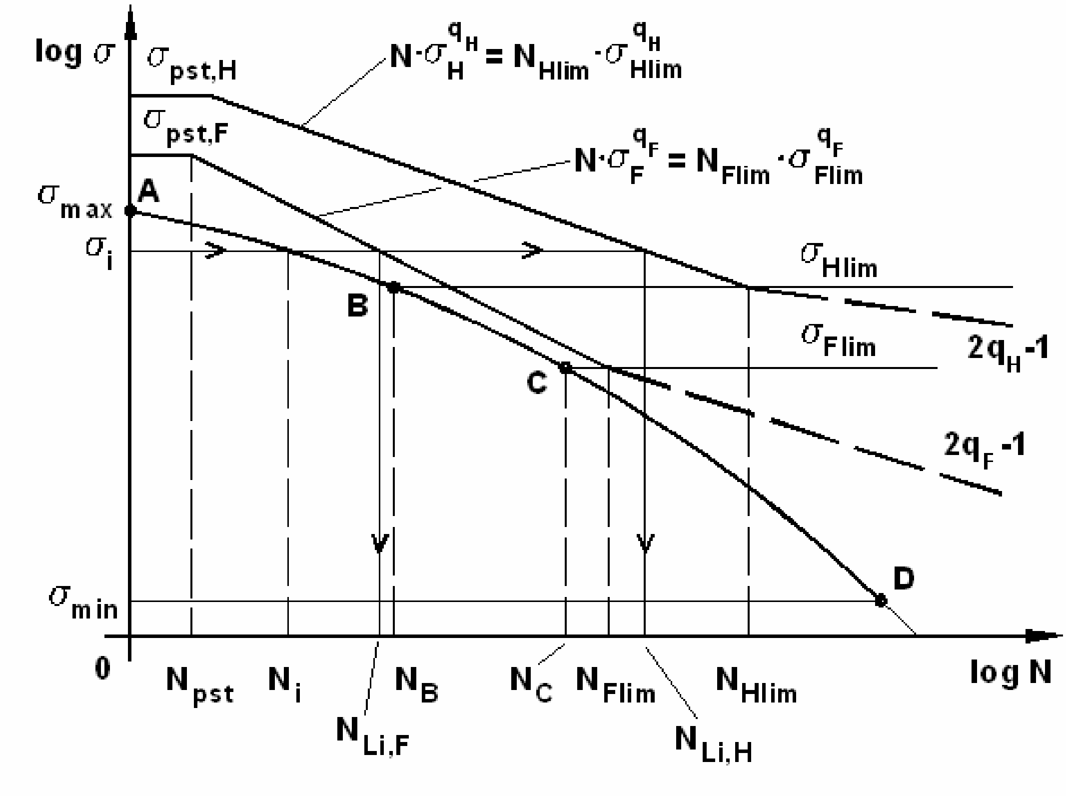


Fig. 8. Wöhler's curve and Haibach's theory of a cumulation of damages.

$$D_{CF,H} = \sum_{i=C(B)}^{A} D_{iF,H} + \sum_{D}^{i=C(B)} D_{iF,H} \tag{14}$$

Where $i = C(B)...A$ are cycles over the fatigue limit σ_{lim} , and $i = D...C(B)$ are cycles under the fatigue limit σ_{lim}.

Partial intensity of damages $D_{iF,H}$ on the i level will be for bending (F):

$$D_{i,F} = \frac{N_i}{N_{Li,F}} = \frac{N_i}{N_{F\lim}} \cdot \left(\frac{\sigma_i}{\sigma_{F\lim}}\right)^{q_F} \quad \text{or} \quad D_{i,F} = \frac{N_i}{N_{Li,F}} = \frac{N_i}{N_{F\lim}} \cdot \left(\frac{\sigma_i}{\sigma_{F\lim}}\right)^{2\cdot q_F - 1} \tag{15}$$

and for a contact (H):

$$D_{i,H} = \frac{N_i}{N_{Li,H}} = \frac{N_i}{N_{H\lim}} \cdot \left(\frac{\sigma_i}{\sigma_{H\lim}}\right)^{q_H} \quad \text{or} \quad D_{i,H} = \frac{N_i}{N_{Li,H}} = \frac{N_i}{N_{H\lim}} \cdot \left(\frac{\sigma_i}{\sigma_{H\lim}}\right)^{2\cdot q_H - 1} \tag{16}$$

Results of our calculations of the accumulate intensity of damages D_{CF} and D_{CH} accordance with the presented technique for a series of differentials are in the Table 4 which results from the Table 2.

Values of accumulate intensities of damages are very small (especially in a contact D_{CH}). It is in virtue of a very low number of high loading cycles. This implies that

Table 4. Results of lifetime calculations of differentials gearings [3].

			Vehicle					
			Personal		Commercial		Truck	
Estimated service life L_D [*km*]			$2.5{\cdot}10^5$		$5{\cdot}10^5$		$2{\cdot}10^6$	
Spectrum total length N_C [*cyc.*]			$7{\cdot}10^5$	$1.3{\cdot}10^6$	$2.5{\cdot}10^6$	$5{\cdot}10^6$	$2{\cdot}10^7$	$4{\cdot}10^7$
Total number of cycles during service life on maximal level N_p [*cyc.*]			7.5	13	25	50	200	400
Intensity of damages by Haibach's theory D_C		Bending	0.002÷0.007	0.04÷0.09	0.07÷0.17	0,14÷0,35	0,57÷1,41	1,13÷2,82
		Contact	0.0001	0.0002	0.0004	0,001	0,004	0,007
Safety factor $S_{F,H}$	Ultimate	Bending S_{Flim}	0.4 ÷ 0.5					
		Contact S_{Hlim}	0.625					
	"Reliably" recommended	Bending S_{FD}	0.5 ÷ 0.6		0.6 ÷ 0.7		> 0.7	
		Contact S_{HD}						

There is no point in calculations, especially in a contact. We recommend to evaluate a differential gearing by means of ultimate safety factors by the Table 4.

- bending – $S_F \le S_{F\lim} = \dfrac{\sigma_{F\lim}}{\sigma_{pst,F}} = 0.4 \div 0.5$
- Contact – $S_H \le S_{H\lim} = \dfrac{\sigma_{H\lim}}{\sigma_{pst,H}} = 0.625$

Values of safety factor in bending S_F and in a contact S_H are set for a hypothetic loading spectrum using a technique by [4-5]. Hereinafter there are our recommended values of safety factors in the Table 4.

CONCLUSION

Calculation of a service life of automobile differential gearings should result from a operating loading spectrum (a function between a number of load cycles or revolutions and a load). Determination of such a spectrum is a very tough task. We recommend for this determination to choose the hypothetic loading spectrum by the part 2.2 of this article. We recommend to make a calculation of a contact stress and a bending stress in the root by the standard ISO 6336 [4], which is nearly similar like the standard DIN3990-T41 [5]. From our calculation results of differential gearings in bending and a contact follows that their intensity of damages in case of personal and commercial cars (with demanded service life L_c<1·10^6 *km*) is very low. So there's no point in checking the service life of gearing by this way. We recommend designing differential gearings in bending and a contact with a sufficient reliability using ultimate or reliable recommended safety factors by equations from the part 3 of this article and on the basis of their values stated in the Table 4. Service life calculation results for heavy trucks (L_c= 1·10^6 *km*) are more or less comparable to demanded safety factors (Table 4). The paper is performed at VSB-Technical University of

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