



EM-1

Series Solution For A Peristaltic Flow Of a Magneto-fluid Through A Taper Eccentric Cylinders

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Abstract:

The magneto-hydrodynamic peristaltic flow of an incompressible Newtonian fluid was investigated between two eccentric tubes. The problem is measured in cylindrical coordinates. Geometrically, we considered two eccentric tubes in which the inner tube is rigid while the outer tube is tapered and have a sinusoidal wave generated on its walls. The governing equations are observed nonlinear second order partial differential equations. Under the conditions of long wavelength approximation the problem has been solved with the help of the homotopy perturbation method (HPM). The obtained results are then plotted to see the influence of the different physical parameters on the velocity profile, pressure gradient, pressure rise and frictionless force expressions. The velocity profile is drawn in the two and three dimensions. The trapping bolus phenomena is also discussed through the contour plot of the streamlines. Keywords Peristaltic Flow ; Magneto Fluid; Taper Eccentric Cylinders; HPM.

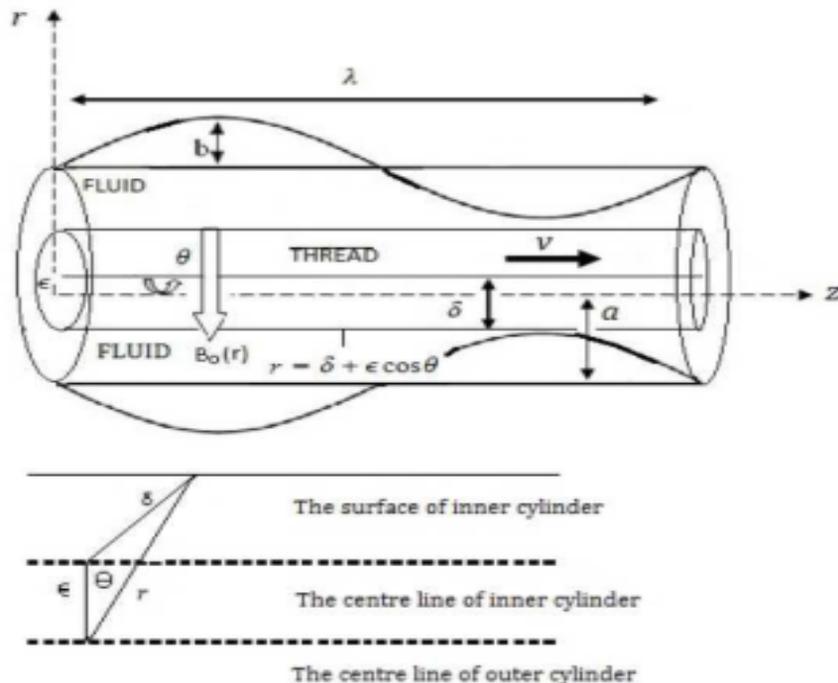
3. 1 Introduction

Peristalsis pumping is a phenomenon in which fluid transport happens when a gradual wave of area contraction or expansion propagates along the length of distensible duct. Peristalsis is one of the topics highly important in applied mathematics, engineering, physiological world and its have many applications in real life. It is an automatic and vital process that moves food through the digestive tract, transport from kidney to bladder, in the vasomotion of small blood vessels such as venues, capillaries and arterioles. And also the mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, corrosive fluids, a toxic liquid transport in the nuclear industry. The peristaltic transport with long wavelength at low Reynolds number [1,2] with all Reynold numbers [3] or for long and short wavelength [4] have been analyzed. Tsui, Yeng-Yung, et al.[5] Pumping flow in a Channel with a peristaltic wall RACHID, Het al.[6] The effect of a pulsatile flow on the Peristaltic Output: Case of a Newtonian Fluid. And with through a porous medium by Afifi NAS et

al.[7].Mekheimer [8] have shown the effect of the induced magnetic field on peristaltic flow of a couple stress fluid. N.Nagendra[9] MHD flow of a Newtonian fluid through a porous medium in an asymmetric channel with peristalsis. The non-Newtonian effects of different types of fluids [1--13] or of the Nonlinear and curvature effects [14] on the peristaltic flow have been studied. Investigations are available in the literature to study the effect of an endoscope on peristaltic motion of Newtonian and non-Newtonian fluids [15-16]. The eccentric annulus is normally not easy to discuss even without peristalsis. There are only a few studies that take into attention the effect of the eccentricity with peristalsis [16]. In the present work we discuss the peristaltic flow of Newtonian fluids under the effect of the magnetic field in an eccentric cylinder, which the inner tube is rigid while the outer has sinusoidal waves generated on its tapered walls. Peristaltic pumping characteristics are discussed in detail. The flow analysis is developed in the unsteady state by using the long wavelength approximation. The problem is first modeled and then solved analytically for the axial velocity, axial pressure gradient and pressure rise. This analysis gives a better judgement for the speed of injection and the fluid flow characteristics within the syringe. Also, the injection can be carried out more proficiently and pain of the patient can be extenuated.

2 Mathematical formulation of the problem:

Consider a MHD flow of an incompressible Newtonian fluid through eccentric tubes. The inner tube is rigid (endoscope or catheter) and the outer has sinusoidal waves generated on the walls of tapered. The radius of the inner tube is δ' but we need to reference the fluid motion to the center of the outer tube. The center of the inner tube is now at position $z' = \epsilon', y' = 0$ where r and θ are coordinates in the cross-section of the pipe as shown in figure 1.b. Then the boundary of the inner tube is described to order ϵ' by $r_1' = \delta' + \epsilon' \cos[\theta']$. (obtained by using the cosine rule) where ($\epsilon' \ll \delta'$) is the parameter that controls the eccentricity of the inner tube position. The geometry of the walls surface is described in fig.(1)



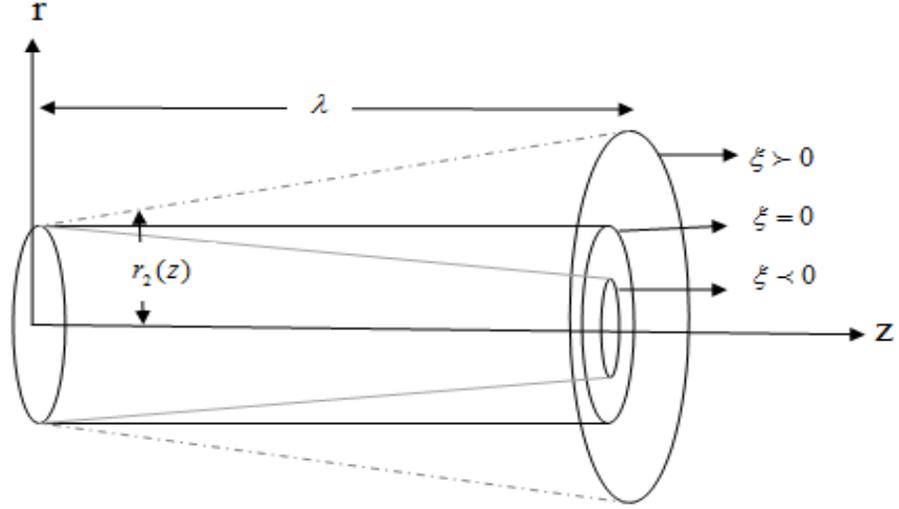


Fig 3.1 The simplified model of geometry of the problem

$$\begin{aligned}
 r_1' &= \delta' + \varepsilon' \cos[\theta'], \\
 r_2' &= a_1'(z) + b' \cos\left[\frac{2\pi}{\lambda}(z' - ct')\right], \\
 a_1'(z) &= a + z'\xi'(z')
 \end{aligned} \tag{1}$$

where $a_1'(z)$ is the radius of the tapered tube segment in the annulus region, a is the radius of tube of the non-tapered wall, ξ is the tapering parameter, δ' is the radius of the inner tube, b' is the amplitude of the wave, λ' is the wavelength, c' is the propagation velocity and t' is the time. The problem has been studied in cylindrical coordinate system (r', θ, z') radial, azimuthal and axial coordinates respectively.

The equation of motion of the flow in the gap between the inner and the outer tubes are

$$\frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial r'} + \frac{v'}{r'} + \frac{1}{r'} \frac{\partial w'}{\partial \theta'} = 0 \tag{2}$$

$$\rho \left[\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial z'} + v' \frac{\partial u'}{\partial r'} + \frac{w'}{r'} \frac{\partial u'}{\partial \theta'} \right] = -\frac{\partial p'}{\partial z'} + \mu \left[\frac{\partial^2 u'}{\partial z'^2} + \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} + \frac{1}{r'^2} \frac{\partial^2 u'}{\partial \theta'^2} \right] - \frac{\sigma a^2 B_0^2(r)}{\rho} u' \tag{3}$$

$$\begin{aligned}
 \rho \left[\frac{\partial v'}{\partial t'} + u' \frac{\partial v'}{\partial z'} + v' \frac{\partial v'}{\partial r'} + \frac{w'}{r'} \frac{\partial v'}{\partial \theta'} - \frac{w'^2}{r'} \right] = \\
 -\frac{\partial p'}{\partial r'} + \mu \left[\frac{\partial^2 v'}{\partial z'^2} + \frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'} \frac{\partial v'}{\partial r'} + \frac{1}{r'^2} \frac{\partial^2 v'}{\partial \theta'^2} - \frac{v'}{r'^2} - \frac{2}{r'^2} \frac{\partial^2 w'}{\partial \theta'^2} \right]
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \rho \left[\frac{\partial w'}{\partial t'} + u' \frac{\partial w'}{\partial z'} + v' \frac{\partial w'}{\partial r'} + \frac{w'}{r'} \frac{\partial w'}{\partial \theta'} + \frac{w' v'}{r'} \right] = \\
 -\frac{\partial p'}{\partial \theta'} + \mu \left[\frac{\partial^2 w'}{\partial z'^2} + \frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'} \frac{\partial w'}{\partial r'} + \frac{1}{r'^2} \frac{\partial^2 w'}{\partial \theta'^2} - \frac{w'}{r'^2} + \frac{2}{r'^2} \frac{\partial v'}{\partial \theta'} \right]
 \end{aligned} \tag{5}$$

Where v', w', u' are the velocity components in r', θ' and z' -directions respectively, ρ is the density, p' is the pressure and μ is the viscosity, $B_0(r)$ is the [magnetic field](#), σ is the electrical conductivity.

The boundary conditions are:

$$\mathbf{u}' = \mathbf{0}, \quad \text{at} \quad \mathbf{r}' = \mathbf{r}'_2, \quad (6)$$

$$\mathbf{u}' = \mathbf{V}', \quad \text{at} \quad \mathbf{r}' = \mathbf{r}'_1, \quad \text{It is convenient to non-}$$

dimensionalize the variables appearing in equations (2-5) and introducing Reynolds number Re , wave number ratio δ as follows:

$$p = \frac{a^2}{\mu c \lambda} p', \quad u = \frac{u'}{c}, \quad V = \frac{V'}{c}, \quad \theta = \theta', \quad t = \frac{c}{\lambda} t', \quad Re = \frac{\rho c a}{\mu}, \quad r_1 = \frac{r'_1}{a} = \frac{\delta'}{a} + \frac{\varepsilon'}{a} \cos(\theta')$$

$$\phi^* = \frac{b}{a}, \quad \delta = \frac{\delta'}{a}, \quad \varepsilon = \frac{\varepsilon'}{a}, \quad \delta_0 = \frac{a}{\lambda}, \quad z = \frac{z'}{\lambda}, \quad r_2 = \frac{r'_2}{a} = 1 + \frac{b'}{a} \cos(2\pi(z-t)),$$

$$\overline{M}^2(r) = \frac{\sigma B_0^2(r) a^2}{\mu}, \quad \xi(z) = \frac{\lambda}{a} \xi'(z)$$

(7)

where Re is the Reynolds number, M is the Hartmann number, where the parameter of taper is ($\xi = \tan \phi$), ϕ is called taper angle and for the converging tapering ($\xi < 0$), non-tapered artery ($\xi = 0$) and the diverging tapering ($\xi > 0$) as shown in (fig.3.1). δ_0 is the dimensionless wave number and ε is eccentricity parameter. ϕ^* is the amplitude ratio. After using the above assumption and the long wavelength approximation ($\delta_0 \rightarrow 0$) then

taking $\overline{M} = \frac{M}{r}$, the equations of motion in the dimensionless form become :

$$\frac{\partial u}{\partial z} = 0, \quad (8)$$

$$\frac{\partial p}{\partial z} = \left[\delta_0^2 \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right] - \frac{M^2}{r^2} u \quad (9)$$

$$\frac{\partial p}{\partial r} = 0, \quad (10)$$

$$\frac{\partial p}{\partial \theta} = 0, \quad (11)$$

Eqs. (10, 11) show that p is not a function of r and θ . The corresponding boundary conditions in non-dimensional form are

$$u = 0, \quad \text{at} \quad r = r_2 = 1 + \xi z + \phi^* \cos[2\pi(z-t)], \quad (12)$$

$$u = V, \quad \text{at} \quad r = r_1 = \delta + \varepsilon \cos[\theta] \quad (13)$$

3 Solution of the problem

Solution of the above boundary value problem is obtained by series solution method [21]. The deformation equation for the given problem is defined as

$$H(u, f) = (1-f)(\ell[\tilde{u}] - \ell[\tilde{u}_0]) + f(\ell[\tilde{u}] + \frac{1}{r^2} \frac{\partial^2 \tilde{u}}{\partial \theta^2} - \frac{\partial p}{\partial z}) = 0 \quad (14)$$

where ℓ , the linear operator is assumed to be $\ell = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{M^2}{r^2}$. We define the following initial guess satisfying the boundary conditions

$$\tilde{u}_0 = V \frac{\text{Sinh}(M (\text{Log}[\frac{r}{r_2}]))}{r_2} \text{Csch}[M (\text{Log}[\frac{r_1}{r_2}])], \quad (15)$$

Now we describe

$$\tilde{u}(r, \theta, z, t, f) = u_0 + f u_1 + f^2 u_2 + \dots \quad (16)$$

Using the above equation into Eq. (14) and then finding the terms of first two orders of embedding parameter f , we get the following problems including boundary conditions

Zeroth order system:

$$\ell[u_0] - \ell[\tilde{u}_0] = 0 \quad (17)$$

$$u_0 = 0, \text{ at } r = r_2 \quad (18)$$

$$u_0 = V, \text{ at } r = r_1 \quad (19)$$

The solution of the above zeroth order system can be obtained by using Eq. (15) and is simply found as

$$u_0(r, \theta, z, t, f) = \tilde{u}_0 = V \frac{\text{Sinh}(M (\text{Log}[\frac{r}{r_2}]))}{r_2} \text{Csch}[M (\text{Log}[\frac{r_1}{r_2}])] \quad (20)$$

First order system:

$$\ell[u_1] - \ell[\tilde{u}_0] - \frac{1}{r^2} \frac{\partial^2 u_0}{\partial \theta^2} + \frac{dp}{dz} = 0$$

or

$$\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} - \frac{M^2}{r^2} u_1 + \frac{1}{r^2} \frac{\partial^2 u_0}{\partial \theta^2} - \frac{M^2}{r^2} u_1 = \frac{dp}{dz} \quad (21) \quad u_1 = 0, \text{ at } r = r_2 \quad (22)$$

$$u_1 = 0, \text{ at } r = r_1 \quad (23)$$

The solution of the above linear ordinary differential equation is given in appendix

$$\begin{aligned} u_1 = & \frac{1}{4M(-4+M^2)} \text{Csch}[M \text{Log}[\frac{r_1}{r_2}]] (-4M \frac{\partial p}{\partial z} r^2 \text{Sinh}[M(\text{Log}[\frac{r_1}{r_2}])] (4r_2^2 M \frac{\partial p}{\partial z} \\ & - g(-4+M^2)(\text{Log}[\frac{r_1}{r_2}]) \text{Sinh}[M(\text{Log}[\frac{r_1}{r_2}])] - 4r_1^2 M \frac{\partial p}{\partial z} \text{Sinh}[M(\text{Log}[\frac{r_2}{r_1}])]) \\ & + g(-4+M^2)(\text{Log}[\frac{r_1}{r_2}]) \text{Sinh}[M(\text{Log}[\frac{r_1}{r_2}])]) \end{aligned}$$

Where

$$g = \frac{1}{r_1^2} M V \varepsilon \text{Csch}[M(\text{Log}[\frac{r_1}{r_2}])] r_1 \text{Coth}[M(\text{Log}[\frac{r_1}{r_2}])] +$$

$$M \varepsilon (1 + 2 \text{Csch}^2 [M(\text{Log}[\frac{r_1}{r_2}]) \text{Sin}^2 [\theta]]$$

Finally, for $f \rightarrow 1$, we approach the final solution. So from Eq. (16),

we get

$$\tilde{u}(r, \theta, z, t) = u_0 + u_1 \quad (24)$$

where u_0 and u_1 are defined in Eq. (20).

The instantaneous volume flow rate $\bar{Q}(z, t)$ is given by

$$\bar{Q}(z, t) = 2\pi \int_{r_1}^{r_2} r u dr, \quad (25)$$

$$\frac{\bar{Q}}{2\pi} = \frac{1}{4M(-4+M^2)^2} \left((4+M^2)(2g r_2^2 + (r_1^4 - r_2^4)M \frac{\partial p}{\partial z}) + 2(r_1^2 \text{Cosh}[M(\text{Log}[\frac{r_1}{r_2}])]$$

$$(-g(4+M^2) + 2M(g \text{Coth}[M(\text{Log}[\frac{r_1}{r_2}])] + M(-4+M^2) \text{VCsch}[M(\text{Log}[\frac{r_2}{r_1}])])$$

$$+ M(\text{Coth}[M(\text{Log}[\frac{r_1}{r_2}])])(-2(r_1^4 + r_2^4)M \frac{\partial p}{\partial z} + g r_2^2(-4+M^2)(\text{Log}[\frac{r_1}{r_2}])$$

$$+ r_1^2 \text{Csch}[M(\text{Log}[\frac{r_1}{r_2}])])(-2g + 4r_2^2 M \frac{\partial p}{\partial z} + g(-4+M^2)(\text{Log}[\frac{r_2}{r_1}])$$

$$+ 2g r_1^2 \text{Sinh}[M(\text{Log}[\frac{r_1}{r_2}])] - 2(-4+M^2) \text{VCsch}[M(\text{Log}[\frac{r_2}{r_1}])]$$

$$(r_2^2 M + 2r_1^2 \text{Sinh}[M(\text{Log}[\frac{r_1}{r_2}])])$$

26)(

The mean volume flow rate Q over one period is given as[22]

$$\bar{Q}(z, t) = \frac{Q}{\pi} - \frac{\phi^{*2}}{2} + \xi z \phi \text{Cos}[2\pi(z-t)] + 2\phi^* \cos[2\pi(z-t)] + \phi^{*2} \cos^2[2\pi(z-t)] \quad (27)$$

Now pressure gradient $\frac{dp}{dz}$ will be evaluated by using Eqs.(26) and (27) and is defined

in appendix

$$\frac{\partial p}{\partial z} = (2M(-4+M^2)^2(\bar{Q} - \frac{1}{M(-4+M^2)^2} \pi(g r_2^2(4+M^2) +$$

$$\begin{aligned}
& r_1^2 \text{Cosh}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right](-g(4+M^2) + 2M^2(-4+M^2)\text{VCsch}\left[M\left(\left(\text{Log}\left[\frac{r_2}{r_1}\right]\right)\right)\right]) \\
& + g r_1^2 M \text{Csch}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right](-2 + 2\text{Cosh}\left[2M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right] - (-4 \\
& + M^2)\text{Log}[r_1] + (-4 + M^2)\text{Log}[r_2]) + (-4M \\
& + M^3)(g r_2^2 \text{Coth}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right]\left(\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right) \\
& - 2\text{VCsch}\left[M\left(\text{Log}\left[\frac{r_2}{r_1}\right]\right)\right](r_2^2 M \\
& + 2r_1^2 \text{Sinh}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right])\right)/(\pi((2gr_2^2 + (r_1^4 - r_2^4)N)(4 \\
& + M^2) - 2gr_1^2(4 + M^2)\text{Cosh}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right] \\
& + 2M \text{Coth}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right](-2(r_1^4 + h^4)M + g r_2^2(-4 \\
& + M^2)\left(\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right) + 2r_1^2 M \text{Csch}\left[M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right](-2g + 4r_2^2 M \\
& + 2g \text{Cosh}\left[2M\left(\text{Log}\left[\frac{r_1}{r_2}\right]\right)\right] + g(-4 + M^2)\left(\left(\text{Log}\left[\frac{r_2}{r_1}\right]\right)\right)))
\end{aligned} \tag{28}$$

The pressure rise $\Delta p(t)$ in non-dimensional form is defined as

$$\Delta p(t) = \int_0^1 \frac{\partial p}{\partial z} dz \tag{29}$$

4 Results and discussion

The analytical and numerical results obtained above for the given analysis are discussed graphically. The graphical treatment for the data of pressure rise Δp , pressure gradient $\frac{\partial p}{\partial z}$ and velocity profile $u(r, \theta, z, t)$ with the variation of all emerging dimensionless parameters like time t , flow rate Q , the taper parameter ζ , the velocity of the inner tube V , the eccentricity parameter ε and the MHD parameter M has been analyzed. In the end, the stream lines observing the peristaltic flow are drawn for the parameters M , Q and ζ while other parameters remain fixed. The comparison graph for the values obtained in present work with the results of R. Ellahi et. al. [68] is displayed in figures. The graphs for the pressure rise $\Delta p(t)$ versus flow rate Q under the effects of given parameters are drawn in figs 2-6. These graphs show the pumping regions, that is, the peristaltic ($Q > 0, \Delta p > 0$), the augmented pumping ($Q > 0, \Delta p < 0$) and the retrograde pumping ($Q < 0, \Delta p > 0$). The pressure gradient $\frac{\partial p}{\partial z}$ against the coordinate z with the variation of pertinent parameters are shown in figs 7-10. The velocity field $u(r, \theta, z, t)$ versus the radial coordinate r is plotted in figs 11-15 for both two and three dimensions. The stream line graphs are shown in figs 16-18. Fig 2 is plotted to see the variation of pressure rise for different values of the eccentricity parameter ε and the angle θ while all other parameters are kept fixed. It is observed that peristaltic pumping region is in between $Q \in [0, 0.7]$, augmented pumping is in $Q \in [0.7, 2]$ and retrograde pumping part is $Q \in [-1, 0]$. It is also observed from this graph that the pressure rise increases with the variation of ε but decreases with the angle θ in between the region $Q \in [-1, 0.7]$ and opposite behavior is seen in the remaining part. The graph of pressure rise for the parameter M and δ is plotted in fig 3. Fig 4. shows that the peristaltic pumping part is Q

$\in [0, 0.3]$ when $\phi^* = 0.1$ and varies values of ε , while augmented and retrograde pumping regions are $Q \in [0.3, 2]$ and $Q \in [-1, 0]$, respectively. that the pressure rise in fig 5. decreases with parameter ζ in case ($\zeta > 0$ and $\zeta < 0$) when it is increases with the taper parameter ζ in case ($\zeta = 0$), The variation of pressure rise ∇p for V is similar to that of M (See fig6) The pressure gradient $\frac{\partial p}{\partial z}$ for the parameters M and δ is drawn in fig 3.6. It is measured from this figure that pressure gradient is in linear relation with both of the parameters in narrowest parts of the cylinders but inverse relation is seen in the wider parts. The variation of pressure gradient with the parameters ϕ and ε is very much similar to that of the parameters M and δ and is shown in fig8. The only difference is that the pressure gradient is minimum on the left and right sides of the cylinder while appears maximum at the centre. It means that flow can easily pass without imposition of large pressure gradient in the two sides of the channel while much pressure gradient is required to maintain the flux in the central part near $z = 0.8$. This is in good agreement with the physical condition. Also, for a diverging tapering with angle ζ . Figs(8a) extending case $\zeta > 0$ figs(8b), the pressure gradient values higher than all other existing results corresponding to converging tapering case $\zeta < 0$. It can be observed from figs9 and 10 that the pressure gradient increases with the parameters Q and V , while when δ is increased the pressure gradient decreases on the left and right sides but increases at the centre of the cylinders. It is also seen that the variation of pressure gradient remains same in the two sides of the channel and become different at the central part with changing V but this variation remains same throughout for the parameter Q . The fig11 shows that the velocity field is an increasing function of the parameter δ while decreasing with the parameter M . The velocity in inverserelation with Q but have a direct variation with ε (see fig12). It is also observed that the presence of magnetic field for fluid causes to slow down the flow. It is observed from fig13 that the velocity distribution is increasing with δ and ϕ^* while reducing for t . Fig14 shows that the velocity profile is linearly changing with ϕ^* and V . shows the Fig 15 that velocity in the case $\zeta > 0$ at (15.a) higher than reported in the case $\zeta \leq 0$ at (3.15.b). Fig 3.16 is drawn to see the stream lines for the parameter M . It is measured from this figure that numbers of bolus are not changing but size is increasing with the increasing effects of M in the bottom of the cylinder, while bolus are lessened in number when seen in the upper part. The boluses are reduced both in size and number when seen for the parameter Q in both parts of the geometry (see fig17). Fig 16 is drawn to see the stream lines for the parameter M . It is measured from this figure that numbers of bolus are not changing but size is decreasing with the increasing effects of M in upper part of the cylinder, while bolus are disappears when seen in the the bottom part. The boluses are reduced both in size and number when seen for the parameter Q in both parts of the geometry (see fig 17). It is seen from fig 18 that the numbers of bolus are decreasing with different values of the parameter ζ in both sides of the cylinder but in the lower half of the tube, the bolus becomes smaller with increasing magnitude of the parameter $\zeta > 0$ and fade out with less values of $\zeta < 0$.

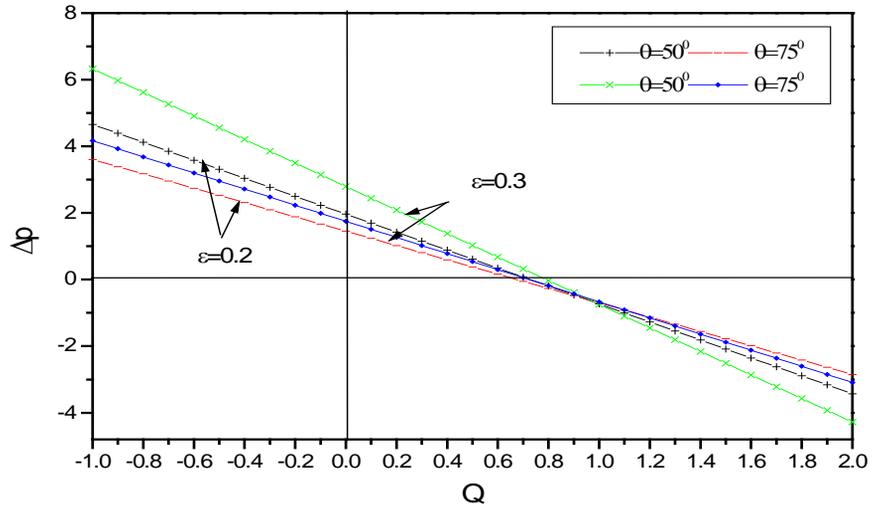


Fig 2 pressure rise versus flow rate for fixed parameters
 $\phi^* = 0.2, \zeta = 0, t = 0.1, \delta = 0.1, M = 0.5, V = 0.5$

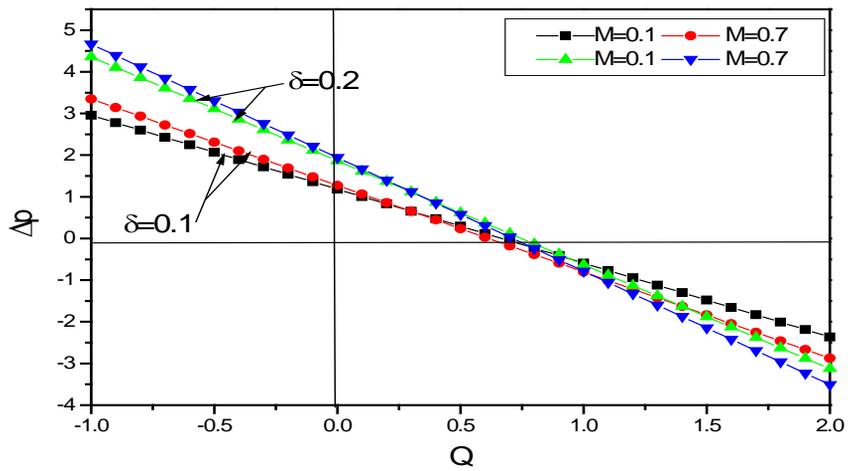


Fig 3 pressure rise versus flow rate for fixed parameters
 $\phi^* = 0.2, \zeta = 0, t = 0.1, \varepsilon = 0.01, \theta = 50^\circ, V = 0.5$

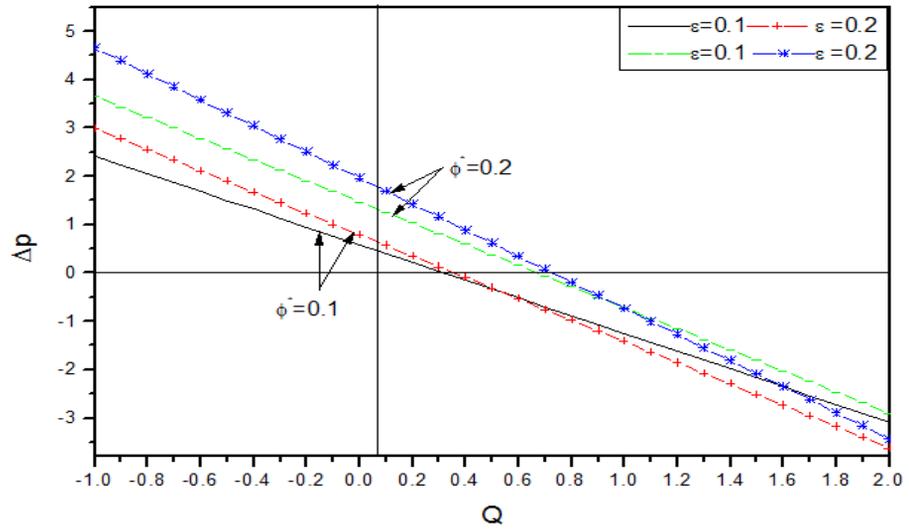


Fig 4 pressure rise versus flow rate for fixed parameters
 $\theta = 50^\circ, \zeta = 0, t = 0.1, \delta = 0.1, M = 0.5, V = 0.5$

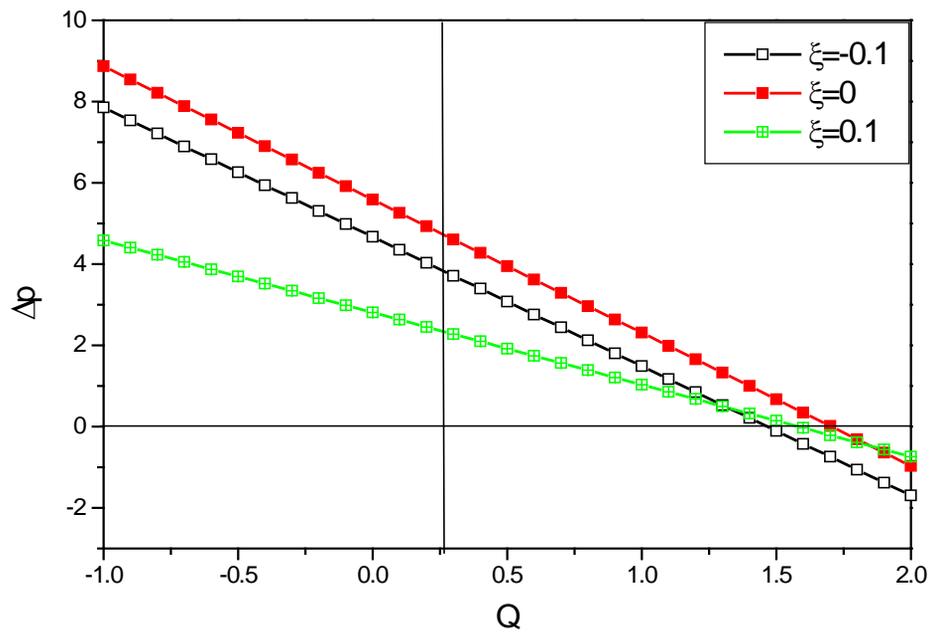


Fig 5 pressure rise versus flow rate for fixed parameters
 $\theta = 50^\circ, \varepsilon = 0.01, t = 0.1, \delta = 0.1, M = 0.5, V = 0.5$

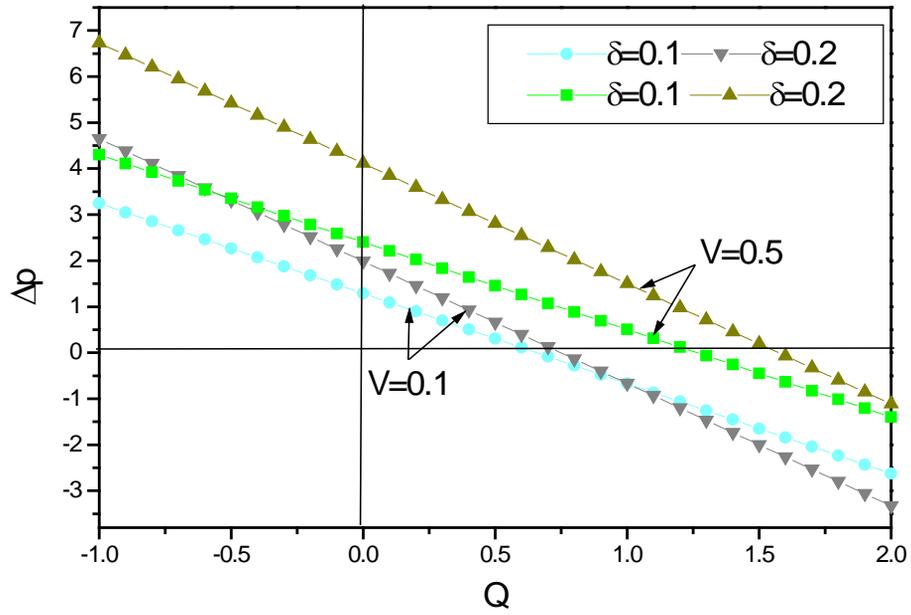


Fig 6 pressure rise versus flow rate for fixed parameters $\phi^* = 0.2, \zeta = 0, t = 0.1, \varepsilon = 0.01, M = 0.5, \theta = 50^\circ$

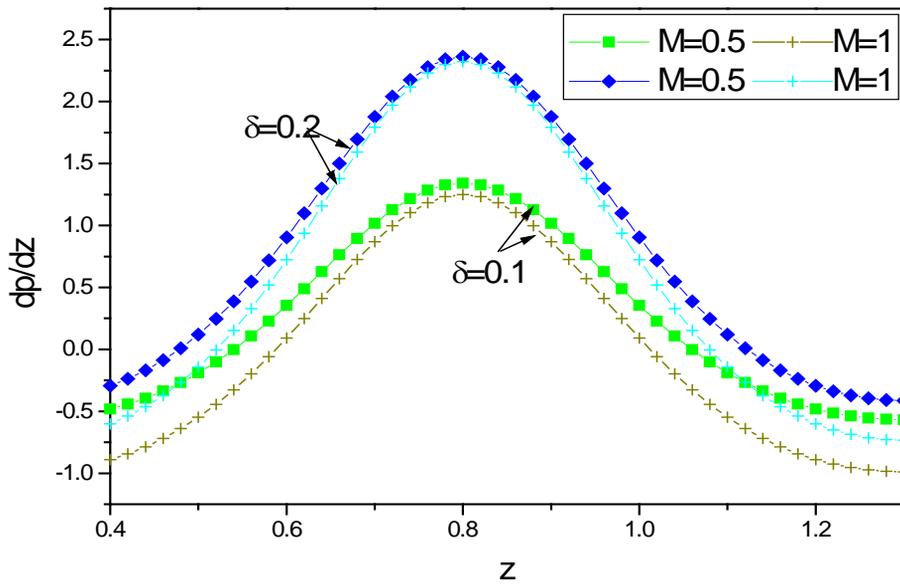
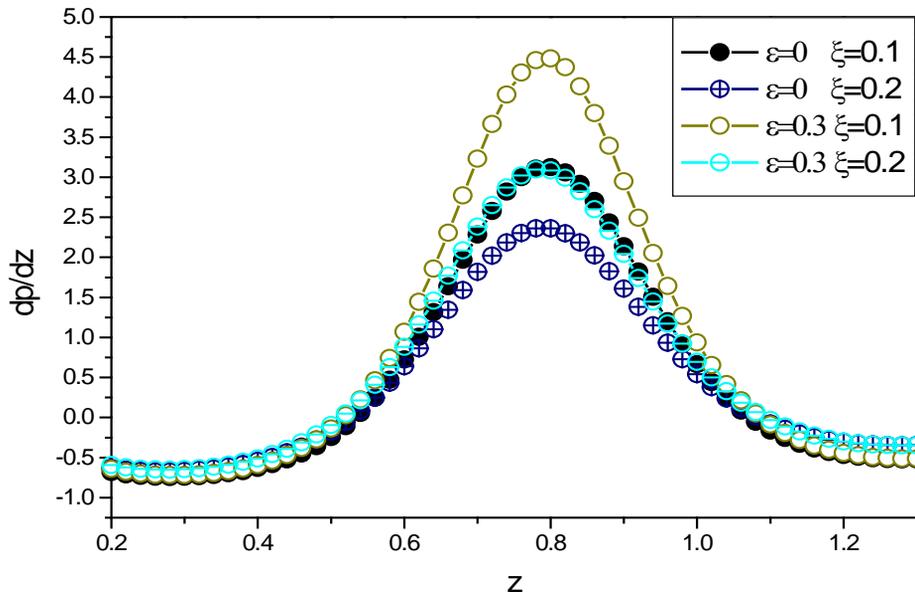
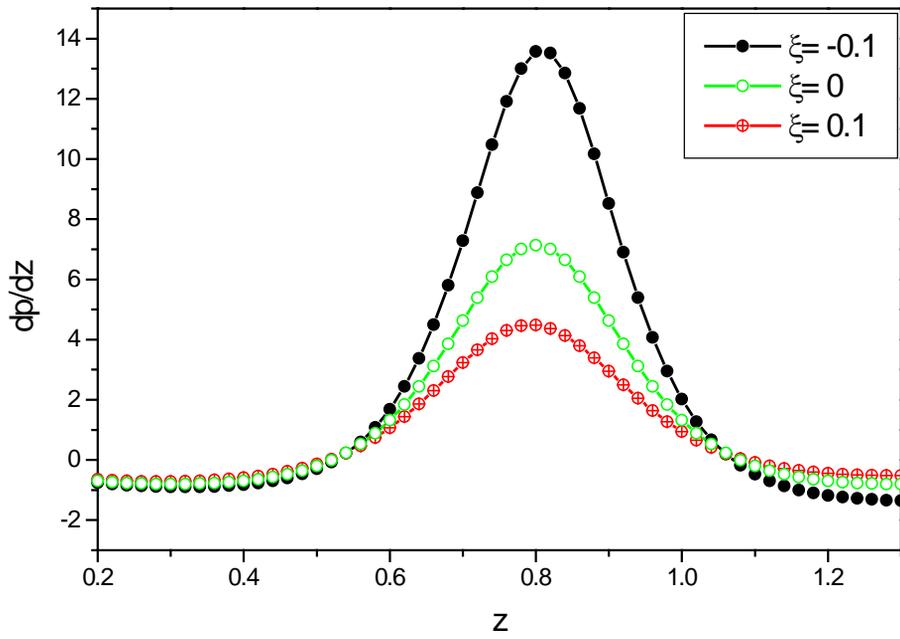


Fig 7 Variation of pressure gradient $\frac{dp}{dz}$ with z for fixed parameters $\phi^* = 0.1, \zeta = 0, t = 0.3, \theta = 50^\circ, \varepsilon = 0.01, V = 0.3, Q = 0.5$



a



b

Fig8 Variation of pressure gradient $\frac{dp}{dz}$ with z
for fixed parameters $M = 0.5, t = 0.3, \theta = 50^\circ, \varepsilon = 0.01, V = 0.3, Q = 0.5$

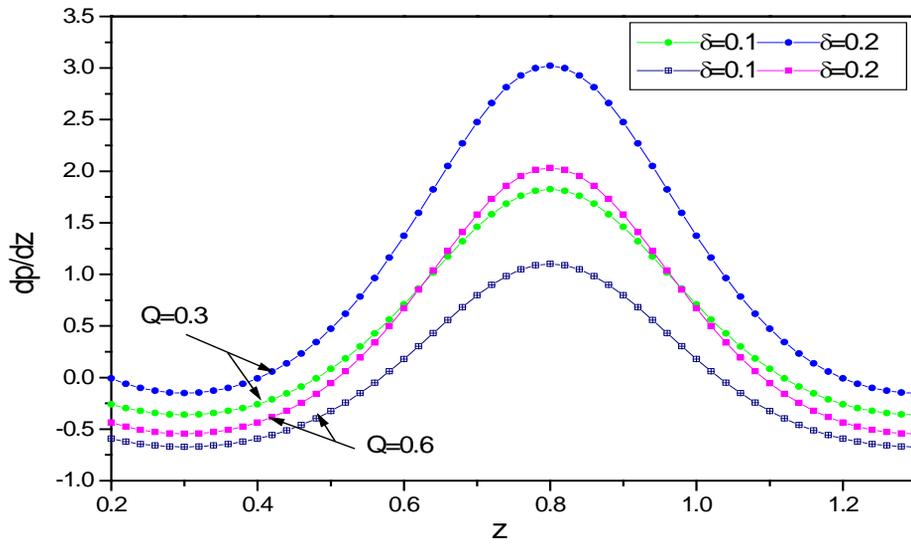


Fig9 Variation of pressure gradient $\frac{dp}{dz}$ with z for fixed parameters $\phi^* = 0.1, \zeta = 0, t = 0.3, \theta = 50^\circ, \varepsilon = 0.01, V = 0.3$

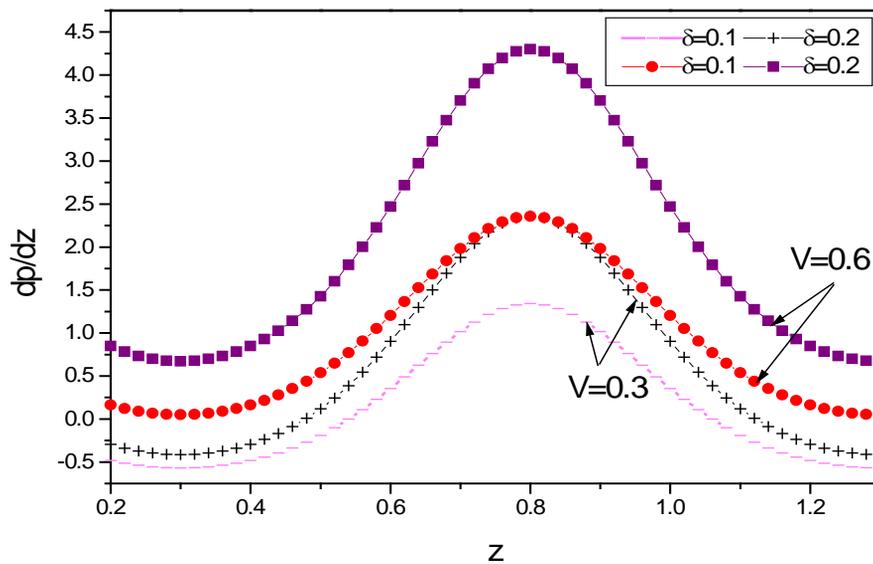


Fig10 Variation of pressure gradient $\frac{dp}{dz}$ with z for fixed parameters $\phi^* = 0.1, \zeta = 0, t = 0.3, \theta = 50^\circ, \varepsilon = 0.01, M = 0.5, V = 0.3$

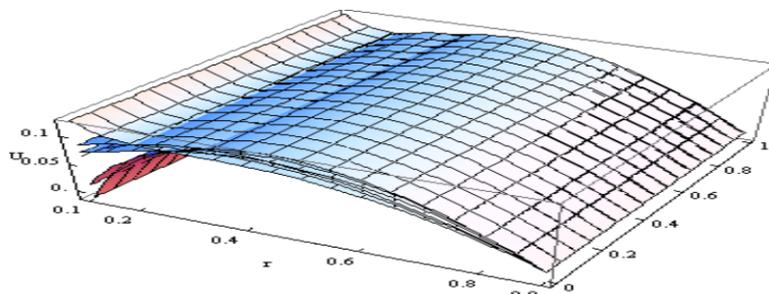
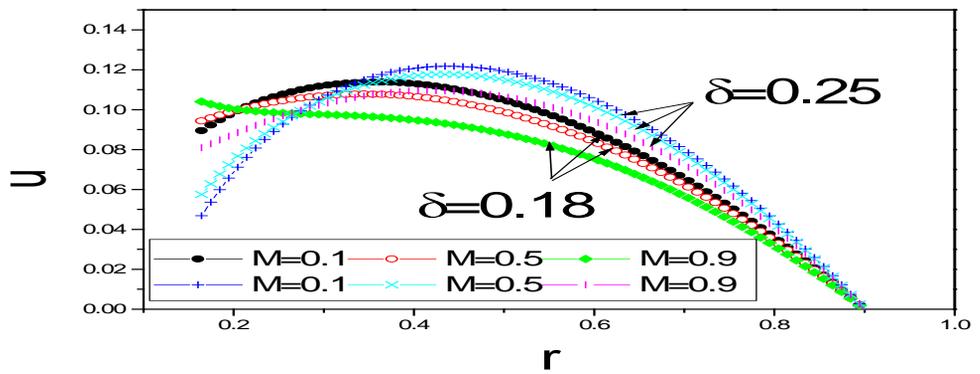


Fig11 Variation of velocity profile u with r for fixed parameters
 $z = 0, \phi^* = 0.1, \zeta = 0, \theta = 75^\circ, V = 0.1, Q = 0.5, t = 0.5, \varepsilon = 0.1$

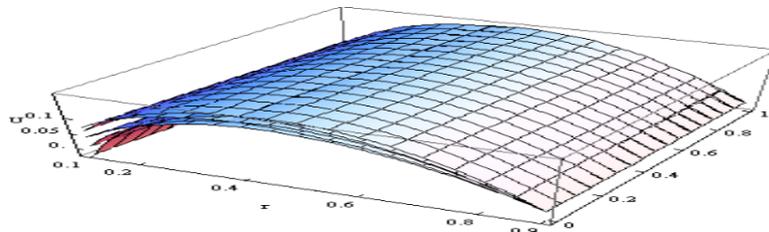
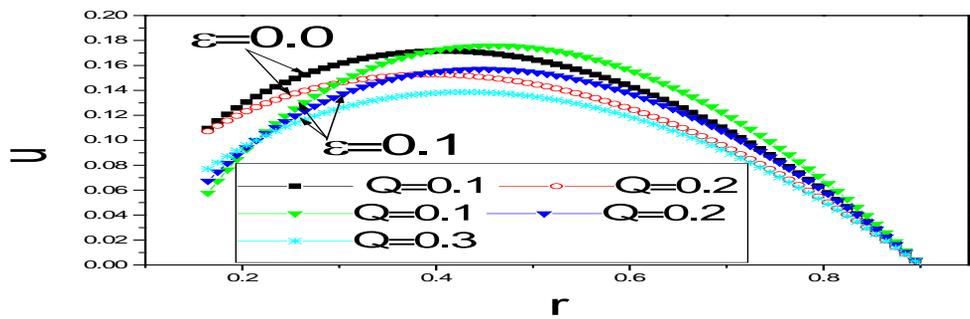


Fig12 Variation of velocity profile u with r for fixed parameters
 $z = 0, \phi^* = 0.1, \zeta = 0, \theta = 75^\circ, V = 0.1, Q = 0.5, t = 0.5, \varepsilon = 0.1$

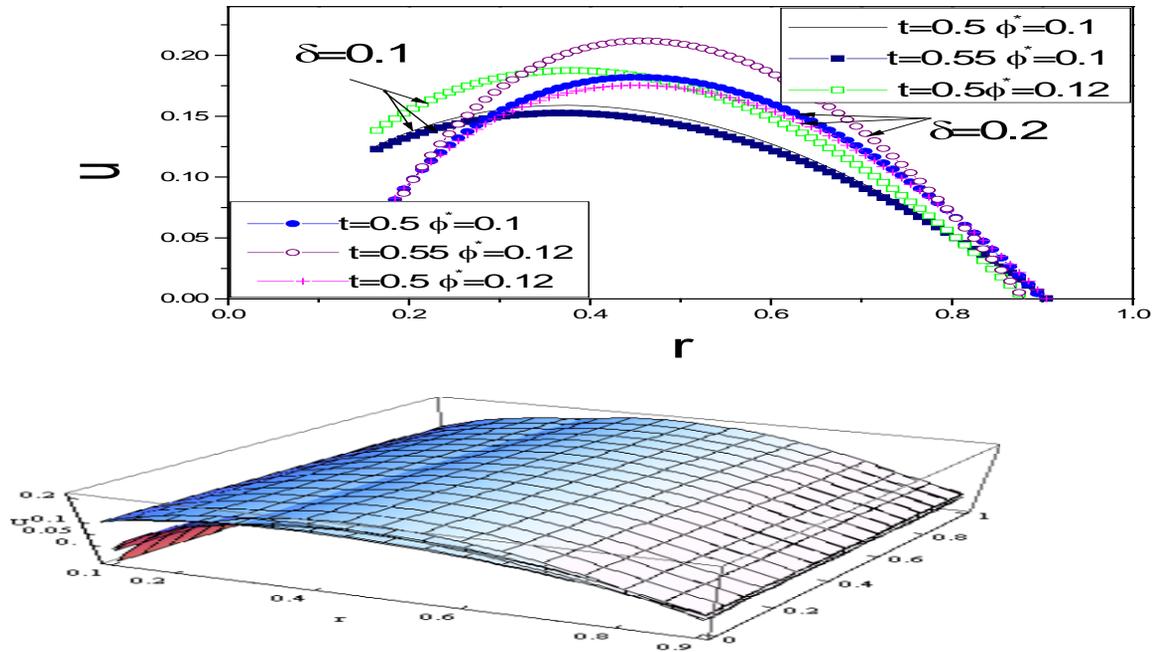


Fig13 Variation of velocity profile u with r for fixed parameters $z = 0, \phi^* = 0.1, \zeta = 0, \theta = 75^\circ, V = 0.1, Q = 0.1, \varepsilon = 0.01, M = 0.5$

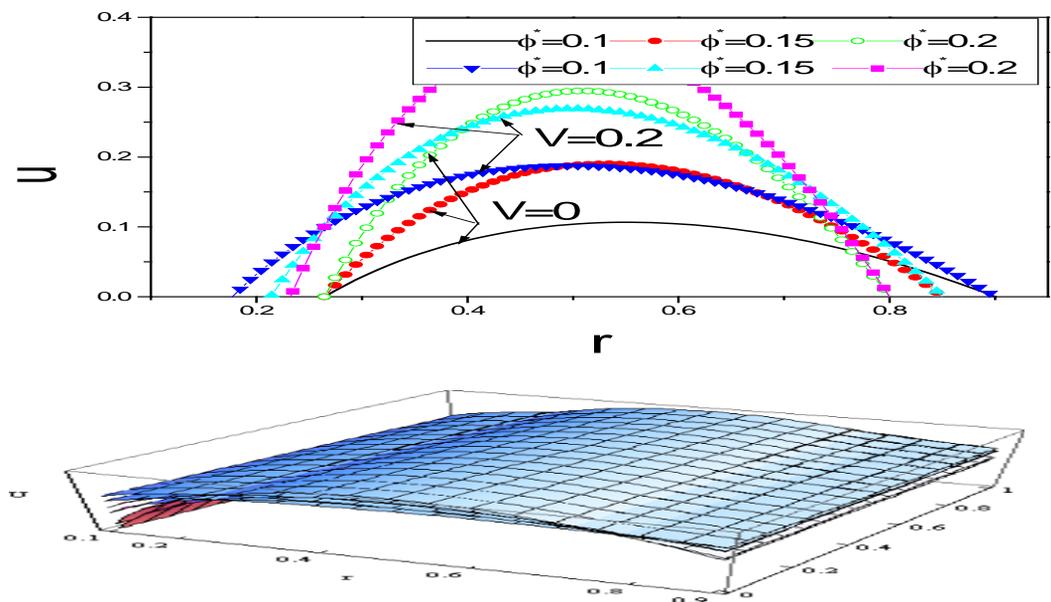


Fig14 Variation of velocity profile u with r for fixed parameters $z = 0, \zeta = 0, \theta = 50^\circ, Q = 0.1, t = 0.5, \delta = 0.2, \varepsilon = 0.1, M = 0.5$

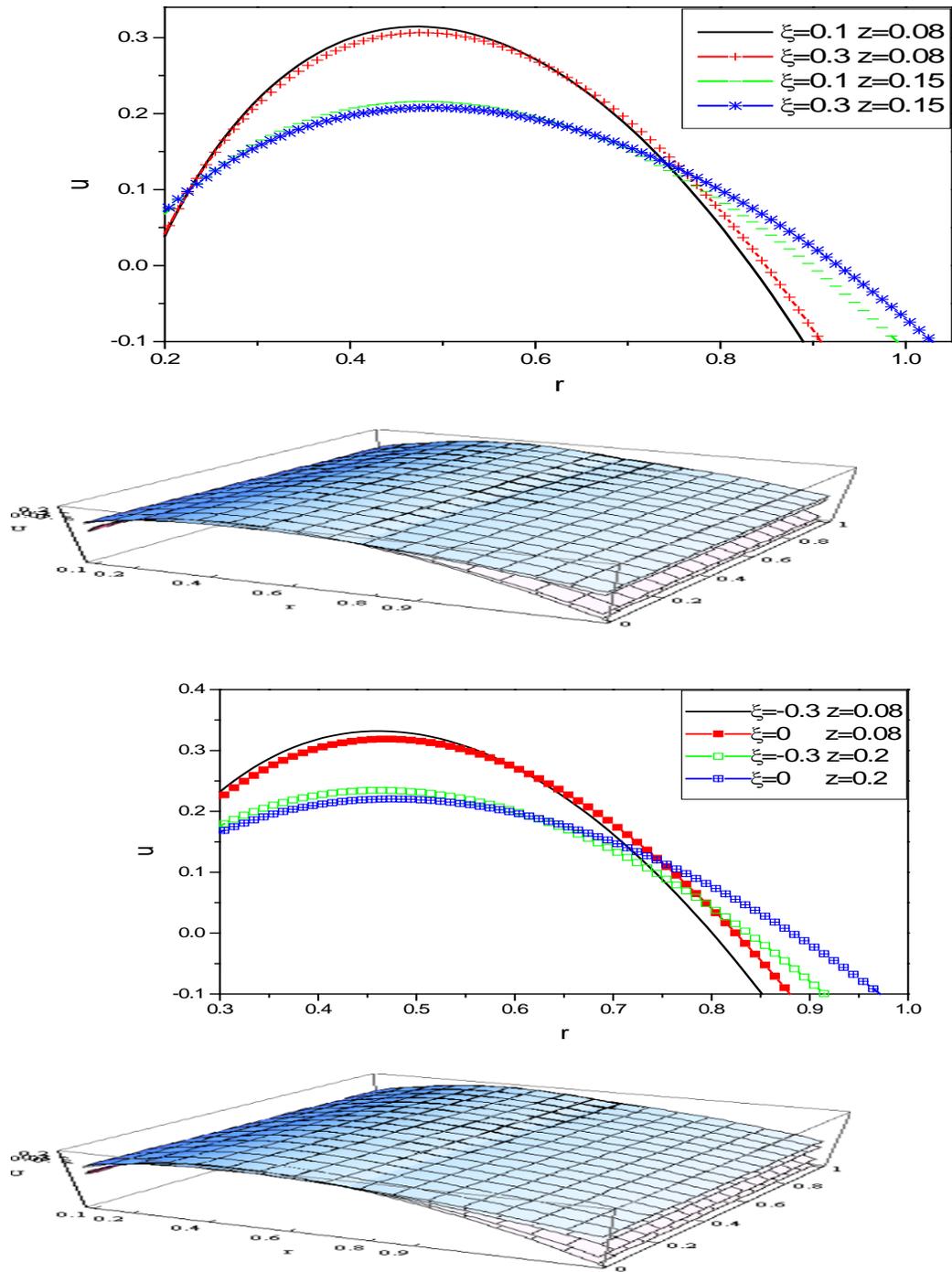


Fig 15 Variation of velocity profile u with r for fixed parameters $\theta = 50^\circ$, $\phi^* = 0.2$, $Q = 0.1$, $t = 0.5$, $\delta = 0.2$, $\varepsilon = 0.1$, $M = 0.5$

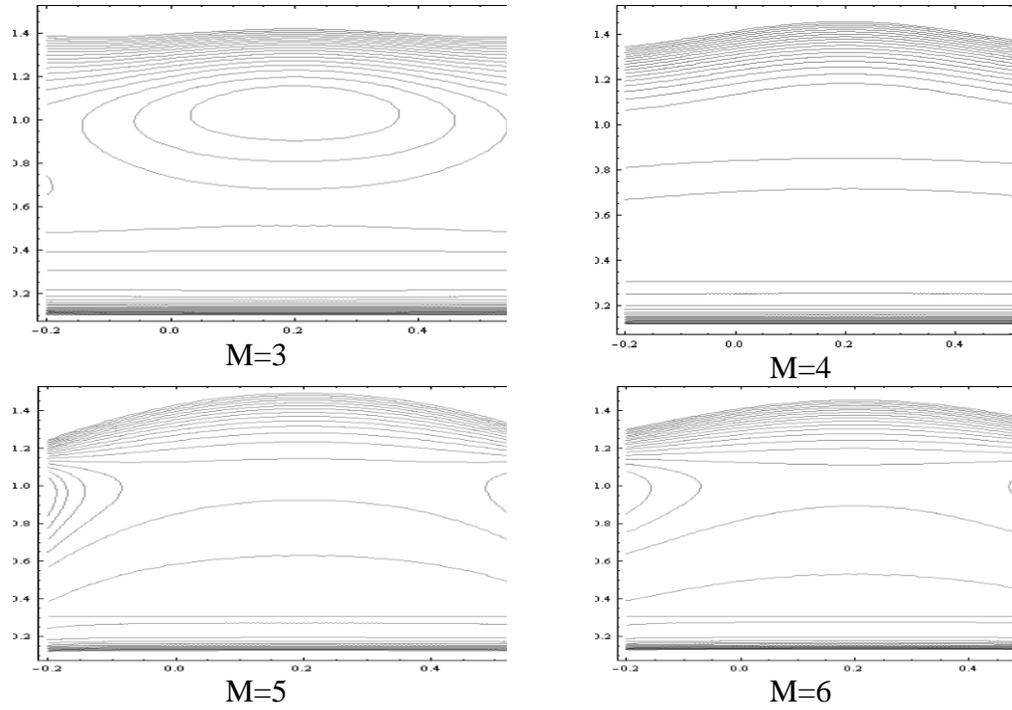


Fig 16: Stream lines for different values of M for The other parameters are $\theta = 50^\circ, \phi^* = 0.02, V = 0.3, Q = 1, t = 0.2, \delta = 0.05, \varepsilon = 0.4, \zeta = 0$

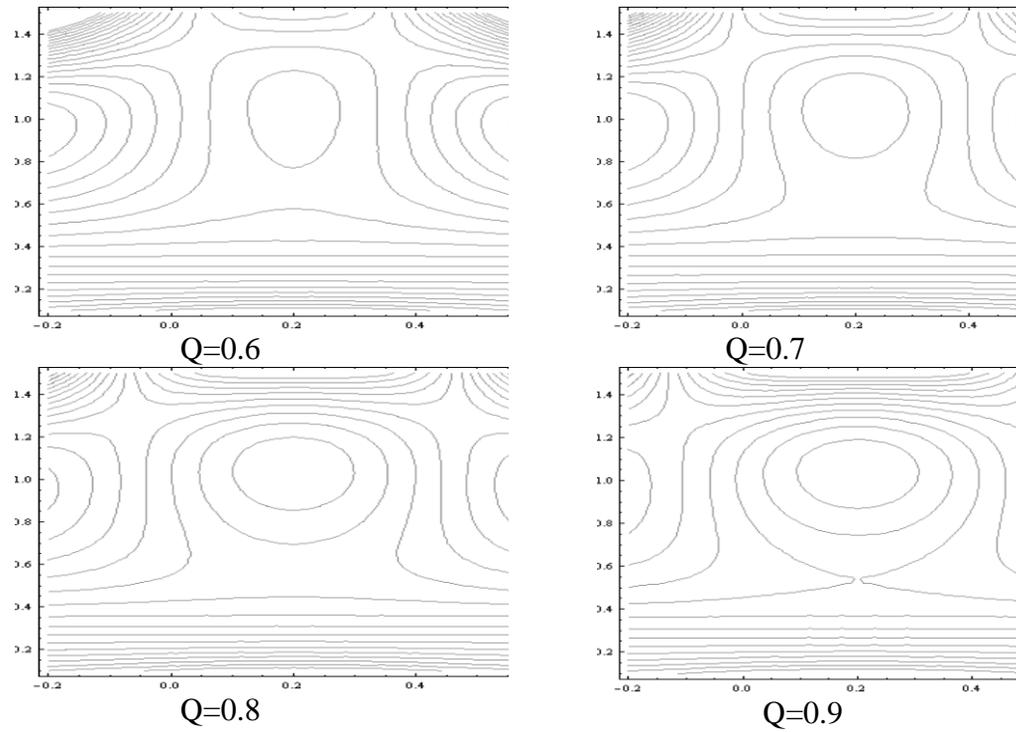


Fig 17: Stream lines for different values of Q for The other parameters are $\theta = 50^\circ, \phi^* = 0.05, V = 0.3, M = 1, t = 0.5, \delta = 0.05, \varepsilon = 0.4, \zeta = 0$

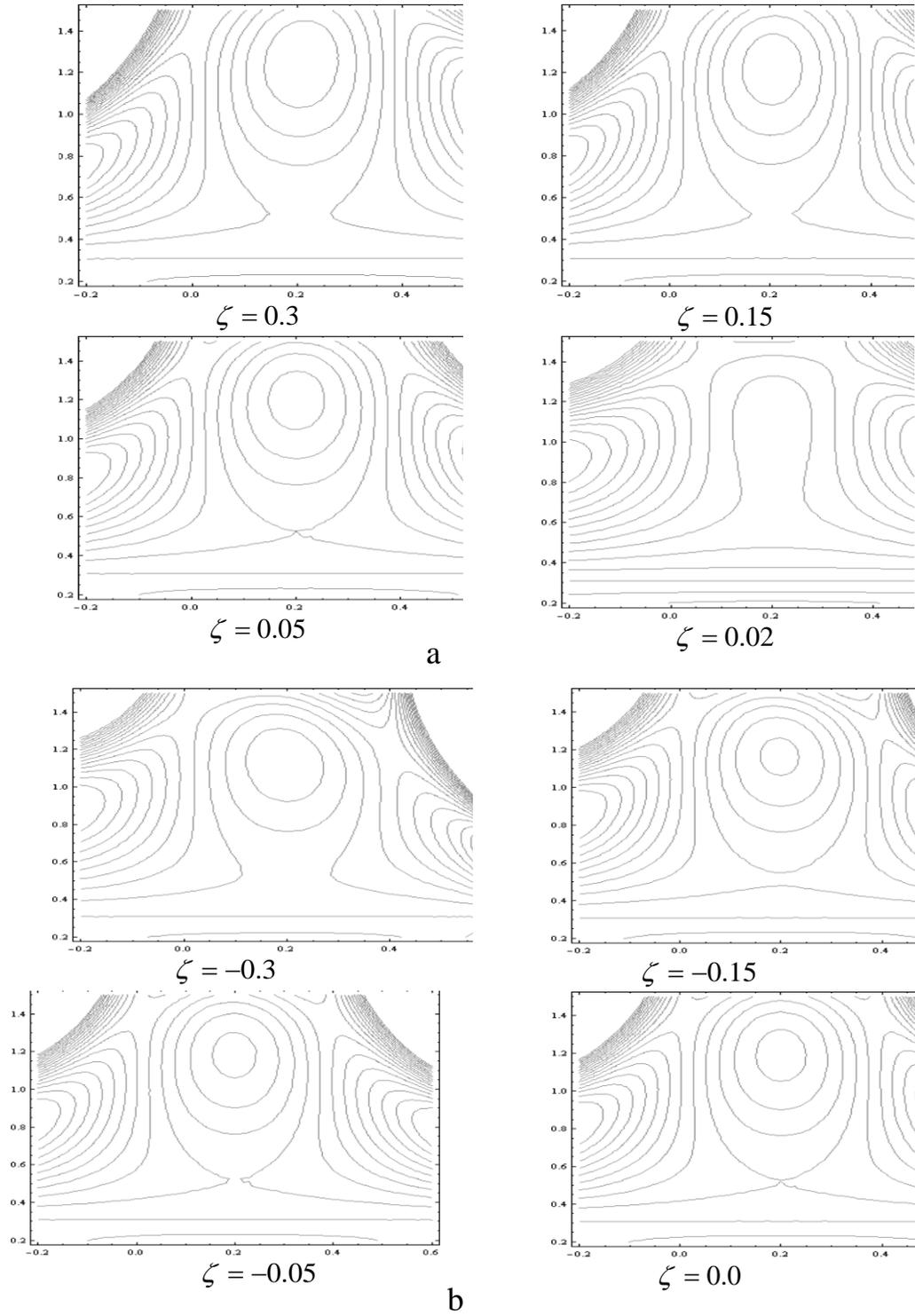


Fig 18: Stream lines for different values of a for ($\zeta > 0$), b for ($\zeta \leq 0.0$) The other parameters are $\theta = 50^\circ, \phi^* = 0.2, V = 0.3, Q = 0.6, t = 0.5, \delta = 0.05, \varepsilon = 0.4, M = 0.3$

5 Concluding remark

In the present investigation Homotopy Perturbation method solutions are presented for the peristaltic flow of MHD fluid between two eccentric tubes. The inner tube is rigid and the outer have sinusoidal waves generated on the walls of tapered. The problem is measured under the assumptions of long wave length and low Reynolds number. The following observations have been found:

- It is observed that pressure rise is a decreasing function of taper parameter ζ and θ while increasing function of radius δ , magnetic field M
- pressure rise is a increasing function of eccentricity parameter ε , amplitude ratio ϕ^* and the velocity of the inner tube V .
- The pressure gradient increases with radius δ , eccentricity parameter ε and the velocity of the inner tube V while decreases with flow rate Q , taper parameter ζ and magnetic field M .
- The velocity profile is increase with the increase in radius δ , amplitude ratio ϕ^* and eccentricity parameter ε while decreases with the flow rate Q , magnetic field M and taper parameter ζ ,
- The velocity in the case of eccentric cylinders higher than concentric one
- It is depicted that number of bolus is changing inversely with taper parameter ζ and flow rate Q while not changing increasing effects of magnetic field M but dimensions.

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