The use of mathematical modeling in architectural design to provide sustainable housing

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ABSTRACT

. The research deals with the problem of housing in developing countries, especially the increasing costs of constructing housing units and the inability of governments to provide adequate   housing   for   citizens.
The research works to find a way to help reduce construction costs, taking into account design standards and user requirements, by studying the concept of housing in Egypt, its types and development and the role of the state in providing sufficient and sustainable housing that meets the needs and aspirations of users, reviewing the design criteria according to the Unified Egyptian Building Code to ensure that housing projects meet the needs of users, and how to achieve sustainable housing by learning about the concept of sustainability   and   its   principles.
The research used the inductive approach to build a mathematical model using linear programming aimed at reducing the wasted spaces within the housing unit and thus reducing construction costs, and then using the experimental method to apply and test the mathematical model on one of the government housing models "Dar Misr" in order to evaluate   it.
The research helped provide a tool that helps architectural designers in designing and evaluating sustainable housing projects, which reduce construction costs taking into account the design criteria required for the housing unit.

Keywords: Sustainable housing - Egyptian building law - Mathematical modeling - Linear programming

1.INTRODUCTION:

Rapid urban growth in many developing countries has led to many problems (economic, social, health, security, ...) in the absence of governments to meet citizens' housing needs, and slum areas emerged that lack the necessary services such as electricity, clean drinking water and sanitation networks. Also, crime rates increased and environmental health deteriorated in these areas(Kenawy, 2007).

Where obtaining adequate housing represents a basic requirement for every family, because it provides them with psychological stability, social standing, and economic aspiration for families to raise their social and economic level, and adequate housing is one of the essential needs of the human being, as it comes second after providing food and clothing, as recognized by scholars The meeting where housing is one of the basic pillars of community security, and was affirmed by the United Nations in the Istanbul Declaration on Human Settlements: "Affirming commitment to the right to housing for citizens with the guarantee of the legal guarantor of tenure, protection from discrimination, and equal opportunities to obtain adequate and affordable housing, While ensuring the participation and cooperation of the public, private and non-governmental sectors in order to provide this"(Al-Kafri, 2003).

Where the Millennium Conference in 2000 referred to a set of development goals related to the housing sector, and the most important of those goals were (environmental management by supporting housing with safe drinking water, sanitation networks and basic services - attention to housing the poor, given the many

problems he faces), as one of the main challenges to sustainability in the housing sector(UN-Habitat, 2001).

In April 2002, the United Nations Housing Rights Program appeared as a joint initiative between UN-Habitat and the Office of the United Nations High Commissioner for Human Rights, which focused on governments' focus on taking appropriate measures to promote, protect and guarantee the right to adequate housing(UN-Habitat, 2002).

In 2008, UN-Habitat announced that the development goal of the United Nations Housing Rights Program is the right to adequate housing by trying to consider affordability, accessibility, support and empowerment of housing reform and achieving sustainability(UN-Habitat, 2008).

A framework was developed for developing countries to achieve sustainable housing in sustainable cities through the principles proposed by the United Nations Human Settlements Program in Nairobi that include environmental, economic, social and cultural dimensions that must be applied and tested in new residential areas(UNEP, 2011).

Therefore, the state played a major role in monitoring and providing adequate, sustainable and low-cost housing to be able to meet the needs of all users, despite the great development in housing and the state's responsibility to provide a huge amount of housing units to meet the increasing demand for them due to the increasing number of the population, it represents a great burden on the state budget due to the increasing prices of building and construction materials(Al-Kafri, 2003).

Therefore, we find that many of those concerned with providing housing, including architects, urbanists and decision-makers in the field of housing, lack relevant studies and concepts of sustainability that work to reduce the cost of constructing housing units in a manner that does not affect the design standards and the needs of different groups of users, and helps the state provide the largest possible number sustainable housing units to meet the needs of citizens(Mahmoud, 2004).

1.1. Research problem:

Architects do not consider the basic requirements of users and the optimal use of architectural spaces and reduce unused spaces, which leads to an increase in the cost of construction of a residential unit and the inability of the user to purchase it.

1.2. Research objectives:

Providing a tool that helps architects and decision-makers in the field of housing to provide sustainable housing units in accordance with the architectural design standards of the unified Egyptian Building Law and consider the needs and requirements of the target groups of the population in Egypt.

1.3. Research Methodology:

To achieve this goal, the research followed the deductive methodology to defining housing and extracting the design requirements for housing according to the architectural design standards of the Egyptian Building Law and the concepts of sustainability as an entry point to study sustainable architectural spaces for the formation of the housing unit, and the most important requirements of users to meet their functional needs within the housing unit, then use the inductive methodology to study mathematical modeling Especially linear programming and how to use it to reduce wasted areas, where we will built a mathematical model using linear programming consisting of a target function and a set of constraints and it will be solved by LP Solve IDE- 5.5.2.11 program, then use the experimental methodology to apply and test the mathematical model on one of the medium governmental housing models "Dar Misr" in order to evaluate The effectiveness of architectural designs to be exploited. Plots of land and achieving the concept of sustainability in terms of reducing construction costs and optimal use of resources.

2.DEFINITION OF HOUSING:

The concepts of housing varied, which began with the material concept, considering the house as a shelter for protection from natural conditions and the content of some activities that take place inside the housing unit as shown in Figure (1), which is the concept adopted by the theories of architecture and planning from the middle of the twentieth century and its application coincided with the global trend towards industrial thought And the modularity of the product, and thus the housing theories, policies and projects are absent from the basic needs of users (the human dimension) when providing housing(Studies, 2013).

Then the eighties witnessed a development in the field of urbanization to respect the human dimension that led to the definition of housing as an organizer of social relations between the family and society and between family members each other, in addition to achieving psychological needs such as feeling safety, belonging and expressing identity, including the social status and cultural identity of the family. Needs are important and necessary for all social groups without exception(Al-Shehri, 2008).

Thus, the broader concept of housing is the place in which all daily residential activities and interactions with others take place and the shape of this range varies according to the lifestyle and type of dwelling in which the family lives, and these activities are not limited within the boundaries of the private residential unit, but may extend outside it to include common spaces and sometimes the street, as shown in Figure (2). Therefore, the concept of

housing covered by the research paper includes the planning and design concept and includes the material, social, economic and environmental content(Studies, 2013).

2.1. Architectural design standards for housing:

When we talk about housing in its concept the first thing that comes to mind is the fact that this housing is whether from a health or social point of view or in terms of its containment of modern civilization means and with the least areas it can solve the growing housing problem.

The research will focus on studying the following(Al-Shehri, 2008):

a. Standard unit of resident.
b. Standard unit of housing.
C. Design requirements for residential spaces according to the Egyptian Building Law.

2.1.1. Standard unit of resident:

The standard unit of the inhabitant may be taken as an individual or as a group, but the relationship between the number of housing and the number of the population as individuals gives only population density and it is an unclear and accurate picture of the housing situation in the same country. These relationships may differ from one region to another, and the dwelling may house a single individual or shelters. Ten individuals, as the social reality is at the same time the resident as one individual and the dwelling of a group of individuals who wish to share in one housing and so that a clear

No. of Residential unit	Main Rooms	No. of users	Total of users in residential unit
101	Bedroom no. 1	1	4
	Bedroom no. 2	1	
	Master bedroom	2	
102	Bedroom no. 1	2	5
	Bedroom no. 2	1	
	Master bedroom	2	
201	Bedroom no. 1	2	6
	Bedroom no. 2	2	
	Master bedroom	2	
202	Bedroom no. 1	1	3
	Bedroom no. 2	0	
	Master bedroom	2	
301	Bedroom no. 1	0	2
	Bedroom no. 2	0	
	Master bedroom	2	
302	Bedroom no. 1	0	Not used
	Bedroom no. 2	0	
	Master bedroom	0	

Table No. 1 shows way of distribution of users to the main rooms in the residential units within the housing.

Source: the researcher.

picture can be given about the housing situation in a particular place, drawing a table in which the number of users who represent the inhabitants of each housing is distributed vertically and the number is distributed horizontally, the main rooms
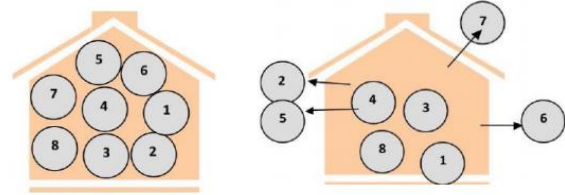

Figure No. 1 shows housing in its specific concept

Figure No. 2 shows the comprehensive concept of housing

Source: (Studies, 2013)

located in each residence(Yousef, 2002).

2.1.2. Standard unit of housing:

The human being is the basis for every measure, and therefore the standard unit of the dwelling depends on the scale of the human being, where the person makes the necessary things that he needs so that their dimensions match the human scale, and architects must respect the human scale when designing the dwelling where the dimensions of the dwelling are determined according to the scale of the human being and its uses of furniture and appliances Hygienic and cars, and the human scale depends on the measurements of a normal person, both in terms of height and size, in his various positions and during his movement(Mahmoud, 2004).

It is necessary to know the dimensions of tools, clothes and the different uses of the human being in order to be able to determine accordingly the appropriate dimensions of the rooms, designs and spaces that must be left between the furniture without leaving wasted areas while respecting the aesthetic aspect of the residence and not neglecting it, whether in designs or painting and lighting operations(Yousef, 2002).

Therefore, countries have taken care to issue strict laws that regulate housing construction processes to achieve the necessary design standards to achieve comfort for the user inside the housing unit and meet his basic requirements, as we find in Egypt a unified Egyptian building law has been issued to ensure housing quality and to achieve the human dimension and its basic requirements. Designing housing units according to the Unified Egyptian Building Law promulgated by Law No. 119 of 2008(Law, 2009).

2.1.3. Design requirements for residential spaces according to the Egyptian Building Law:

According to Article No. 94 of the Egyptian Building Law, several design requirements have been defined that must be adhered to by the architectural designer when designing residential architectural spaces to ensure the quality of housing, as these requirements have been set by architectural specialists with respect for the human dimension and the basic requirements of the users as shown in Table No. 2 (shmff, 2021):

Use	Minimum dimension (meters)	The minimum of area (meter square)
Living hall	3.00	10.00
Residential rooms	2.50	7.50
Kitchen	1.50	3.00
Bathroom	1.20	1.50
W.C	0.80	0.80
Guard room	2.00	5.00

Table No. 2 shows the design requirements for residential spaces according to the Egyptian Building Law
Source: (shmff, 2021)

3. SUSTAINABILITY:

Sustainability was defined in 1987 AD by the Portland Commission (the World Commission for Development and the Environment (WCED)) as "the path of human progress that meets the needs and requirements of current generations without prejudice to the ability of future generations to meet their needs"(un-habitat, 2021), meaning that every generation must meet its current needs without incurring debts. It can pay it without prejudice to the needs of future generations, and according to this definition, the principles of justice between one generation, justice between current and future generations, and preserving the integrity of the environment are the main guides for sustainable development((HABITAT), 1994).

3.1. dimensions of sustainability:

According to the United Nations definition of shelter UN-Habitat, it is a dynamic process (effective and changing), with multiple dimensions, and requires the concerted efforts of all workers in the field of urban development (the state, the private sector, non-governmental organizations and users), and at all levels (central and local) in order to achieve urban development For current generations without prejudice to the needs of future generations within a stable institutional, political, economic and social framework capable of empowering the actors of that development(UN-Habitat, 2002).

In the definition of the United Nations Human Settlements Program, UNCHS, the city that achieves sustainable urban development is the city that achieves economic and social development side by side with urban development, so that such development takes into account the avoidance of negative impacts on the surrounding
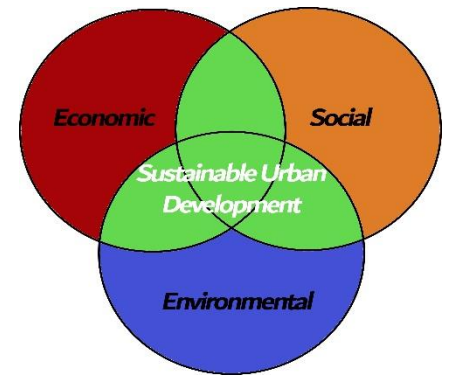

Figure 3 shows the dimensions of sustainable urban development.
Source: (UN-Habitat, 2012)

environment resulting from that development(UN-Habitat, 2012).

3.2. Sustainable housing:

The dimensions of achieving sustainable housing are an integral part of the main dimensions of sustainability (environmental, economic and social) that were presented at the global level, which aim to achieve balance with each other in order to achieve sustainability, and by applying to the housing system, sustainable housing refers to housing that meets the needs of its residents in The present time through the good utilization of the available resources to provide shelter that meets the material and aesthetic needs and which is consistent with the environment in which it is located and also ensures the sustainability of resources to meet the needs of future generations in obtaining environmentally, economically and socially appropriate housing(UN-Habitat, 2002).

Thus, the main principles of sustainable housing, whether at the planning or design level, are a set of environmental principles in the efficient utilization of natural resources in the spatial scale without causing a negative impact on the environment, and a set of economic principles that are mainly concerned with reducing implementation costs to minimum levels, especially when dealing with limited income groups that do not have sufficient purchasing power without reducing the quality and efficiency of housing, and social principles represented in caring for the destitute groups, achieving their basic needs and improving their standard of living(H., 1998).

The principles of sustainable housing can be achieved from the beginning of architectural design through:

- The importance of accommodating the largest possible number of (living) jobs in the dwelling

13

within the lowest possible void spaces, without compromising the level of well-being and considerations of decent living, and paying attention to determining the dimensions of the spaces in the residential unit (length, width and height) according to its functions, the way it is furnished, and highlighting the methods of architectural treatments In increasing the absorption of vacuum functionally and visually(UN-Habitat, 2012).

- Reducing unused spaces, as the current housing units suffer from the abundance of unused spaces and waste in the spaces, as the movement surfaces (corridors, entrances, and halls) constitute more than 30% of the total building area(rcrc, 2021).

- One of the most important entrances to sustainability is the saving in spaces, and the rationalization of spaces is a basic and direct factor in reducing the cost of housing, and rationalization does not mean austerity, but rather the search for the least appropriate dimensions of activities(Mahmoud, 2004).

- Simplicity is a basic methodology for achieving sustainability and it appears in the various design stages, starting from setting the spatial program, calculating functional needs, passing through initial and then final designs, and developing structural and mechanical plans, as it aims to reduce costs in establishment, maintenance, and operation, without prejudice to functional aspects , And the aesthetic of the housing(Mahmoud, 2004).

- The importance of the flexibility of the spatial design of the dwelling, the ease of its future growth, and its role in shaping the house according to the family's desire and capabilities, and among the means of achieving flexibility is the fragmentation of the implementation, such as dividing the stages of finishing and cladding, or the implementation of some roles, or the elements and delaying the rest of the elements until they are needed, and the availability of capacity To complete it(Diab, 2007).

- Adopting a method of value engineering at work, which is concerned with the balance between job performance, the total cost (visible and invisible) associated, and the level of quality achieved, with the aim of giving smart alternatives more efficient and effective and thus achieving the principles of sustainable housing(UN-Habitat, 2012).

3.3. The role of the state in solving the housing problem:

Any government and any country should be concerned primarily with the basic living issues of citizens and facilitate them for them, and the most important of those responsibilities is to provide housing for all citizens with affordable means and in accordance with a long-term strategy(Afifi, 2002).

The state's housing policy is linked to and affected by the state's economic system, as the housing market represents one of the important markets within the economic system. Housing as a commodity is affected by supply, demand and exports of construction materials, and the housing system is linked to the state's political system as the housing policy is part of the state's general policy(Mahmoud, 1992).

This effect may be direct, such as an increase in projects, constructions, and housing. The effect may be indirect, as is the case when the state's political crises occur with other countries and the occurrence of an economic blockade and the resulting reduction or prevention of imports of basic construction materials such as cement and iron if the local production is not sufficient to cover the required needs(Mahmoud, 1992).

Therefore, the role of the state is based on providing adequate housing for citizens with low and middle incomes, suitable for their limited financial capabilities, in a way that preserves their humanity and human dignity, and guarantees them a safe and decent life, where we find, for example, that the Egyptian government issued a law establishing the Social Housing Fund and supporting real estate financing in accordance with Republican Decree No. 93 of 2018, to be based on social housing affairs and real estate financing support, and is considered a public service body, with a legal personality, and this comes from the Egyptian state's sense of the importance of housing as one of the basic pillars of the security of any society as the Egyptian constitution states that "the state guarantees citizens the right to Regarding adequate, safe and healthy housing in a manner that preserves human dignity and achieves social justice, "noting that Egypt is a signatory to the Universal Declaration of Human Rights (the right to adequate housing).

The Ministry of Housing has prepared a draft housing strategy in Egypt, and it is currently being discussed in the community with all concerned parties and approved to be binding on all sectors. Among its most important principles is the consolidation of human rights principles in the right to housing, social justice, and support for those who are unable(cabinet, 2021).

In addition, "one of the principles of this strategy is the necessity for citizens to enjoy the right to obtain adequate housing, not to distinguish any sector from others in housing policies and programs, and not to marginalize any sector, to create housing

policies and programs for dynamic spaces that allow different segments of society to interact and develop over time, the commitment of the state." With the help of those who are unable to find housing, including the marginalized and disadvantaged groups due to market economy failures, support housing policies for sustainable national economic development plans, whether directly or indirectly, which means that housing contributes to achieving effective economic development for current and future generations, and housing policies and programs are based on correct information. On the sector, through adequate studies of housing markets and its dynamics"(UN-Habitat, 2012).

In the field of legislation, the Egyptian Building Law and its implementing regulations have been issued to ensure the establishment of a sound, safe and healthy building that meets the requirements of safety, safety, health rules, provisions for lighting, ventilation, and courtyards, as well as requirements for securing the building and its occupants against the dangers of fire, including (the Egyptian Code for Ethics and Conducts for Practicing the Engineering Profession and the Egyptian Code for Quality Indicators Life), and the Social Housing Law was issued as part of the state's efforts to enable every citizen to obtain his right to housing(Law, 2009).

The Ministry of Housing launched the largest project in the history of Egypt, which is (the Social Housing Project), with the aim of providing decent housing units for people with low incomes and plots of land prepared for construction with a maximum of 400 square meters throughout the republic within the governorates and within the scope of new cities, in addition to the fact that the Council of Ministers issued several decisions regarding This project includes expanding the scope of the target group of applicants for the project by raising the age up to 50 years instead of 45 years, allocating 5% of the project units for people with special needs according to the type and classification of disability so that the housing units are allocated on the ground floor for people with mobility disabilities. Allocating housing units to them without discrimination, taking into account the psychological and social dimension, integrating them into society naturally and eliminating any form of discrimination(cabinet, 2021).

Among these decisions, taking into account equality between men and women in the conditions that must be met by applicants for units in the project, determining percentages of housing units for cases of urgent accommodation and administrative evacuation, setting a list of priorities for those applying for units in the project (lower-income family, married and dependent, married,

Single, disabled persons, revolution victims, and martyrs' families)(UN-Habitat, 2012).

A widowed or divorced woman who supports her family (the breadwinner woman) was added to the list in the context of preserving women's rights in society(presidency, 2021).

Support was also provided from the Social Housing Fund and real estate financing support for every citizen to provide his housing unit according to his income, provided that the citizen pays a serious reservation and down payment amounting to 15% of the unit value, and the rest of the unit price is paid through a soft loan over a period of 20 years in a monthly instalment. He has a maximum interest of 7%, as a maximum, which is decreasing annually, and the state bears the value of the land and the provision of utilities (electricity - water - sewage) and the provision of required services (playgrounds, schools, hospitals, commercial services, etc.)(mhuc, 2021).

The cost of the development projects that have been and are being implemented has reached more than 20 billion pounds, namely (Dar Misr - Asmarat - Al Mahrousa 1 and 2 - Al Sayeda Kindergarten (formerly Tal Al Aqrab) - Zarzara - Mahfouz - Ezbet shantyh, Judaism and others) in order to provide a decent life worthy of the citizens(mhuc, 2021).

3.4. Mathematical modelling:

Mathematical modeling is defined for any economic, administrative, scientific or military problem. It is nothing but the simplified form of this problem, which most often takes the form of equations, inequalities, or dependencies that represent the relationship that can be quantified for the various factors that have a relationship to the problem(Zeina, 2017).

Linear programming greatly contributes to finding an optimal solution for problems in which certain data or information are available, as linear programming determines the maximum or minimum value of a specific function called the objective function, which depends on a final number of variables, and these variables may be independent of each other, Or they may be bound together by so-

$$min \quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$
$$s.t.$$
$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$$

$$x_i \geq 0 \quad \text{Where} , \quad i = 1, 2, 3, \ldots, n$$

called constraints, they may in all cases be expressed in the following standard form(Dantzig, 1963):

And we mean that the linear system above is a system of linear equations in the standard form, and the parameters are fixed numbers a_{ij}, b_j, c_i, while x_i they are the variables that must be assigned (Dantzig, 1963).

Note: If the objective function is (maximization), it can be returned to a (minimization) function by making $z' = -z$ (Dantzig, 1963).

3.5. Using mathematical modeling to reduce wasted design space:

As in this part of the research, mathematical modeling will be used with the aim of helping the architectural designer to reduce the architectural spaces and thus the optimal use of building area, to achieve sustainable housing while ensuring that all design standards are met in accordance with the design requirements of the Egyptian Building Law and the requirements of the users by building a mathematical model using linear programming to achieve the desired goal.

The goal of formulating the mathematical model is to analyze the division of rooms and dimensions in a particular housing model, to calculate and know the dimensions and areas of rooms that achieve the least total area of a residential unit.

4. THE COMPONENTS OF THE RESIDENTIAL MODEL TO BE STUDIED.
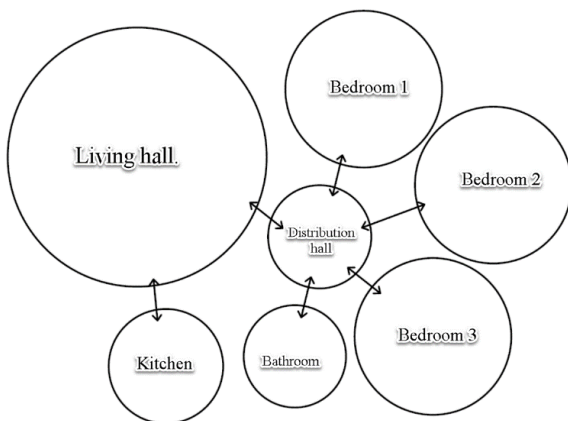
Figure 4 shows the spatial relationships for a residential unit model
Source: the researcher

One of the medium governmental housing models was chosen "Dar Misr" model B, where the housing model consists of a number of architectural spaces:
- Living hall.
- Kitchen.
- Bathroom.
- Distribution hall.
- Bedroom 1.
- Bedroom 2.
- Bedroom 3.

Figure No. 3 indicates the spatial relationships between the architectural spaces of an average housing apartment model. Figure No.4 shows the horizontal projection of the residential apartment model to be studied and analyzed.
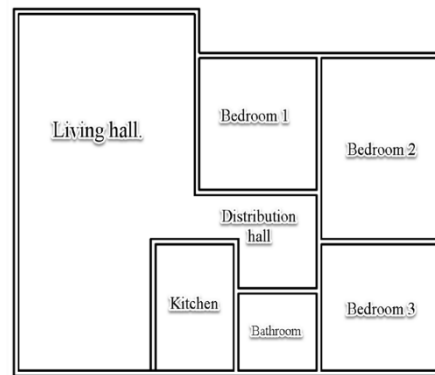

Figure No. 5 shows the Architectural plan of a residential unit model
Source: the researcher

The design problem of the residential unit model will be formulated as shown in Figure 5, where it was found that the decision in determining the dimensions and the areas of the rooms is variable and affects the ability to optimize the use of building area and the cost of construction.

4.1. Data collection plan:

The data required in the problem analysis to formulate the mathematical model was collected by linear programming as follows:
- As for the Architectural plan of the unit model, the decision variables will be imposed on all dimensions of the vertical and horizontal axes of the residential model.
- The design requirements were compiled according to the unified Egyptian building law in terms of dimensions and room areas for the residential model, as shown in Table 1.

4.2. Building the mathematical model:
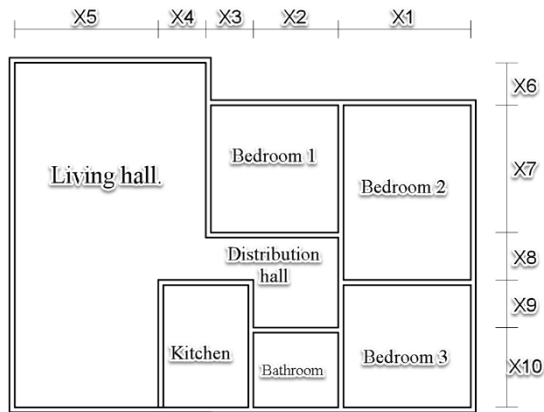
The mathematical model consists of three main parts:

16

Figure 6 shows the decision variables relative to the dimensions of the spaces

4.2.1. Determination of decision variables:

In this part, a set of variables (X1, X2, X3, ……… Xn) will be imposed that represent the dimensions of the architectural spaces with respect to the vertical and horizontal axes, as shown in Figure 6.

Variables	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Value	3	2	1	1	4	1	3	1.5	1.5	2
Total area of the unit	87 m2									

Table No. 3 shows the existing dimensions of spaces for the residential unit.　　Source: the

4.2.2. Defining the objective function:

The objective function, represented here in the aim of the research, will be determined, which is to achieve the least area of the housing unit while meeting the requirements of the users through the fulfillment of the design requirements of the unified Egyptian building law.

Objective Function (Minimum Residential Unit Area) = (Area of Living Hall) + (Area of Kitchen) + (Area of Bathroom) + (Area of Distribution Hall) + (Area of Bedroom Number 1) + (Area of Bedroom Number 2) + (Area of Bedroom Number 3)

MIN. Area = ((X5+X4) (X6+X7+X8)) +(X5(X9+X10)) +((X4+X3) (X9+X10)) +X2X10+(X3X8+X2(X8+X9)) +(X3+X2) X7+X1(X7+X8) +X1(X9+X10).

4.2.3. Determination of limitations (design requirements):

Restrictions have been put in place to achieve the design requirements of the Unified Egyptian Building Law for each architectural vacuum as follows:

A. Living Hall.

The restrictions have been set for the living hall to meet the design requirements so that minimum dimension is less than 3 m, and the minimum of area is not less than 10 m².

$X_5+X_4 \geq 3$

$X_6+X_7+X_8+X_9+X_{10} \geq 3$

$(X_5+X_4) *(X_6+X_7) + (X_5+X_4) *X_8 + X_5*(X_9+X_{10}) \geq 10$

B. Kitchen.

As for the kitchen, restrictions have been set so that the requirements are met that the minimum dimension should not be less than 1.5 m, and the minimum of area is not less than 3 m².

$X4+X3 \geq 1.5$

$X9+X10 \geq 1.5$

$(X4+X3) *(X9+X10) \geq 3$

C. Bathroom.

And for the bathroom, the curbs were placed so that minimum dimension was less than 1.2 m, and the minimum of area was not less than 1.5 m².

$X2 \geq 1.2$

$X10 \geq 1.2$

$X2*X10 \geq 1.5$

D. Distribution Hall.

As for the distribution hall, its dimensions were calculated to allow the bedrooms and the bathroom to open, considering that the bedroom opening = 0.90 m and the bathroom door opening = 0.80 m. For X2, a restriction has been set for it that it is not less than 1.2 m, as for the other side (X8 + X9) Which serves the bedrooms, numbers 2, 3, were taken into account the door openings and after spoken from each side of the door opening and the thickness of the wall separating the two rooms with the clam layer of the two sides,

(X8 + X9) = 0.1 + 0.9 + 0.1 + 0.15 + 0.1 + 0.9 + 0.1 = 2.35 m.

$X8+X9 \geq 2.35$

E. Bedrooms.

As for bedrooms, numbers 1, 2, and 3, restrictions have been put in place that meet the required requirements, so that minimum dimension is less than 2.5 m, and the minimum of area of any room is not less than 7.5 m 2.

Bedroom 1:

$X3+X2 \geq 2.5$

$X7 \geq 2.5$

$(X3+X2) *X7 \geq 7.5$

Bedroom 2:

$X1 \geq 2.5$

$X7+X8 \geq 2.5$

$X1*(X7+X8) \geq 7.5$

Bedroom 3:

$X1 \geq 2.5$

$X9+X10 \geq 2.5$

$X1*(X9+X10) \geq 7.5$

4.2.4. Finding the best solution:

The mathematical model was built by means of linear programming to create the least space for the residential unit, according the respect of the design requirements of the unified Egyptian building law, which ensure that the design meets the requirements of the users, and the ideal values for the dimensions of the architectural spaces will be found by using the computer program LPSolve IDE- 5.5.2.11 to process The data and finding the best solution for the mathematical model for the residential unit, as shown in Figure 6.

Figure 7: The program used to process the data
Source: the researcher

Figure 8 shows the values of the variables after processing them
Source: the researcher

5.RESULTS AND DISCUSSION:

- The research concludes that the mathematical model is changing the values of the variables, which represent the dimensions of the residential unit, and thus a modification in the architectural design.
- The modified Architectural plan of the residential unit achieved the least total area, as

the previous total area was 87 m2, and after using the mathematical model and treating it, the total area became 62.31 m2 by 28 % reduction in area, as shown in Table no. 5.
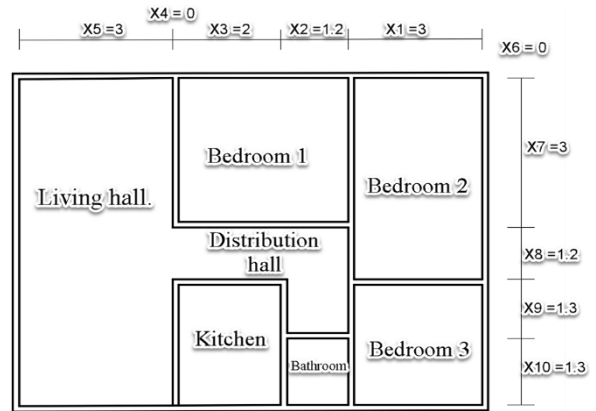

Figure No. 9 shows the dimensions of Architectural spaces after adjusting it according to the new values of the variables -
Source: the researcher

Architectural Spaces	Area before use Mathematical Modeling	Area after use Mathematical Modeling
Living Hall	37	20.1
Kitchen	6	5
Bathroom	4	1.56
Distribution Hall	7	5.65
Bedroom 1	9	9.9
Bedroom 2	13.5	12.6
Bedroom 3	10.5	7.5
Total area of housing unit	87	62.31

Table No. 5 shows the existing and new area of Architectural spaces.
Source: the researcher

- The research concludes mathematical modelling help to reduce the area of Architectural spaces of the residential unit according the design
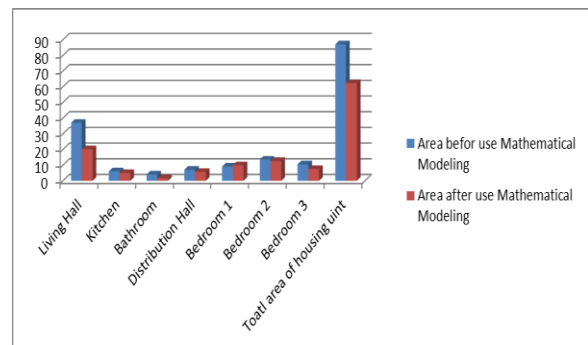

Figure No. 10 shows the existing and new area of Architectural spaces before and after using mathematical modelling   Source: the researcher

requirements of the Egyptian Building Law and requirements of the users as shown in Figure 9.

- The research concludes that the use of the mathematical model will to help architects in Reducing the dimensions and areas of Architectural spaces to the required extent.
- Reducing wasted areas in the residential unit will reduce construction costs and consumption of building materials, thus reducing the rate of environmental pollution and achieving the concept of sustainability.
- The use of mathematical modelling in the design will help the state to establish the largest number of housing units to meet the needs of citizens and thus will help solve the housing problem.

6.CONCLUSION:

- Through the research study, the research reached a method that helps architects to design sustainable housing by using mathematical modelling to build a mathematical model with linear programming in order to reduce wasted areas when designing residential units, and thus reduce the total area of residential units, which results in the optimal use of building area to create a larger number of housing units to accommodate the growing demand from citizens.
- The application of the concept and principles of sustainability in all fields of life helps to preserve the environment, optimize the use of resources, and respect the needs of users and their various activities.
- Solving the housing problem and achieving sustainability, in which all segments of society must join hands with the state, especially architects and urbanites, by creating mechanisms that help decision-makers to create sustainable urban environments that meet all users' requirements and aspirations.

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