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Analytical and Finite Element Stress Analysis of Pressure Vessels under Constant and Cyclic Loading

Anwar Kandil[1], Ali El-Baghdady[2] and Mahmoud El-shenawy[3]

ABSTRACT

Pressure vessels are wildly used in many fields, such as chemical, petroleum, military industries as well as in nuclear power plants. Pressure vessels should be designed with great care because rupture of pressure vessel may cause catastrophic accident. The common problem to the pressure vessel's designer is the accurate evaluation of stresses due to the applied mechanical and /or thermal loads. The finite element method (FEM) is one of the numerical stress analysis for many subjects. In this paper a complete stress analysis through the wall of pressure vessel under the effect of constant and cyclic loading is presented. Hoop, radial, axial and effective stresses for cylindrical pressure vessels have been evaluated analytically as well as using the program package ANSYS as a numerical finite element method. A comparison between the analytical and numerical solution is presented, and it is found a good agreement between them.

KEYWORDS: Pressure vessels, Stress analysis, Finite element method, Cyclic loading, Thermal stresses.

NOTATION

$E \ldots\ldots$ Young's modulus of material (Mpa)

$f \ldots.$ Frequency of pressure oscillation (s^{-1})

$H = \dfrac{E\,\alpha\,\Delta T}{1-v} \ldots.$ Thermal load (Mpa)

$K = {}^{r_o}/_{r_i} \ldots\ldots$ Diamter ratio

$K_i = {}^{r_c}/_{r_i} \ldots$ Diameter ratio for inner cylinder

$K_o = {}^{r_o}/_{r_c} \ldots$ Diameter ratio for outer cylinder

$P_a = \dfrac{P_{i_{max}} - P_{i_{min}}}{2} \ldots$ Pressure amplitude (Mpa)

$P_c \ldots\ldots$ Contact pressure (Mpa)

$P_i \ldots\ldots$ Applied internal pressure (Mpa)

$P_m = \dfrac{P_{i_{max}} + P_{i_{min}}}{2} \ldots$ Mean pressure (Mpa)

$P_o \ldots$ External pressure (Mpa)

$P_{(r_i,t)}$. Pressure at inner surface r_i and after time t (Mpa)

$r \ldots\ldots$ Desired radius (mm)

$r_c \ldots.$ Contact radius of the cylinder (mm)

$r_i \ldots\ldots$ Inner radius of the cylinder (mm)

$r_o \ldots.$ Outer radius of the cylinder (mm)

$R = {}^{r}/_{r_i} \ldots$ Normalized radius

$t \ldots\ldots$ Time (s)

$T \ldots.$ Temperature at any radius of the cylinder (°C)

$T_i \ldots.$ Temperature at inner surface of the cylinder (°C)

$T_o \ldots$ Temperature at outer surface of the cylinder (°C)

$\Delta T = T_i - T_o \ldots\ldots$ Temperature difference (°C)

$\alpha \ldots\ldots$ Coefficient of linear thermal expansion $(mm^{-1}\,°C^{-1})$

$\sigma \ldots.$ Stress (Mpa)

$\sigma_a \ldots.$ Stress amplitude (Mpa)

$\sigma_h \ldots.$ Hoop stress (Mpa)

$\sigma_m \ldots.$ Mean stress (Mpa)

$\sigma_r \ldots.$ Radial stress (Mpa)

$\sigma_v \ldots.$ Equivalent stress (Mpa)

$\sigma_z \ldots.$ Axial stress (Mpa)

$v \ldots\ldots$ Poisson's ratio

$\bar{t} = ft \ldots\ldots\ldots$ Normalized time

$\bar{\sigma} = {}^{\sigma}/_{P} \ldots\ldots$ Normalized stress

$\bar{\sigma}_h = {}^{\sigma_h}/_{P} \ldots\ldots$ Normalized hoop stress

$\bar{\sigma}_r = {}^{\sigma_r}/_{P} \ldots\ldots$ Normalized radial stress

$\bar{\sigma}_v = {}^{\sigma_v}/_{P} \ldots\ldots$ Normalized equivalent stress

$\bar{\sigma}_z = {}^{\sigma_z}/_{P} \ldots\ldots$ Normalized axial stress

1. INTRODUCTION

Pressure vessels are widely used in a wide range of technological applications. Examples of great interest are found in nuclear engineering military industries and the field of metal forming. They are usually subjected to high pressure which may be constant or cyclic. In practice many pressure vessels are subjected also to high internal temperature.

Evaluation of stress analysis in cylindrical pressure vessel is important for principal design consideration. Therefore many investigators have direct their effort to study the pressure vessel in order to predict the required

[1]*Department of Production Engineering and Mechanical Design, Faculty of Engineering, Port Said University, Egypt. Email:Anwarkandil@hotmail.com*

[2] *Department of Production Engineering and Mechanical Design, Faculty of Engineering, Port Said University, Egypt.*

Mechanical Engineer

Email:Elshenawy47518@yahoo.com

working capacity. In order to achieve better use of material or to allow higher working capacity, many attempts were made using different types of analysis.

Alzaharnah et al. [3] have determined the thermal stresses due to temperature gradient developed in the pipe wall. You et al. [4] presented an accurate method to carry out elastic analysis of thick-walled spherical pressure vessels subjected to internal pressure. Kandil [6] presented a complete analysis of stresses within the wall of a cylindrical pressure vessel subjected to cyclic internal pressure and temperature. Gupta et al. [7] studied the effect of non-homogeneity on elastic-plastic transition in a thick walled circular cylinder under internal pressure and temperature using Seth's transition theory. Lee [8] presented the use cylindrical pressure vessel subjected to internal high pressure in manufacture. Shao [9] used a multi-layered approach based on the theory of laminated composites to evaluate the temperature, displacements and thermal / mechanical stresses in a functionally graded circular hollow cylinder. Duncan et al. [10] recently determined the effect of cross hole on the elastic response by considering the shakedown and ratcheting behavior of a plain thin & thick walled cylinder with radial cross hole subjected to constant internal pressure and cyclic thermal loading.

Kandil [12] presented a stress analysis of shrink-fitted compound cylinders subjected to cyclic internal pressure and temperature and carried out a complete evaluation of the time-dependent mechanical and thermal stresses distributions across the wall thickness of duplex cylinder using a numerical model on the basis of the forward finite difference technique. Heckman [13] sat out to explore applicable methods using finite element analysis in pressure vessel analysis. He studied pressure vessels, especially in determining stresses in local areas such as penetrations, O-ring grooves and other areas difficult to analyze by hand. Yang et al. [14] carried down shakedown the residual stress analysis of an autofrettaged compound cylinder is designed using a Mat lab graphical user interface (GUI) and program design technique. Ayob et al. [15] calculated the effect of autofrettage on thick-walled cylinders under high internal pressure. Verijenko et al. [16] determined stress analysis of multilayered pressure vessels under internal, external and inter laminar pressures.

The main aim of this paper is to investigate and determine the stress analysis through the wall of cylindrical pressure vessels subjected to different loads. These loads are formulated as follow:

- Constant pressure (internal and /or external pressure).
- Internal temperature.
- Internal pressure and temperature.
- Variable pressure.
- Variable pressure and constant temperature.

Hoop, radial, axial and equivalent stress distribution for monoblock and compound cylindrical pressure vessels have been evaluated by theoretical analysis and finite element analysis. A comparison between the analytical and numerical solutions is carried out.

2. THEORITICAL ANALYSIS

2.1 Thick Walled Cylinder Under Pressure

Figure 2.1 shows a cross section of a thick walled cylinder subjected to internal and external pressure (P_i, P_o). The hoop, radial and longitudinal stresses across the cylinder wall can be determined by the following equations [1, 2]:

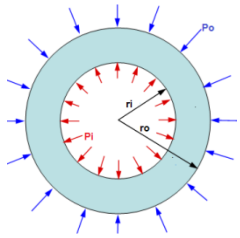


Fig. 2.1: Thick walled cylinder under

$$\sigma_h = \frac{1}{r_o^2 - r_i^2}\left[r_i^2 P_i - r_o^2 P_o + \left(\frac{r_i r_o}{r}\right)^2 (P_i - P_o)\right] \quad 2.1$$

$$\sigma_r = \frac{1}{r_o^2 - r_i^2}\left[r_i^2 P_i - r_o^2 P_o - \left(\frac{r_i r_o}{r}\right)^2 (P_i - P_o)\right] \quad 2.2$$

A longitudinal stress for closed ended cylinder,

$$\sigma_z = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} \quad 2.3$$

A longitudinal stress for open ended cylinder

$$\sigma_z = 0 \quad 2.4$$

The equivalent stresses (σ_v) are obtained by Mieses theory,

$$\sigma_V = [\sigma_h^2 + \sigma_r^2 + \sigma_z^2 - \sigma_h \sigma_r - \sigma_h \sigma_z - \sigma_r \sigma_z]^{0.5} \quad 2.5$$

2.2 Thick Walled Cylinder under Temperature

In the present analysis the temperature is considered to be symmetrical about the axis of the cylinder and independent on the axial coordinate. The cylinder is assumed with free ends such that the axial displacement is zero. The linear and logarithmic temperature distributions are usually calculated by [3, 4]:

$$T = T_i \left(\frac{r_o - r}{r_0 - r_i}\right) \quad 2.6$$

$$T = T_i \frac{\mathrm{Ln}\left({}^{r_o}/_{r}\right)}{\mathrm{Ln}\left({}^{r_o}/_{r_i}\right)} \quad 2.7$$

The stress components due to temperature difference between inner and outer surface of thick cylinder are given by [1]:

$$\sigma_h = \frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 + \frac{r_o^2}{r^2}\right)\ln K\right) \quad 2.8$$

$$\sigma_r = \frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1-\frac{r_o^2}{r^2}\right)\ln K\right) \qquad 2.9$$

$$\sigma_z = \frac{H}{2\ln K}\left(1 - 2\ln\frac{r_o}{r} - \frac{2}{K^2-1}\ln K\right) \qquad 2.10$$

2.3 Thick Walled Cylinder under Internal Pressure And Temperature Difference

The stress distributions for closed ended cylinder subjected to internal pressure (P_i) and temperature deference (ΔT) can be obtained by superposition the stress components in Eqs. 2.1 to 2.3 with the corresponding Eqs. 2.8 to 2.10 [5: 7]:

$$\sigma_h = \left[\frac{P_i}{(K^2-1)}\left(1+\frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1+\frac{r_o^2}{r^2}\right)\ln K\right)\right] \qquad 2.11$$

$$\sigma_r = \left[\frac{P_i}{(K^2-1)}\left(1-\frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1-\frac{r_o^2}{r^2}\right)\ln K\right)\right] \qquad 2.12$$

$$\sigma_z = \left[\frac{P_i}{K^2-1}\right] + \left[\frac{H}{2\ln K}\left(1 - 2\ln\frac{r_o}{r} - \frac{2}{K^2-1}\ln K\right)\right] \qquad 2.13$$

2.4 Compound Cylinder under Internal Pressure

The stress analysis of the multilayer walled cylinder shown in Fig. 2.2 is somewhat different from the thick walled cylinder. This may be attributed to the effect of the shrink-fit pressure. Thus the resulting stresses in overlapping stress due to internal and contact pressure are given by [8]:

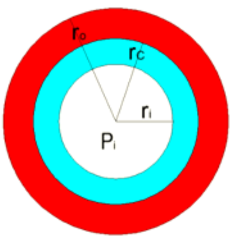


Fig. 2.2: Compound cylinder under

For inner cylinder

$$\sigma_h = \left[\frac{-P_C K_i^2}{K_i^2-1}\left(1+\frac{r_i^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2-1)}\left(1+\frac{r_o^2}{r^2}\right)\right] \qquad 2.14$$

$$\sigma_r = \left[\frac{-P_C K_i^2}{K_i^2-1}\left(1-\frac{r_i^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2-1)}\left(1-\frac{r_o^2}{r^2}\right)\right] \qquad 2.15$$

The longitudinal stress for closed ended cylinder

$$\sigma_z = \frac{P_i}{K^2-1} \qquad 2.16$$

For outer cylinder

$$\sigma_h = \left[\frac{P_C}{K_o^2-1}\left(1+\frac{r_o^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2-1)}\left(1+\frac{r_o^2}{r^2}\right)\right] \qquad 2.17$$

$$\sigma_r = \left[\frac{P_C}{K_o^2-1}\left(1-\frac{r_o^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2-1)}\left(1-\frac{r_o^2}{r^2}\right)\right] \qquad 2.18$$

the longitudinal stress for closed ended cylinder

$$\sigma_z = \frac{P_i}{K^2-1} \qquad 2.19$$

Longitudinal stress for free ended compound cylinder is equal to zero.

2.5 Compound Cylinder under Temperature Difference

For compound cylinder with free ends and subjected to internal temperature the resulting stress due to contact pressure (P_c) and temperature deference (ΔT) is described by [8, 9]:

for inner cylinder

$$\sigma_h = \left[\frac{-P_C K_i^2}{K_i^2-1}\left(1+\frac{r_i^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1+\frac{r_o^2}{r^2}\right)\ln K\right)\right] \qquad 2.20$$

$$\sigma_r = \left[\frac{-P_C K_i^2}{K_i^2-1}\left(1-\frac{r_i^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1-\frac{r_o^2}{r^2}\right)\ln K\right)\right] \qquad 2.21$$

For outer cylinder

$$\sigma_h = \left[\frac{P_C}{K_o^2-1}\left(1+\frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2-1}\left(1+\frac{r_o^2}{r^2}\right)\ln K\right)\right] \qquad 2.22$$

$$\sigma_r = \left[\frac{P_C}{K_o^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right)\ln K\right)\right] \quad 2.23$$

A longitudinal stress for closed ended compound cylinder subjected to internal temperature can be calculated by:

$$\sigma_z = \frac{H}{2\ln K}\left(1 - 2\ln\frac{r_o}{r} - \frac{2}{K^2 - 1}\ln K\right) \quad 2.24$$

2.6 Compound Cylinder under Pressure and Temperature Difference

For compound cylinder subjected to internal temperature and internal pressure the stress components are expressed as follows [10, 11]:

For inner cylinder

$$\sigma_h = \left[\frac{-P_C K_i^2}{K_i^2 - 1}\left(1 + \frac{r_i^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2 - 1)}\left(1 + \frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 + \frac{r_o^2}{r^2}\right)\ln K\right)\right] \quad 2.25$$

$$\sigma_r = \left[\frac{-P_C K_i^2}{K_i^2 - 1}\left(1 - \frac{r_i^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2 - 1)}\left(1 - \frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right)\ln K\right)\right] \quad 2.26$$

For outer cylinder

$$\sigma_h = \left[\frac{P_C}{K_o^2 - 1}\left(1 + \frac{r_o^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2 - 1)}\left(1 + \frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(1 - \ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 + \frac{r_o^2}{r^2}\right)\ln K\right)\right] \quad 2.27$$

$$\sigma_r = \left[\frac{P_C}{K_o^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right)\right] + \left[\frac{P_i}{(K^2 - 1)}\left(1 - \frac{r_o^2}{r^2}\right)\right] + \left[\frac{H}{2\ln K}\left(-\ln\frac{r_o}{r} - \frac{1}{K^2 - 1}\left(1 - \frac{r_o^2}{r^2}\right)\ln K\right)\right] \quad 2.28$$

A longitudinal stress for compound cylinder with free ends and subjected to internal pressure and temperature can be calculated according to Eq. 2.24.

2.7 Thick Walled Cylinder under Variable Pressure

Internal variable pressure, which affects on the inner surface of thick wall cylinder, is fluctuated between two values P_{max} and P_{min}. The resulting of hoop, radial, axial and equivalent stresses can be determined similarly as calculated in thick walled cylinder under constant internal pressure by replacement P_i in Eqs. 2.1: 2.3 with $P_{(r_i,t)}$ which determined as follow [6, 10]:

$$P_{(r_i,t)} = P_m + P_a \sin(2\pi f t) \quad 2.29$$

The mean and amplitude of hoop, radial and axial stress components are calculated by:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad 2.30-a$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad 2.30-b$$

2.8 Thick Walled Cylinder under Variable Pressure and Constant Temperature Difference

In this case the tangential, radial and axial stress for thick walled cylinders with closed ended can be determined as Eqs. 2.11 to 2.13. The resulting stresses are calculated by superposition of stress due to internal variable pressure $(P_{(r_i,t)})$ and the stress due to the applied temperature difference ΔT [4: 6].

2.9 Compound Cylinders under Variable Pressure

For compound cylinders subjected to internal variable pressure the resulting stresses can be determine by superposition of stresses due to shrinkage fit (P_c) and the stresses due to the internal variable pressure $(P_{(r_i,t)})$ as calculated in Eqns. 2.14 : 2.19 by replacement the constant pressure (P_i) with variable pressure $(P_{(r_i,t)})$ [8, 12].

2.10 Compound Cylinder under Internal Variable Pressure and Constant Temperature

For free ended compound cylinders subjected to internal variable pressure and constant temperature the stress components for inner and outer cylinders can be determined similarly as calculated for compound cylinder under constant internal pressure and temperature by replacement constant pressure P_i with internal variable pressure $(P_{(r_i,t)})$. The resulting stresses are calculated by superposition of stresses distribution due to P_c, $P_{(r_i,t)}$ and ΔT [12].

The longitudinal stress σ_z for closed ended compound cylinder is obtained by Eq. 2.24.

3. FINITE ELEMENT ANALYSIS

Finite element analysis (FEA) is a numerical method of constructing a complex system into very small pieces called elements. The software implements equations that govern the behavior of these elements and solves them all creating a comprehensive explanation of how the system acts as a whole. These results can be presented in tabulated or graphical forms. This type of analysis is typically used for the design and optimization of systems which complex to analyze by hand [13, 14].

Basic steps in finite element method:

Like solving any problem analytically, we need to define (1) the solution domain, (2) the physical model, (3) boundary conditions and (4) the physical properties. Then we solve the problem and present the results. In numerical methods the main difference is an extra step called mesh generation. This step divides the complex model into small elements that become solvable in an otherwise too complex situation. The procedures that have to be carried out are:

Building Geometry: Constructing of three-dimensional representation of the object to be modeled and tested using the work plane coordinates system within ANSYS.

Defining of Material Properties: Now that the part exists, define a library of the necessary materials that compose the object (or project) being modeled. This includes thermal and mechanical properties of listed materials.

Applying Loads: Once the system is fully designed the last task is to burden the system with constraints, such as physical loadings or boundary conditions.
Generating Mesh: At this point ANSYS understands the makeup of the parts which defines how the modeled system should be broken down into finite pieces.

Reading the Solution: This is the difficult step, because ANSYS needs understanding within, what state (steady state, transient), the problem must be solved.

Presentation of the Results: After the solution has been obtained, there are many ways to present ANSYS' results such as tables, graphs, or contour plots.

3.1 Finite Element Individualize of a Model

In the finite element method, the structure of interest is sub-divided into discrete shapes called elements as shown in Fig. 3.1. The most common element types include one dimensional beam, two dimensional elements, or three dimensional bricks or tetrahedrons.

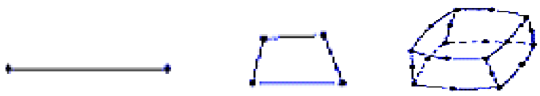


Fig. 3.1: Common element types used in ANSYS simulation.

Types of Elements:

Shell Element:

Structures composed of thin walls can be modeled using shell elements. Shell elements are treated as two dimensional, with a thickness for the element entered, but not shown on the model. Testing included linear elements (SHELL63) and quadratic elements (SHELL91). The linear elements have four nodes, one node at each corner of the element (Fig.3.2-a). The quadratic shell has a node midway between corner nodes adding up to eight nodes per element (Fig.3.2-b).

Solid Elements

The eight node brick (SOLID45) element was used for comparison with other elements. In the eight node brick there is a node at each corner of the brick (Fig.3.2-c).

Contact Elements

A point to point (CONTAC52) element joins two nodes. If contact between two nodes is expected then a contact element is created between the nodes. This element, when set properly, will enable the nodes to touch but prevent them from passing each other.

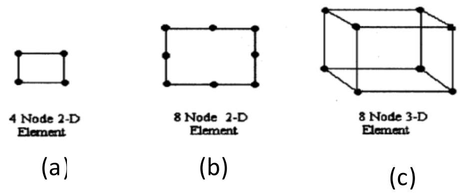


Fig. 3.2: Types of elements.

3.2 Types of Analyses

Structural analysis will be either linear or non-linear analysis.

- Linear analysis assumed that the cylindrical pressure vessels [Monoblock and compound] are subjected to constant internal pressure and /or temperature.
- Non-linear analysis depend on internal pressure variable between two values P_{max} and P_{min}.

Heat Transfer analysis shows the conductivity or thermal properties of the model. This may be either steady state or transient transfer

- Steady state transfer refers to constant temperature affect on the inner and the outer surface of the cylinder.

- Non-linear will usually involve time, radiation, and/or changing thermal properties in the material or through convection.

3.3 Geometrical Modeling

For the pressure vessels generally there are three ways to run the model: three dimensional, symmetric and ax-symmetric as shown in Fig. 3.3.

Three Dimensional Models

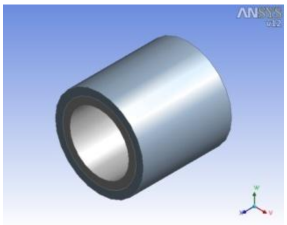
This model is the complete pressure vessel model. One end is pinned along the edge in the Z direction. One node is also pinned in the X and another in the Y direction.

Symmetric Model

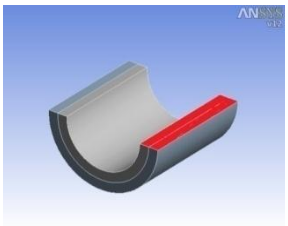
The symmetric model only models half of the pressure vessel. Symmetric boundary conditions are applied along the edges. One end is pinned along the symmetry edge. One node is pinned in the X direction. The model is pinned in the Y direction due to the symmetry of the model. Symmetric models offered an impressive improvement in run time.

Axi-symmetric Model

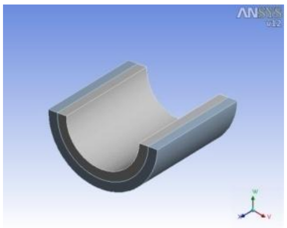
The Axi-symmetric model takes a two-dimensional cut to model the pressure vessel since it is symmetrical about the Z-axis. Symmetry boundary conditions are applied along the end caps. One node is pinned in the Y direction to prevent the model from being under constrained.



(a) Three Dimensional



(b) Symmetric



(c) Axi-symmetric

Fig. 3.3: Ways to run the model.

3.4 Finite element analysis of pressure vessel

Thick walled cylindrical pressure vessel under steady load

The finite element analysis for thick walled cylindrical pressure vessel is developed in plan-strain condition by taking advantage of three dimensions. The geometry and finite element model of the cylinder with free ends is shown in Fig. 3.4. The model is subjected to steady load. This load is usually high pressure and / or internal temperature. The pressure may be internally (P_i) or externally (P_o) [13].

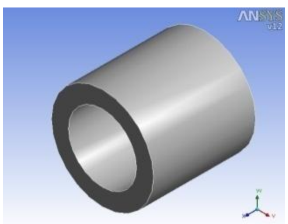


Fig.3.4: Model of thick walled cylinder.

Thick walled cylinder under pressure

Figure 3.5 illustrates the finite element analysis using ANSYS software to study the effect of internal constant pressure for thick walled cylinder, Figure 3.5-a defines the type of material, the type and place of loading. In this case type of loading is internal pressure (P_i) affects on inner surface of cylinder. Figure 3.5-b presents the meshing process for the model. To generate this process it must be define the type of meshing and the size of element, In Fig.3.5-c illustrates the division of the cylinder into small parts to study the effect of internal pressure. Figure 3.5-d solves the models by using equivalent von Mises to clarify the effect of internal pressure.

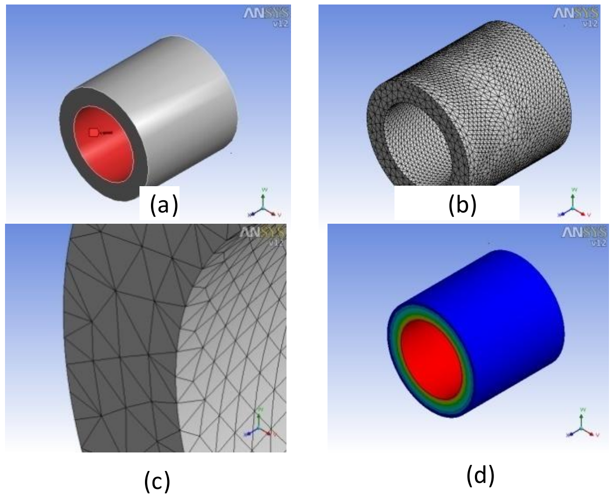


Fig. 3.5: Thick walled cylinder under internal pressu

In the previous passage we explained the use of FEA for the pressure vessel under the effect of constant pressure. Similarly ANSYS software is used to know FEA the resulting stress analysis for monoblock and compound cylindrical pressure vessel subjected to different conditions of constant and /or cycling pressure and constant temperature.

4. RESULTS AND COMPARISON

A complete evaluation of stress analysis for cylindrical pressure vessels are obtained by solving the proposed mathematical model analytically as well as numerically employing the forward finite element technique as given the previous sections. The mathematical model contains several working parameters. They are constant and cyclic pressure, as well as temperature. In this paper the previous parameters on thick walled and compound cylindrical pressure vessels are studied. The effect of shrink-fit pressure is introduced. Also the optimum location of the interference surface is obtained according the working conditions.

4.1 Steady Loads

Figure 4.1 shows the stress distribution of normalized hoop, radial, axial and equivalent stresses for thick walled open ended cylinder with diameter ratio K = 1.5 and subjected only to internal pressure.

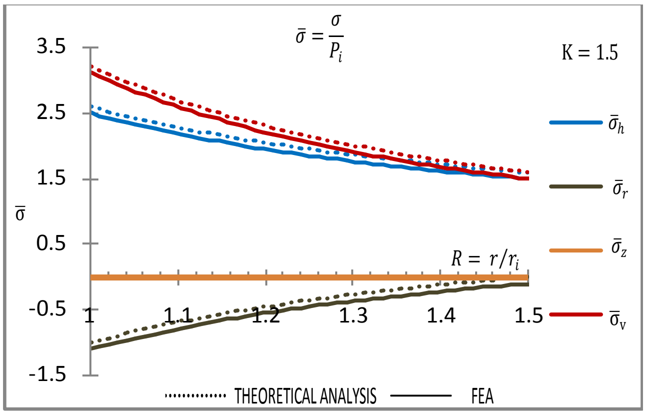


Fig. 4.1: Stresses distribution on monoblock cylinder due to constant internal pressure.

Figure 4.2 illustrates the thermal stresses distribution through the wall thickness of monoblock cylinder with diameter ratio K = 1.5 under the effect of internal temperature.

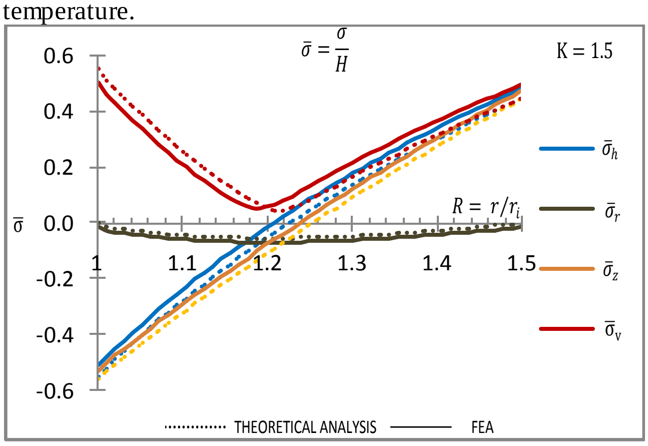


Fig. 4.2: Stresses distribution on monoblock cylinder due to constant internal temperature.

The normalized hoop, radial, axial and equivalent stresses distribution due to both constant internal pressure and temperature for a diameter ratio K = 1.5 and $\left({}^{H}/_{P_i} = 5\right)$ are illustrated in Fig. 4.3.

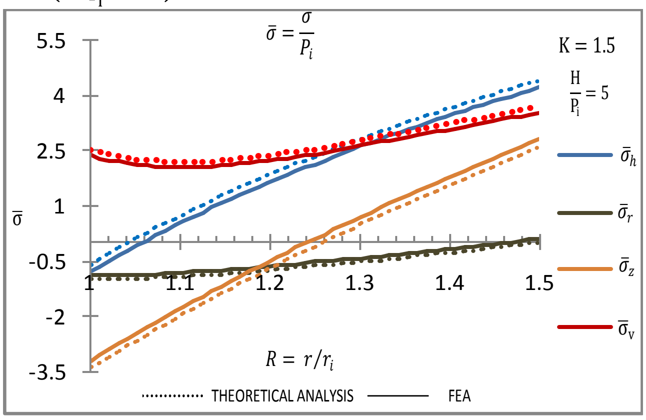


Fig. 4.3: Stresses distribution on monoblock due to constant internal pressure and temperature.

The compound cylindrical pressure vessels are used to obtain more uniform tangential stress distribution over the wall thickness. Figure 4.4 presents stresses distribution for compound cylindrical pressure vessel under the effect of internal constant pressure with $\left(K = 1.5\,,\ K_i = 1.25 \text{ and } \frac{P_c}{P_i} = 0.1\right)$.

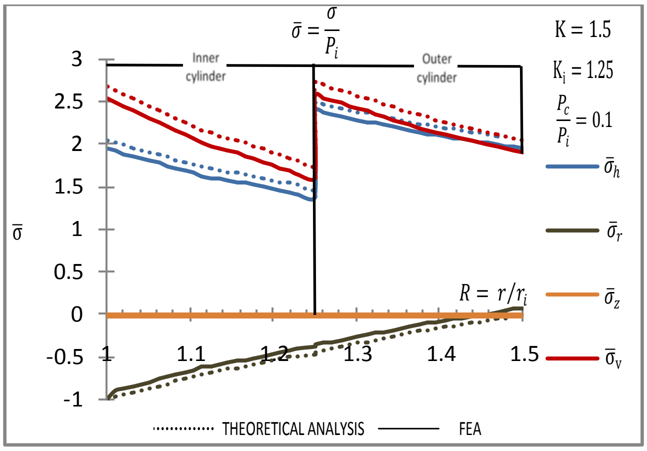


Fig. 4.4: Stresses distribution for compound cylinder due to constant internal pressure.

Figure 4.5 illustrates the resulting stresses for the same compound cylindrical pressure vessel subjected only to internal temperature with $\left(\frac{P_c}{H} = 0.1\right)$.

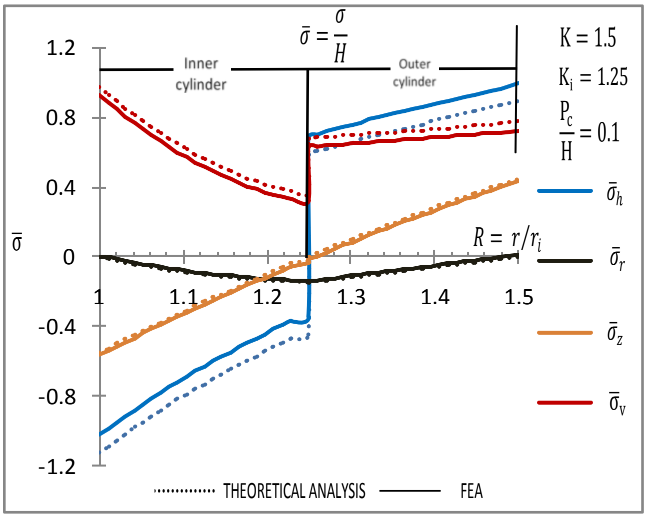


Fig. 4.5: Stresses distribution for compound cylinder due to constant internal temperature.

The stresses distribution on open ended compound cylinder with $(K = 1.5,\ K_i = 1.25)$ under the effect of steady loads of pressure and temperature are plotted in Fig. 4.6.

4.2 Cyclic Loading

Hoop, radial, and equivalent stresses for open ended monoblock cylinder $(K = 1.5)$ subjected to cyclic pressure $(P_a/P_m = 0.5)$ are plotted in Fig. 4.7 against the time $(\bar{t} = ft)$ for different normalized radius (R).

Figure 4.8 presents the tangential, radial, axial and equivalent stresses for thick walled cylinder with free ends and diameter ratio $(K = 1.5)$. The cylinder is subjected to cyclic pressure with $(P_a/P_m = 0.5)$ and constant temperature $H/P_m = 5$.

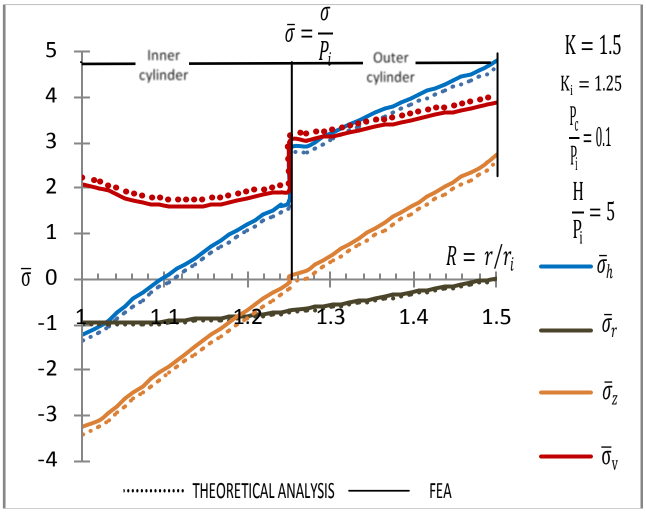
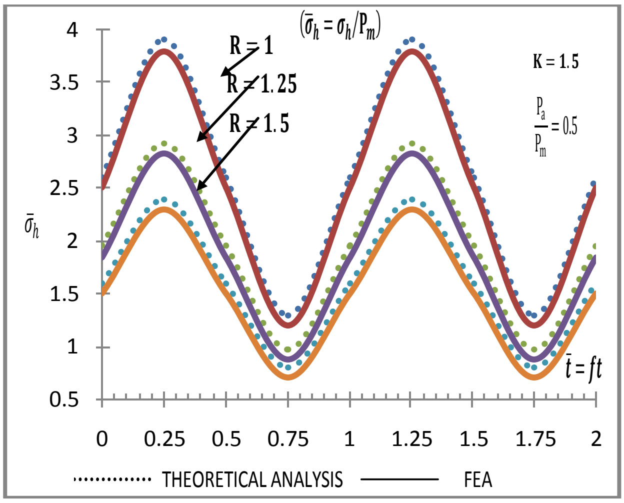
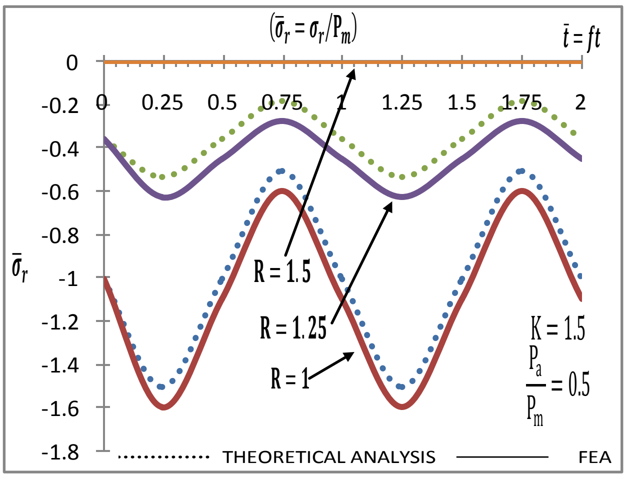


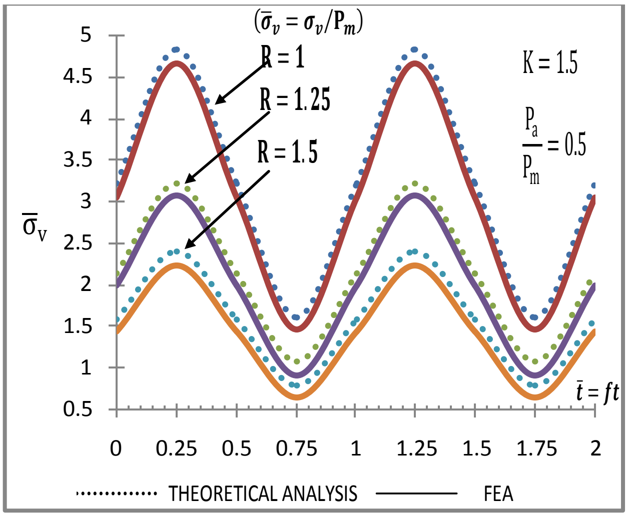
Fig. 4.6: Stresses distribution for compound cylinder due to constant internal pressure and temperature



(a) Normalized hoop stress distribution.

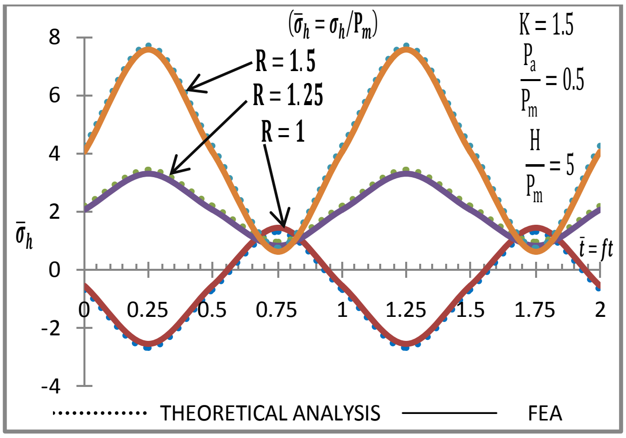


(**b**) Normalized radial stress distribution

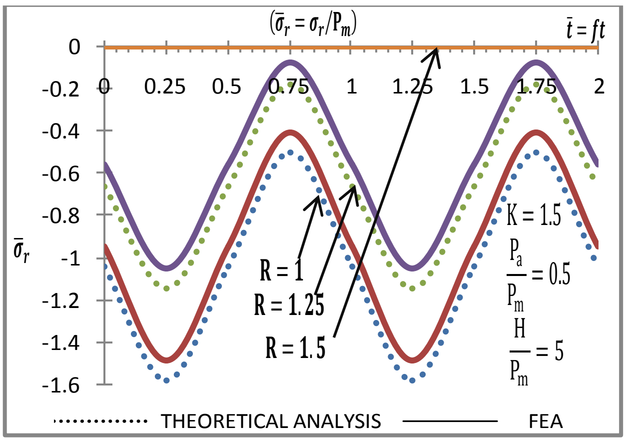


(**c**) Normalized equivalent stress distribution

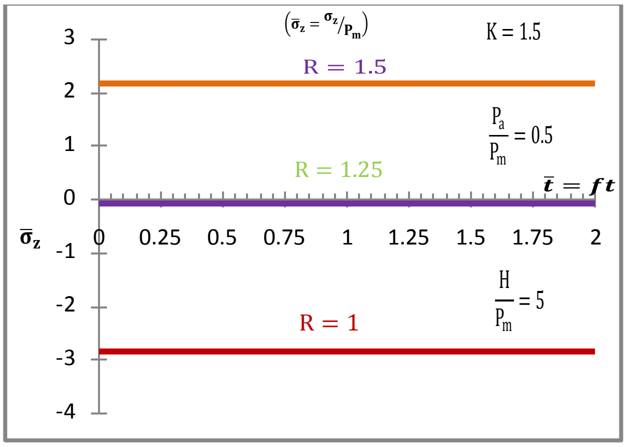
Fig. 4.7: Stresses distribution on monoblock cylinder due to internal cyclic pressure.



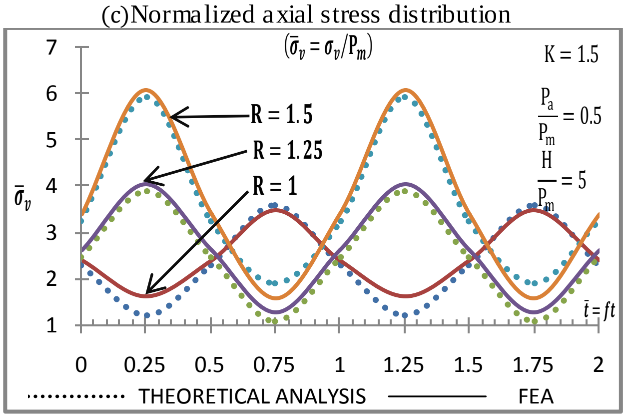
(**a**) Normalized hoop stress distribution



(**b**) Normalized radial stress distribution



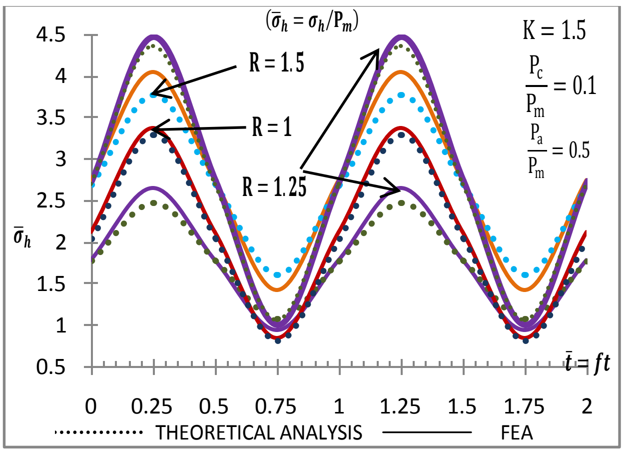
(c)Normalized axial stress distribution



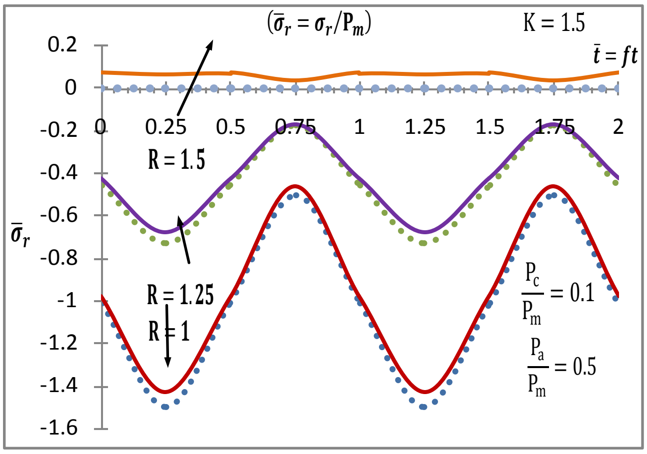
(d)Normalized equivalent stress distribution

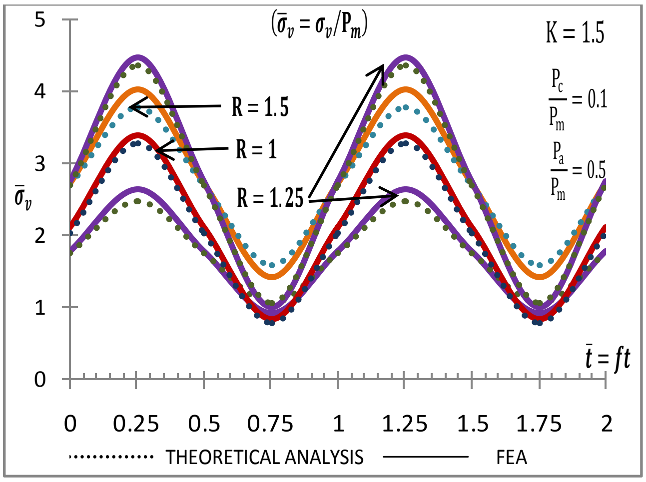
Fig. 4.8: Stress distribution on monoblock cylinder due to internal cyclic pressure and constant temperature.

The resulting stresses for open ended compound cylinder with (K = 1.5, K_i = 1.25) subjected to shrinkage pressure of $P_c/P_m = 0.1$ and cyclic internal pressure of $P_a/P_m = 0.5$ are plotted in Fig. 4.9 against the normalized time $\bar{t} = ft$ at different passion of (R).



(a) Normalized hoop stress distribution

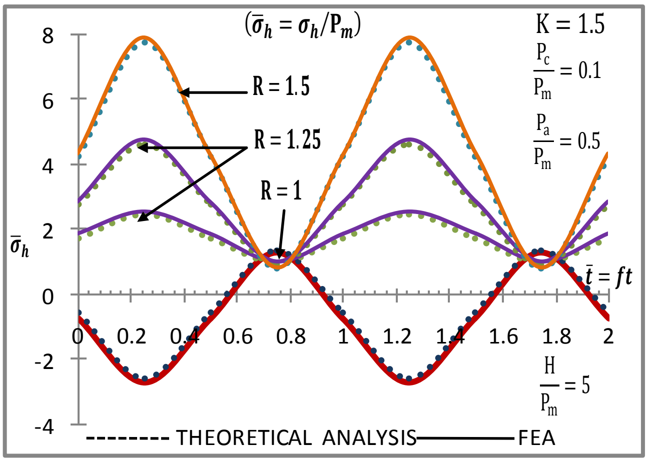




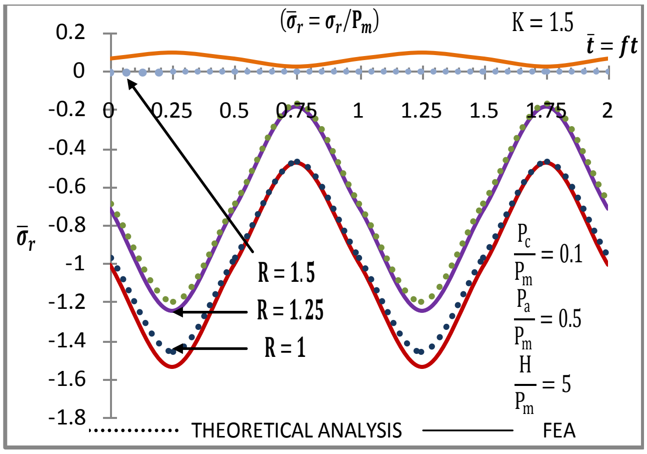
(c) Normalized equivalent stress distribution

Fig. 4.9: Stress distribution for compound cylinder due to internal cyclic pressure

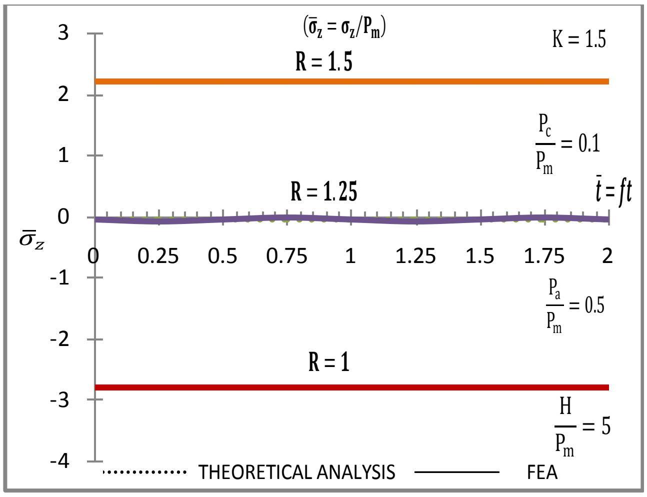
Figure 4.10 illustrates the resulting stresses for compound cylinder with (K = 1.5, K_i = 1.25) under the effect of shrinkage pressure $P_c/P_m = 0.1$, cyclic internal pressure $P_a/P_m = 0.5$, and constant temperature of $H/P_m = 5$. Also the stresses are plotted against the normalized time $\bar{t}$=f t at different positions (R).



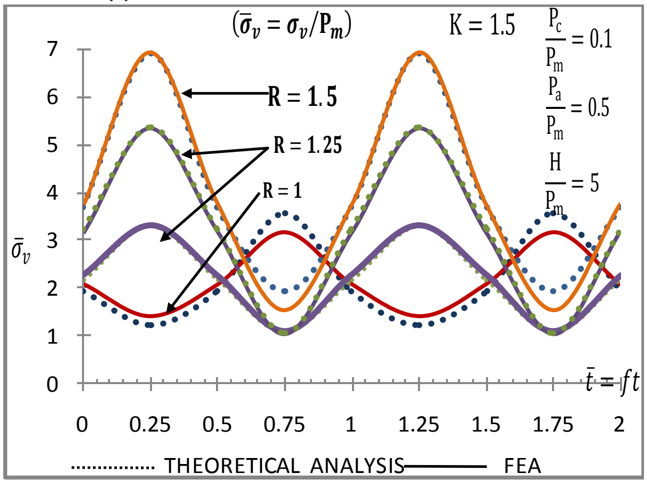
(a) Normalized hoop stress distribution.



(b) Normalized radial stress distribution



(c) Normalized axial stress distributio



(d) Normalized equivalent stress distribution

Fig. 4.10: Stress distribution for compound cylinder due to internal cyclic pressure and constant temperature.

5. CONCLUSIONS

In this paper theoretical analysis was carried out for monoblock and compound cylindrical pressure vessels subjected to steady and dynamic loads. Mathematical models have been developed and solved by using the program package ANSYS as a numerical finite element method. The prior analysis of hoop, radial, longitudinal, and equivalent stresses for cylinder subjected to different loads and working conditions were evaluated theoretically and numerically. The comparison between the two methods of analysis was presented in graphical forms.

From the present study it can be founded that finite element analysis is an extremely powerful tool for pressure vessel analysis when used correctly. Tested models were run and it is found a good agreement with analytical analysis and run in a relatively short time.

Therefore we recommend using finite element method for stress analysis of cylindrical pressure vessel under different loads and conditions.

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