Modeling and Simulation of Small UAV Flight Dynamics

Ahmed Ashraf Elbedewy*, Samy Samir Mohamed, and Hesham Omar Hamed
*MTC, Egypt, *Dody3015@gmail.com*

Supervisor: Ibrahim Kapeel, Ehab Safwat, Mohamed Abozeid, Ahmed M. Kamel
MTC, Egypt, e.khattab@mtc.edu.eg

Abstract*– Nowadays Unmanned Aerial Vehicles (UAVs) have become one of the biggest interests in the whole world either in military applications or in the civilian one such as agricultural observation and traffic monitoring, also a major tendency in UAVs modeling has been increased due to the accurate mathematical modeling and open loop simulation are very important to understand the UAV behaviors for various inputs, accuracy of guidance and control design also depends on the accuracy of the open loop model. So, accurate UAV model is a crucial at all. Accuracy of UAV model depends on the accuracy of its subsystems. in this work a small fixed wing UAV Cessna is taken as a case study and an accurate modeling using experimental measurements of geometric and inertia mass model is performed, dynamic model is performed by using 6 DOF equation of motion, the atmospheric model is modeled through the international standard atmosphere (ISA). Finally, aerodynamic model is estimated using a semi-empirical method (DATCOM) by using the aircraft intitutive design (AID) application in MATLAB. These subsystems are integrated in a Simulink model in order to simulate the overall model.

Keywords—small fixed wing UAV, modeling, aerodynamic data, geometric model, MATLAB (Simulink).

I. I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are playing increasingly dominant roles in several programs around the world[1]. UAVs can be generally classified as fixed wing (traditional) aircraft and rotary wing (quadrotors) aircraft. While the latter type is better for maneuvering operations, the former is more efficient and suited for applications involving long distance cruise tasks. Small fixed wing UAVs have been used in several applications in military such as suicide drones or in civilian such as agricultural observation and traffic monitoring. For these applications the reliability of small UAVs should be increased through high fidelity of modeling. The use of mathematical models to describe a fixed-wing aircraft's motion is a well-known field[2]. Due to the importance of small fixed wing UAVs dynamic modeling become a crucial factor. Several reasons describe the importance of modeling such as it may be used within the layout technique of the aircraft through thinking out how one of a kind design changes affect the performance. Every other important use for a correct model is a simulation of flight conditions which gives important helping for control engineer through saving time and money spent in preliminary training and guidance and control system testing. However, the applicability of the tests is limited via the accuracy of the model. A mathematical description of (UAV) is likewise vital in model-based control techniques. Modeling of small UAVs can be classified into two categories the first is a physic-based modeling and the second is modeling using system identification. although the latter is more accurate and give an efficient model but it uses more time and money, the former can save money and time with an acceptable accuracy which can be tuned using one flight test[3]. Airplane dynamic model comprises from sub models, these sub models are geometric mass inertia model which describe the geometry of the case study fixed wing and the mass distribution, dynamic model which describe the motion of the UAV through force and moment equations, atmospheric model which describe the relation between height and air density, actuation model which contain the transfer functions describe the servo motors utilized, propulsion model generally for small fixed wing UAV brushless dc motor with appropriate propeller and aerodynamic model representing aerodynamic force and moment during flight. All of these models can be performed and modeled using several methods such as theoretical method, computational method, semi-empirical method and experimental method. Some of these methods are utilized according to the sun model such as geometric mass inertia sub model is modeled based on experimental method, dynamic and atmospheric sub models are modeled based theoretical method and aerodynamic model is represented according to the data collected from DATCOM which is considered as a semi empirical method[2][4]. The accuracy of the small fixed wing UAV depends on the UAV mathematical model and the accuracy of the sub models estimated based on the previous methods. Each sub model is represented in MATLAB Simulink and integrated to comprise the small UAV whole model. In this work small fixed wing UAV Cessna is utilized as a case study. An experimental work is performed to evaluate the Cessna mass inertia model, DATCOM is used through the aircraft intitutive design MATLAB application to estimate the aerodynamic data which is tabulated in Simulink to generate the aerodynamic model in Simulink, servos which is used in the UAV actuation system is considered as a first order transfer function with time delay predefined in its datasheet. This paper is designed as the following section II describe the airplane mathematical model, section III illustrates the modeling of the case study small fixed wing UAV which contain the aerodynamic model, atmospheric model, actuation model and mass inertia model, results are

5th IUGRC International Undergraduate Research Conference,
Military Technical College, Cairo, Egypt, Aug 9th – Aug 12st, 2021.

presented in section IV. Finally, conclusion and future work is represented in section V.

II. II. AIRPLANE MATHEMATICAL MODEL

The important and first step in designing guidance and control system for UAVs is a six degree of freedom mathematical modeling of the required airplane or developing a non-linear dynamic model[1][2].

A. Kinematic model

It can be represented in two equations which relate the linear and angular positions in inertial frame with those of the body frame and frame transformation matrices. Forces and moments are not considered.

1) Linear motion

It relates the linear velocities in body frame (u, v, w) and the derivates of linear position in the inertial frame (p_n, p_e, p_d)

$$\frac{d}{dt}\begin{pmatrix} p_n \\ p_e \\ p_d \end{pmatrix} = R_b^i \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad (1)$$

R_b^i ...... Transformation matrix from body to inertial frame.

2) Angular motion

It represents the relation between the body angular rates which is described in the body frame (p, q, r) and the angular positions (φ, θ, ψ) which is rotated with these angles with respect to the body frame.

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = R_b^r \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (2)$$

R_b^r ...... Transformation matrix from the rotated frames about the body frame to body frame.

B. Dynamic model

In order to derive the dynamic model, the newton's second law is applied to the translational and rotational motion, these equations must be performed in a fixed frame which is the inertial frame in our work it is assumed to be the earth frame (flat earth model) which is accepted for small UAVs. So, the motion will be translated from body frame to the earth frame through some rotational matrices[1], [2].

1) Translational motion

Starting from applying the newton's second law to the translational motion:

$$\sum F = \frac{d}{dt_i}(linear\ momentum) = m\dot{V}_i \quad (3)$$

F .... the total force (gravity, aerodynamic and propulsion) acting on UAV.

m ...... UAV total mass.

$\dot{V}_i$ ...... velocity derivative in the inertial frame.

Due to the applied forces in the body frame, equation (1) will be translated to body frame:

$$\sum F_b = m\dot{V}_b + w_{b/i} \times V_b \quad (4)$$

$F_b \equiv (f_x, f_y, f_z)$, $V_b \equiv (u, v, w)$, $w_{b/i} \equiv (p, q, r)$,

Hence, the three translational dynamic equations are as the following:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} rv - qw \\ pw - ru \\ qu - pv \end{pmatrix} + \frac{1}{m}\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \quad (5)$$

2) Rotational motion

Applying newton's 2nd law for the rotational motion:

$$\sum M = \frac{d}{dt_i}(angular\ momentum) = \frac{dH}{dt_i} \quad (6)$$

M ...... the total moment applied on the UAV.

Converting this moment from the inertial frame into the body frame:

$$\sum M_b = \frac{dH_b}{dt} + w_{b/i} \times H_b \quad (7)$$

H_b ...... the angular momentum can be defined as the product of the inertia matrix and angular velocity.

$$H_b = Jw_{b/i} \quad (8)$$

Substitute from (6) in (5) then:

$$\sum M_b = \frac{dJw_{b/i}}{dt} + w_{b/i} \times Jw_{b/i} \quad (9)$$

$$\dot{w}_{b/i} = J^{-1}\left[-w_{b/i} \times (Jw_{b/i}) + M\right] \quad (10)$$

$\dot{w}_{b/i} = \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix}, J = \begin{pmatrix} j_{xx} & j_{xy} & j_{xz} \\ j_{yx} & j_{yy} & j_{yz} \\ j_{zx} & j_{zy} & j_{zz} \end{pmatrix}$, for the airframe symmetry about their two planes, then the product of inertia is very small can be assumed to be zero $J = \begin{pmatrix} j_{xx} & 0 & 0 \\ 0 & j_{yy} & 0 \\ 0 & 0 & j_{zz} \end{pmatrix}$

Equations (1), (2), (5) and (10) represents the dynamic model of the UAV, but it's not complete due to the external moments and forces (gravity, aerodynamic and propulsion) are not computed.

C. Force and moment equations

In this section the total forces and moments that affect UAV will be described. There four forces acting on UAV body gravitational, aerodynamic, propulsion and disturbance forces which may appear from the atmospheric effect and it will be neglected, also there are three moments will appear aerodynamic, propulsion and disturbance moments, also the disturbance moment will be neglected[5].

1) Gravity effect

The force which arises from the gravity effect can be modeled as the following in the body frame.

$$f_g = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \quad (11)$$

There is no gravity moment due to its force effect in the center of gravity $c.g.$

2) Aerodynamic effect

It can be divided into two components longitudinal which cause the translational motion in x-z plane as well as the rotational motion in pitch and lateral components which cause a side translational motion as well as roll and yaw rotational motion[6].

- Longitudinal contains the lift and drag forces and pitching moment.

$$F_l = \frac{1}{2}\rho V_{uav}^2 SC_L, F_d = \frac{1}{2}\rho V_{uav}^2 SC_L, m = \frac{1}{2}\rho V_{uav}^2 ScC_m \quad (12)$$

- Lateral contains the side force and roll and yaw moments.

$$F_Y = \frac{1}{2}\rho V_{uav}^2 S C_Y\ ,\ l = \frac{1}{2}\rho V_{uav}^2 S b C_l, n = \frac{1}{2}\rho V_{uav}^2 S b C_n \qquad (13)$$

3) Propulsion effect

The propulsion force will affect the longitudinal axis of the UAV. Also, its moment will affect through the applied axis causing a roll angle effect can be removed by adjusting the UAV aileron deflection[7].

III. MODELING OF THE CASE STUDY SMALL FIXED WING UAV

The previous section derives the necessary equations that describe any UAV dynamics, in this section a MATLAB Simulink model will be built according to the collected data from the Cessna UAV which is chosen as a case study. The UAV geometric and mass inertia model is generated, aerodynamic model is estimated, actuation model is considered as a first order transfer function with the predefined time delay in the datasheet, atmospheric model is built according to the international standard.

A. Cessna geometric model

All the geometric data is measured physically from the Cessna airplane in the laboratory to fully acquire the geometric model in Fig. 1. The top view of the case study airplane Cessna and the geometric data is tabulated Table I.

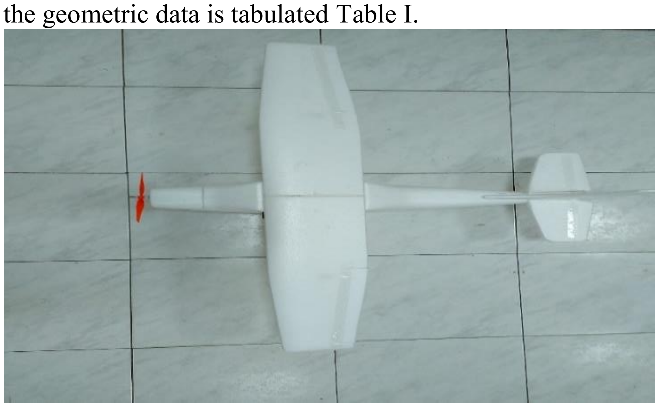


Fig. 1. Cessna case study top view.

TABLE I

Property	value		
	wing	Horizontal tail	Vertical tail
semi-span	1.96 ft	0.65 ft	0.65 ft
Break-span	0.91 ft	0 ft	0.213 ft
Root chord	0.57 ft	0.42 ft	0.82 ft
Break chord	0.57 ft	0.42 ft	0.42 ft
Tip chord	0.393 ft	0.262 ft	0.295 ft
Inboard Dihedral angle	0 deg	0 deg	0 deg
Outboard Dihedral angle	0 deg	0 deg	0 deg
Inboard sweep angle	0 deg	-6 deg	20 deg
Outboard sweep angle	3 deg	0 deg	20 deg

B. Mass inertia model

Mass inertia model give us an important information about mass distribution in the UAV fuselage, these mass inertia data is used to determine the overall moment about the body axes. Experimental work is done by using simple pendulum method in order to estimate the moment of inertia, Cessna is a symmetric airplane about x-z plane. So, the product of inertia $j_{xy} = j_{yx} = j_{zy} = j_{yz} = 0$.

Also, $j_{xy} \& j_{yx}$ *is very small values* ≈ 0 . The pendulum method is cheap, simple and easy to understand. Also, it gives an accurate data for small UAV. The setup is done according to []. the oscillation is done in vertical plane perpendicular to the axis of rotation. The amplitude of the oscillation is small enough to satisfy that $\sin(\theta) = \tan(\theta) = \theta$. The moment of inertia equation which is used in pendulum method is as the following:

$$I = \frac{mgL_{C.G}}{\omega_n^2} - mL_{C.G}^2 \qquad (14)$$

$$\omega_n = \sqrt{R_e^2 + I_m^2}\ , R_e = \frac{5}{T_f},\ I_m = \omega_d = \frac{2\pi}{T}$$

m ……. Airplane mass.

w_n …… natural frequency.

$L_{C.G}$ …... distance from the fixed point to the UAV center of gravity.

T_f ……. Oscillation damps out time.

T ……. Time for one oscillation.

This method is done for long and short length and the average value is taken in order to increase the accuracy of the measurements. The experimental setup is shown in Fig. 2. In Table II the data collected and calculated for the short suspension. Also, in Table III the calculated data for the long suspension both for J_{xx}. These measurements are performed for both J_{yy}, J_{zz}. Also, these measurements for long and short suspensions.

TABLE II

Test no.	No. of oscillations	Total time (sec)	T_f(sec)	T(sec)	w_n
1	10	103.22	20.06	2.006	3.075
2	10	104.69	23.06	2.306	2.725
3	10	157.31	23.13	2.313	2.716
4	10	112.32	23.53	2.353	2.67
5	10	137.63	23.47	2.347	2.677
average	10				2.7726

TABLE III

Test no.	No. of oscillations	Total time (sec)	T_f(sec)	T(sec)	w_n
1	10	105.82	25.81	2.581	2.434
2	10	132.69	25.34	2.534	2.479
3	10	113.03	25.81	2.581	2.434
4	10	103.31	25.69	2.569	2.446
5	10	104.97	27.29	2.729	2.302
Average	10				2.418

(a) J_{xx} measurement (b) J_{xx} measurement (c) J_{xx} measurement

Fig. 2. Moment of inertia measurement setup

C. Aerodynamic model

Computational and experimental methods which can be used to estimate the aerodynamic data of the airplane are expensive and could take more and more time. So, a semi-empirical approach is used to calculate all the aerodynamic characteristics. Datcom is a good approach to do this function. MATLAB have improved its applications one of these is the aircraft intitutive design (AID) which is used to facilitate interfacing with Datcom. The input data of the AID is wing, horizontal, vertical and fuselage characteristics, all geometric data, airfoil standard and finally the flight conditions. All these data are shown in Fig. 3. With AID application. The whole airplane is shown in Fig. 4. From AID graphical interface the file is analyzed with the specified values, it will generate a Datcom file with the same specified data, this input file is run with the Datcom program to generate the output files which contain all the aerodynamic coefficients with the specified angle of attack and the fin deflections for elevator, rudder and aileron. Also, the dynamic derivatives are generated. These aerodynamic data is taken and a MATLAB function is written with these output data in order to make a lookup tables in Simulink model of the whole UAV model to simulate the aerodynamic characteristics and generate the aerodynamic force and moment for each time step according to the input data to each lookup table which is the angle of attack and fin deflections for static part and the derivative of the angle of attack and UAV angular rates for the dynamic part. These aerodynamic force and moment are added with the gravity force and propulsion force and moment to generate all forces and moments acting on the UAV's fuselage. These data are plotted, each aerodynamic coefficients with the angle of attack and the fin deflection in the next section. Simulink model is presented in Fig. 5. Consists of dynamic and static parts.

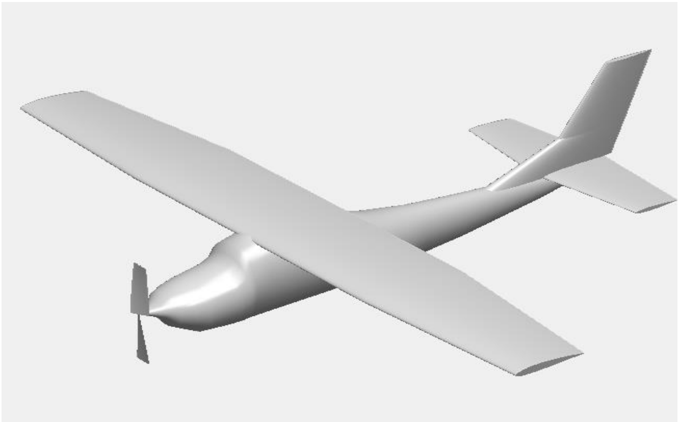


Fig. 4. Aircraft intitutive design of Cessna model

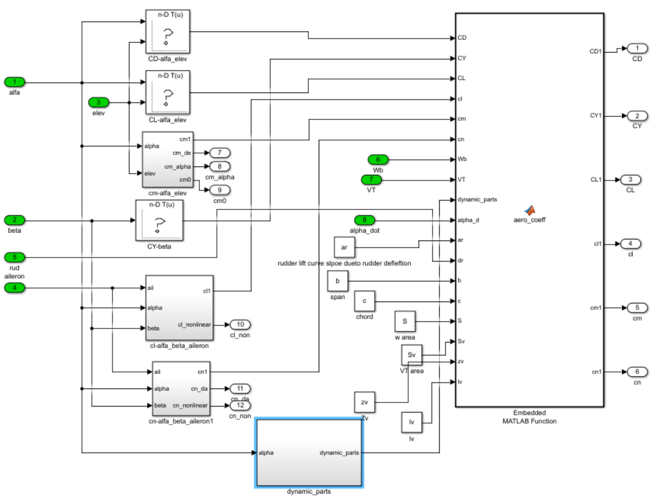


Fig. 5. Simulink aerodynamic model

D. *Atmospheric model*

The international standard atmosphere is used to model the atmospheric model as shown in the following equations. Also, Fig. 6. Shows the Simulink model of the atmospheric model.

$$T_H = T_0 + \frac{dT}{dH}[H - H_0] \tag{15}$$

$$P_H = P_0\left[\frac{T_H}{T_0}\right]^{\left(\frac{-g_0}{R\frac{dT}{dH}}\right)} \tag{16}$$

$$\rho_H = \rho_0\left[\frac{T_H}{T_0}\right]^{\left(\frac{-g_0}{R\frac{dT}{dH}}+1\right)} \tag{17}$$

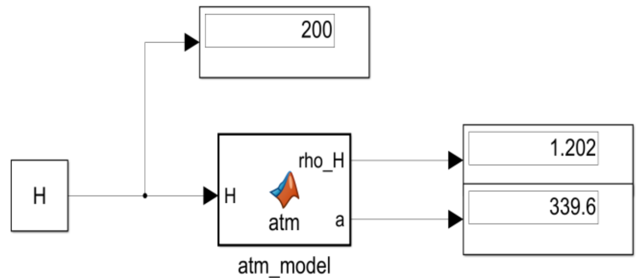


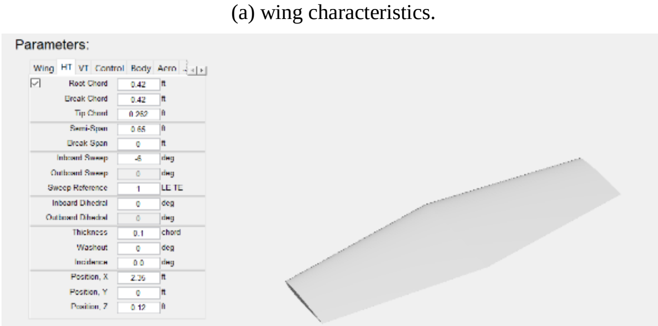
Fig. 6. Simulink atmospheric model

E. *Actuation model*

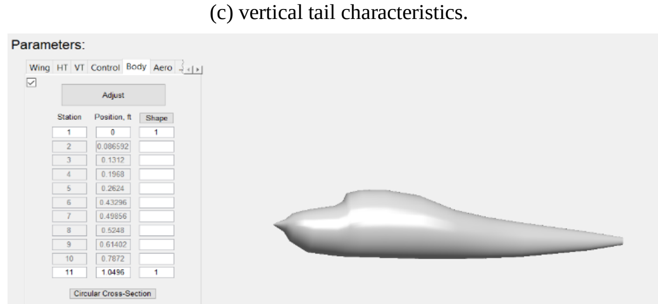
Commercial servo motors are utilized to control elevator, rudder and aileron deflections. Servo motor is a dc motor with potentiometer feedback to make a position control, the required angle is fed to each servo and the achieved angle is the same as the required one but with a time delay of the servo, this time delay is very small according to the several loops time delays in the UAV. So, it can be represented with a delay block in MATLAB Simulink followed by the limiter to

(a) wing characteristics.

(c) vertical tail characteristics.



(b) horizontal tail characteristics.



(d) fuselage characteristics.

Fig. 3. Airplane component characteristics

limit the upper and lower deflection, four delayers for each servo. This servo model is represented in Fig. 7.

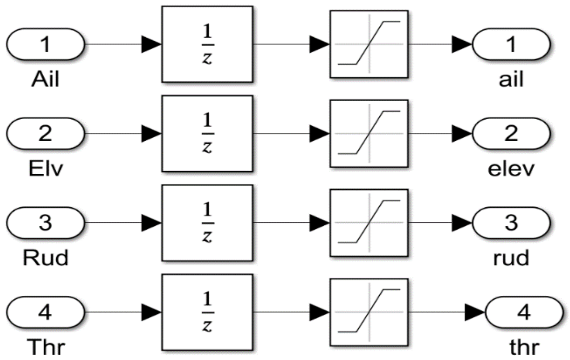


Fig. 7. Servo Simulink model.

IV. SIMULATION AND RESULT ANALYSIS

In this section the results will be presented for each model. Moment of inertia is calculated and tabulated in Table IV and the simulation is done for the aerodynamic model to estimate the lift and drag curves. Also, the pitching moment curve is estimated. These results will be presented as shown in Fig. 8. And Fig. 9. Also, in Fig. 10.

TABLE IV

MOI	long suspension	short suspension	Average
I_{xx}	0.8684	0.0738	0.4711
I_{yy}	0.8633	0.2131	0.5682
I_{zz}	0.7866	0.3239	0.5553

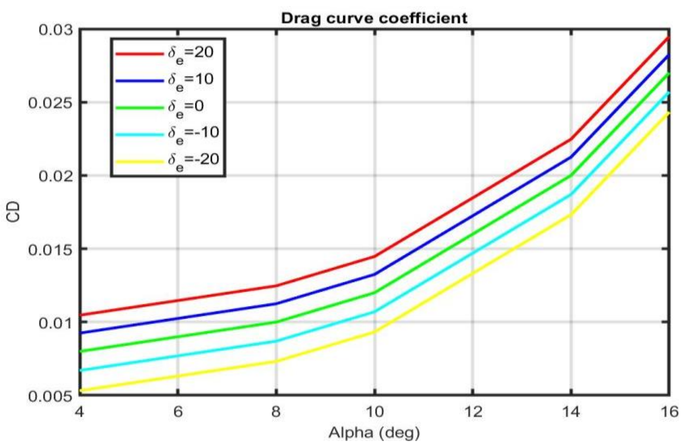


Fig. 8. Drag curve coefficient

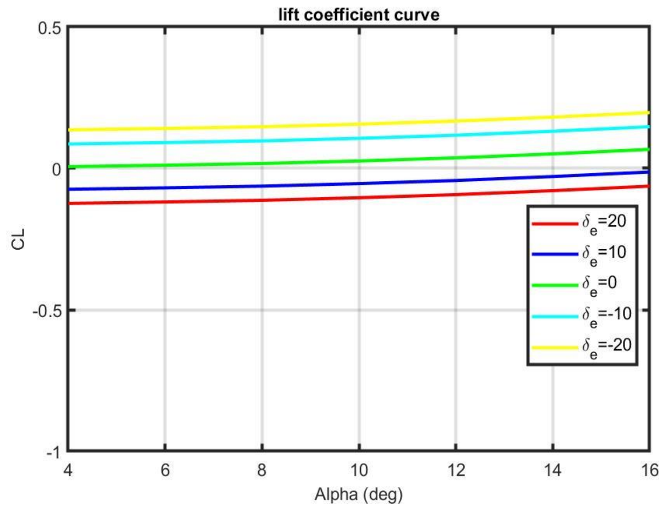


Fig. 9. Lift curve coefficient

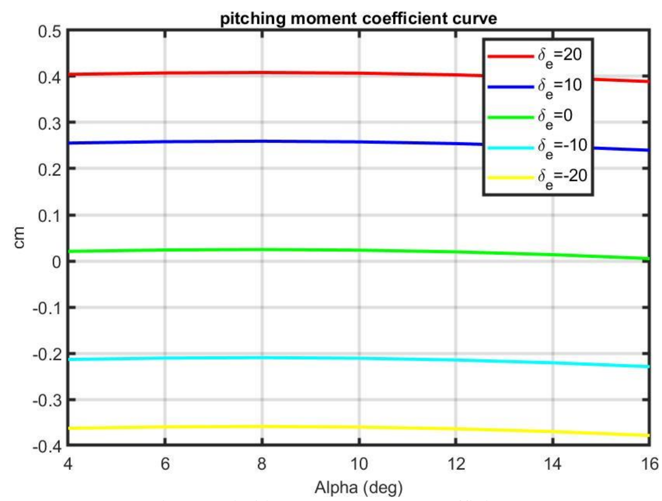


Fig. 10. Pitching moment curve coefficient

V. CONCLUSION AND FUTURE WORK

In this work dynamic model of Cessna small fixed wing UAV is presented, moment of inertia is calculated using experimental

Measurement, aerodynamic characteristics for Cessna model is evaluated using Datcom through AID, atmospheric model is presented. All of these sub models are simulated in MATLAB Simulink. In future work system identification will be used to estimate actuation servos transfer functions. Also, a wind tunnel

Experimental work will be used to estimate the propulsion model and propeller characteristics. Trim conditions will be calculated. Finally, a flight test will be done to validate this physics-based UAV modeling to get a six degree of freedom validated nonlinear model ready to be use as a test UAV for flight guidance and control techniques.

REFERENCES

[1] R. W. Beard and T. W. Mclain, "Small and Miniature Air Vehicles."

[2] D. Branch and D. Committee, "Modeling , Analysis and Identification of Airplane Flight Dynamics Modeling , Analysis and Identification of Airplane Flight Dynamics," 2014.

[3] A. Vuruskan, B. Yuksek, U. Ozdemir, A. Yukselen, and G. Inalhan, "Dynamic Modeling of a Fixed-Wing VTOL UAV," 2014.

[4] W. R. Silva, U. Federal, D. S. Maria, U. Federal, and D. S. Maria, "Modelling , Simulation and Control of a Fixed-Wing Unmanned Aerial Vehicle COBEM-2017-2703 MODELLING , SIMULATION AND CONTROL OF FIXED-WING UNMANNED AERIAL VEHICLE (UAV)," no. January 2018, 2017, doi: 10.26678/ABCM.COBEM2017.COB17-2703.

[5] W. Khan and M. Nahon, "Dynamics Modeling of a Highly-Maneuverable Fixed-Wing UAV DYNAMIC MODELING OF A HIGHLY-MANEUVERABLE FIXED-WING," no. April 2013, 2015.

[6] T. A. Johansen, T. I. Fossen, K. Gryte, R. Hann, M. Alam, and J. Roh, "Aerodynamic modeling of the Skywalker X8 Fixed-Wing Unmanned Aerial Vehicle," 2018, doi: 10.1109/ICUAS.2018.8453370.

[7] F. Uavs, E. M. Coates, A. Wenz, K. Gryte, and T. A. Johansen, "Propulsion System Modeling for Small," *2019 Int. Conf. Unmanned Aircr. Syst.*, pp. 748–757, 2019, doi: 10.1109/ICUAS.2019.8798082.