EA-2	141

MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

FORCE MEASUREMENTS IN WIND TUNNELS

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ABSTRACT

Experimental aerodynamic information of flying vehicles is obtained through force measurements on captive models in wind tunnels. The most common type of wind tunnel balance today is the strain-gauge balance, which in principle is a spring-balance with electric outputs. The paper describes the principles of the resistance strain-gauge, and how it can be applied in balances to measure static and dynamic forces. It also discusses proper bridge-wiring to suppress temperature influences and secondary force components. Strain-gauge balances are analog transducers which must be calibrated against known forces and moments.Multi-component balances are influened by interactions which means that an output of a balance is a function of both a primary and of secondary components. This complicates the process of calibration and a special mathematical model of the balance is used to evaluate calibrations. This model also permits evaluation of aerodynamic forces and moments from wind-tunnel measurements with strain-gauge balances.

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INTRODUCTION

Aerodynamic information at an early time is important in the design of flying vehicles. It may concern calculations of performance, stability, controlability, trajectories or strength. Such information can be obtained from wind-tunnel tests with instrumented models. Total or local static or dynamic forces are measured, static forces on stationary models and dynamic forces on oscillatory or rotating models.

Depending on the speed range of the vehicle it may be necessary to make tests in different types of wind-tunnels. When simulating start and landing conditions in a low-speed tunnel the models must be sufficiently large to permit the necessary imitation of details. High speed models on the contrary do not need this detailing, and the sizes of available transonic and supersonic wind tunnels often limit their scales. Both large and small balances are consequently needed and also balances for large and small force systems.

To measure the current forces and moments different kinds of wind tunnel balances are used. Static measurements on full models require an external or internal 6-component balance, sub-assembly measurements require balances from 1-component (hinge-moment tests) to 6-components (after-body tests). Dynamic forces are measured with 1-component balances in free-oscillation tests and with 5-component balances (the axial force component omitted) in forced-oscillation or in rotation tests (coning motion).

The most common type of balance is the strain-gauge balance (SG-balance), which in principle is a spring-balance with electric outputs. Reasons for its popularity are the possibility to make "homegrown" balances with different geometries and for different sized force systems, and that the necessary electronic equipment is simple. The output from a SG-transducer is small (mV level), but it is nevertheless possible to reach both large sensitivity and noise suppression with proper signal conditioning equipment and to digitize and store the outputs with great speed.

To make an interpretation of its outputs possible, a SG-balance must be calibrated against known forces and moments. The quality of measurements with SG-balances is highly dependent on this calibration process and on the available calibration equipment.

External balances are used in low-speed wind tunnels and internal balances in both low and high speed tunnels in connection with full-model tests. The tendency today is, that internal balances become more and more popular.

BASIC ELEMENTS

Strain-Gauges

Two types of resistive strain-gauges (SG) are used in wind tunnel balances. We will designate them conventional foil gauges (FSG) and semiconductor gauges (SSG).

The FSG consists of a multicrystal metal foil etched into a grid and fixed to an insulating plastic foil. Due to its very small thickness and due to the low elastic modulus of the plastic the FSG has a very low stiffness

6 and can be cemented so intimately to a surface that it follows the surface strain practically totally even if the strain is negative. The strain is transmitted to the SG through shear forces in the adhesive layer. The SG changes its resistance proportionally to the strain in the sublayer.

The SSG may consist of a doped silicon bar fixed to a plastic foil. Depending on type and quantity of doping the resistance of a SSG will increase or decrease with the strain.

When a FSG or SSG is exposed to mechanical strain, its electrical resistance will change. A change in the temperature level of the sublayer also causes a resistance change. Within our needs for accuracy the total effect can be expressed by

$$\frac{\Delta R}{R} = g(\varepsilon + \varepsilon_s) + \kappa \cdot \varepsilon^2 \qquad (1)$$

where ΔR = the resistance change
R = the initial resistance
g = the linear gauge factor
ε = the sublayer strain
ε_s = the apparent strain, or the collected influence of a temperature change on the gauge resistance
κ = the second degree gauge factor

FSGs are characterized by having $g \sim 2$ and $\kappa \sim 10$. If an accuracy of not better than 1% at $\varepsilon \cong 1.5$ mm/m is asked for (1) may be exchanged for

$$\frac{\Delta R}{R} = g(\varepsilon + \varepsilon_s) \qquad (2)$$

Geometrically identical FSG with different ε_s on a special sublayer are available today. It is consequently possible to minimize the temperature influence on an installation by a proper choice of gauges.

The resistance change in a FSG is very small. A 350 Ω gauge gets a resistance change of $\Delta R = 0{,}7\ \Omega$ at $\varepsilon = 1$ mm/m or a change of 0,2 %.

To maximize the output the power must be as large as possible. The experience shows, that a normal installation can dissipate ~2 mW/mm^2 effective gauge surface. The power of a 350 Ω with a surface of 4 x 4 mm should consequently be limited to 3,5 V or 10 mA.

By combining the crystallografic orientation of a silicon bar with different kinds of doping SSGs can be manufactured with positive or negative strain sensitivity.

The strain and the temperature influences on the resistance of a SSG can be described by (1). The value of the linear gauge factor depends on the type and amount of doping and can be

$$-175 < g < 175$$

The second degree gauge factor which is always positive can be

$$4000 < \kappa < 10000$$

where the lower values refer to P-doped gauges and the higher to N-doped gauges.

The SSG's large and non-linear strain sensitivity means that the initial resistance is influenced by the cementing process. It is difficult to know in advance the end up point on the non-linear resistance curve. The SSG presents different ε_s on different sublayers. It is not possible to influence the ε_s of an SSG during the manufacturing process.

The strain sensitivity of a SSG drops with temperature. It can be used within the same temperature range as the FSG. The current loading should not be larger than 10 to 20 mA.

Gauge Installation

The SG requires for its function that the strain in a sublayer can be transmitted to the thin active element of the SG. This is done through shear forces in the adhesive layer between the sublayer and the gauge.

If the molecules in two surfaces are sufficiently close (<5Å) adhesion forces will hold the surfaces together. Cementing is to tie together two solid bodies, the surfaces of which are insufficiently smooth, by use of such forces and a third substance, an adhesive. To fill up all irregularities the liquid adhesive must be thin and be able to wet the surfaces.

Current adhesives for SG installations are practically all of polymerization types. The extremes are the one-component cyanoacrylate adhesive which sets practically momentarily, when it is exposed to a pressure, and the two component epoxy adhesive, which must be heat-cured.

If a SG installation is exposed to humidity water molecules will penetrate the adhesive layer. The initial resistance will increase due to swelling of the adhesive and the adhesion forces between the adhesive layer and the metal sublayer will gradually disappear due to oxidation. A method to control the condition of a SG-installation is to measure the insulation resistance between the SG and the sublayer. A correct installation should have a value of ~10.000 MΩ.

SG-installations which will be used for a long time must be humidity protected.

Wheatstone's Bridge

The resistance change in a SG due to strain is small compared with its initial resistance. This is one reason for using bridge wirings . A bridge consisting of active SGs in all four branches has also the capacity to suppress the influence of changes in temperature level. In force transducers such bridges in many cases also suppress the influences of interfering force and moment components on the outputs.

As shown in Fig.1 the bridge is powered across one of the diagonals. The voltage change across the other diagonal is the output. If the bridge consists of 4 identical SG the application of (1) gives

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$$\frac{\Delta V}{V} = \frac{g}{4} \quad (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) + (\frac{K}{g} - \frac{g}{2})(\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 - \varepsilon_4^2) \quad (3)$$

if ε_3 is neglected next to ε and ε^2 next to 2.

Application of (2) results in

$$\frac{\Delta V}{V} = \frac{g}{4} (\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \quad (4)$$

(4) is a sufficiently good approximation in the design of force transducers, which will be properly calibrated before use.

Introduction of (1) in (3) or (4) shows that the bridge suppresses the apparent strains, if the temperature level is constant within a half or a full bridge. This also means that a bridge does not suppress arbitrary temperature gradients.

In practice a bridge will normally be more complicated. Fig. 2 shows some further elements which could be included in a bridge powered with constant voltage. If the bridge has a temperature dependent zerodrift, despite the fact that all gauges are from the same lot, a resistance must be inserted between e.g. a and b in the figure. This resistance must have a large temperature coefficient, and the resistance value is deduced from measurements and calculations.

Has the bridge an initial unbalance, then it is necessary to deduce from measurements and calculations a resistance with small temperature dependence and insert it between e.g. c and d in Fig. 2. If the bridge sensitivity increases with temperature, a resistance with positive temperature coefficient may be positioned in series with the power cable. Decreases the sensitivity with the temperature as is the case with SSGs, then a thermistor may be wired in series with the power cable. Is the bridge powered with constant current, then the resistance and the thermistor must be wired in parallel with the bridge.

The wires which connect the gauges within a bridge are also parts of the bridge. To avoid temperature dependence and unbalance the wires within a bridge must be symmetrically arranged.

Stress - Strain

Force transducers are in principle spring balances with electric outputs. When an external force or moment loads the spring body, internal forces or mechanical stresses appear in the whole body and in each point exists corresponding strains.

The relation between stress and strain is linear up to a certain value. When this limit is passed, the material will distort plastically. When unloading only the elastic part recovers.

In a one dimensional stress field the following expression applies

$$\varepsilon = \frac{\sigma}{E} \quad (5)$$

where σ = the mechanical stress
ε = the corresponding strain
E = Youngs modulus

A one dimensional stress field is always accompanied also by strains in a surface perpendicular to the stress. These strains can be expressed with

$$\varepsilon_T = - \nu \cdot \varepsilon \qquad (6)$$

where ν = Poissons number

In a two dimensional stress field applies if x and y are two perpendicular axis

$$\left.\begin{aligned} \varepsilon_x &= \frac{1}{E} \quad (\sigma_x - \nu\sigma_y) \\ \varepsilon_y &= \frac{1}{E} \quad (\sigma_y - \nu\sigma_y) \end{aligned}\right\} \qquad (7)$$

If σ_x and σ_y are principal stresses, then ε_x and ε_y will be principal strains and represent the extreme values of the strains.

A change in the temperature level of the body will change its dimensions without creation of corresponding stresses, Temperature gradients on the contrary produce local internal stresses and corresponding local strains without presence of external forces. Only temperature gradients which cause the same strain distribution in half or full bridges can consequently be accepted.

Youngs modulus decrease with temperature for most metallic materials.

In the spring bodies of wind tunnel balances high strength steels with a yield stress of > 1000 N/mm^2 are almost exclusively used. It's an advantage to use maraging steels instead of ordinary steels, because of the former's lesser warping tendencies. Wind tunnel balances have very small internal damping and are often exposed to dynamic loads. The fatigue limit of a spring body of a high quality maraging steel and manufactured by use of electric-discharge-milling is ~ 600 - 700 N/mm^2.

To get a resonable safety factor against permanent deflections and fatigue, the design strains (for nominal static loads) should not exceed ε = 1 - - 1,5 mm/m (200 - 300 N/mm^2).

FORCE TRANSDUCER PRINCIPLES

In a loaded body internal stresses and corresponding strains are present. However the SGs can only be installed on the external surface of the body. This creates no problems as long as a unic relation exists between the external strains and the internal stresses. This means in practice that the section with the SGs must not be to close to the area, where the external forces are applied.

A common rule in the design of spring bodies is that the strain below the SGs primarily must be caused by the component which one intends to measure. Interfering forces should give as small contributions as possible to the strain. It is also advantageous to avoid large strain gradients, because such gradients make it difficult to get symmetry within a bridge.

Beam in Bending

Consider a cantilever beam (Fig. 3a) loaded in its free end by a force and provided with four SGs in the fixed end wired according to Fig. 3b. The relation between the external force (P) and the strains (ε) in the upper and lower surfaces of the beam may be written

$$\varepsilon = \pm \frac{P \cdot L}{E \cdot B} \qquad (8)$$

where L = the distance between the force and the centre of the SG bridge
E = Youngs modulus
B = the beam's bending stifness at the bridge

Application of (1), (4) and (8) give

$$\frac{\Delta V}{V} = g \cdot \varepsilon \qquad (9)$$

The output is consequently linear even if the strain sensitivity of the SG is non-linear (of second degree). This is an important characteristic when using SSGs.

A change in the temperature level of the transducer produces the same ε_s in all four SGs. Temperature gradients along the beam are suppressed, but not gradients across the same. The influence of a positive or negative force along the beam is the same in all bridge elements. A torsion moment or a bending force perpendicular to the force P do not influence the bridge output as long as the SGs are positioned symmetrically in relation to the centre axis of the beam. On the contrary, the beam can not suppress a moment in the same plane as the force P. Such a moment is added to the bending moment from the force P.

Are the four SGs positioned according to Fig. 4a and wired according to Fig. 4b, then the influence from the moment M will be suppressed, if the beam has a constant cross section. It is consequently possible to write

$$(\varepsilon_1 - \varepsilon_2) = \frac{P}{E \cdot B} (L_1 - L_2) \qquad (10)$$

and

$$\frac{\Delta V}{V} = \frac{g}{2} (\varepsilon_1 - \varepsilon_2) \qquad (11)$$

where L_1 and L_2 are defined in Fig. 4a.

The moment M can be determined from

$$(\varepsilon_2 \cdot \frac{L_1}{L_2} - \varepsilon_1) = (\frac{L_1}{L_2} - 1) \cdot \frac{M}{E \cdot B} \qquad (12)$$

Is the transducer in Fig. 4a provided with a force transmitting mechamism according to Fig. 5a, the result will be a force transducer, which is insensible to the position of the force P.

If the force center is chosen to coincide with a point midway between the half bridges as indicated in Fig. 5a, the following expressions can be deduced

$$\left.\begin{aligned}(\varepsilon_1 - \varepsilon_2) &= \frac{P}{E \cdot B} \cdot a \\ (\varepsilon_1 + \varepsilon_2) &= 2 \cdot \frac{M}{E \cdot B}\end{aligned}\right\} \quad (13)$$

This characteristic is used in internal balances of the bending beam type.

Transducers in external balances are often designed according to Fig. 3. The force P is introduced to the transducer in such a way that no moment M will appear.

Bar in Torsion

Consider a circular bar in torsion according to Fig. 6a. The torsion moment causes on the cylindrical surface of the bar shear stresses perpendicular to the longitudinal axis. These can be expressed as

$$\tau = \frac{M}{W} \quad (14)$$

where M = the torsion moment
W = the torsion stiffness of the bar

The internal shear forces cause twisting but not elongation of the bar. However on the cylindrical surface octogonal principal stresses in $\pm 45^o$ inclination to the longitudinal axis of the bar will appear. These principal stresses are connected with principal strains, which can be expressed as

$$\left.\begin{aligned}\varepsilon_{1,2} &= \pm \frac{M}{W \cdot E} (1 + \nu) \\ \varepsilon_{1,2} &= \varepsilon_{3,4}\end{aligned}\right\} \quad (15)$$

If our goal is to make a torsion transducer we must position the four SGs according to Fig. 6a and b. Since all the SGs are exposed to the same numerical strain, and since the strain in opposite elements has equal sign, we have in principle the same situation as a beam in bending, and get

$$\frac{\Delta V}{V} = g \cdot \varepsilon (1 + \nu) \quad (16)$$

We have consequently the same suppression of each individual SG's nonlinearity, the bridge is compensated for changes in the temperature level and for temperature gradients along the bar. A bending moment or an axial force does not give any contribution to the output of the bridge.

The section of the transducer does not need to be circular.

6 INTERACTIONS

In the previous sections are shown how full-bridges can be arranged to measure a force or a moment and at the same time suppress secondary forces and moments. This suppression will not be complete in practice and we designate the remaining influence interactions.

Linear interactions are a consequence of mechanical and electrical non-symmetries and are caused by:

a) Geometrical incompleteness in the spring body due to design and manufacturing errors and due to manufacturing tolerances.

b) Non-symmetries between the SGs of a bridge due to differences in initial resistances, in gauge factors, in the wiring within a bridge and eventually due to shunting of the branches in the bridge.

c) Non-symmetries in the position of the gauges and different thickness in adhesive layers, causing differences in the gauge installations.

Each mechanical loading of the spring body causes a transfer or a deflection of the origo of the force system in relation to the centre of the transducer bridges. This causes the output of the bridges to be in principle non-linear and we designate this characteristic non-linear interactions. Are the deflections proportional to the loads and small compared to the geometrical dimensions of the transducer, then the non-linear interactions can be superimposed and are in principle of the 2:nd degree (product terms).

2-Component Transducer

This condition may be illustrated by the symbolic 2-component transducer in Fig. 7a. If only the force N (Fig. 7b) loads the transducer we may write

$$\left.\begin{aligned} u_N &= k_{1N} \cdot N + k_{3N} \cdot N^2 \\ u_T &= k_{2T} \cdot N + k_{4T} \cdot N^2 \end{aligned}\right\} \qquad (17)$$

where k_{1N} = the primary sensitivity of the N-bridge

k_{3N} = coefficient for the non-linear influence from N^2 on the N-bridge

k_{2T} = coefficient for the linear influence from N on the T-bridge

k_{4T} = coefficient for the non-linear influence from N^2 on the T-bridge

The coefficients in (17) are determined through calibration against known N-loads.

Is only T loading the transducer (Fig. 7c) we may write

$$\left.\begin{aligned} u_N &= k_{2N} \cdot T + k_{4N} \cdot T^2 \\ u_T &= k_{1T} \cdot T + k_{3T} \cdot T^2 \end{aligned}\right\} \qquad (18)$$

where k_{2N} = coefficient for the linear influence from T on the N-bridge

k_{4N} = coefficient for the non-linear influence from T^2 on the N-bridge

k_{1T} = the primary sensitivity of the T-bridge

k_{3T} = coefficient for the non-linear influence from T^2 on the T-bridge

The coefficients in (18) are determined through calibration against known T-loads.

If we let N and T load the transducer simultaneously (Fig. 7d) and if we add (17) and (18), we will still find differences between outputs calculated from known load combinations and measured outputs. To solve this problem we have to add a further 2:nd degree term (product term) to each of the equations in our system

$$\left.\begin{aligned} u_N &= k_{1N} \cdot N + k_{2N} \cdot T + k_{3N} \cdot N^2 + k_{4N} \cdot T^2 + k_{5N} \cdot N \cdot T \\ u_T &= k_{1T} \cdot T + k_{2T} \cdot N + k_{3T} \cdot T^2 + k_{4T} \cdot N^2 + k_{5T} \cdot T \cdot N \end{aligned}\right\} (19)$$

where k_{5N} = coefficient for the non-linear influence from $N \cdot T$ on the N-bridge

k_{5T} = coefficient for the non-linear influence from $T \cdot N$ on the T-bridge

The two last coefficients in (19) are determined from a calibration, with known combinations between N and T. The result is a 2-component transducer and a system of 2 calibrated equations, which permit simultanous measurements of the two forces N and T with known accuracy.

6-Component Transducer

The common 6-component case (3 forces and 3 moments) will be rather complicated. With given presumptions we have a system of 6 equations with in principle 27 terms each. Six are linear and 21 are non-linear (load squares and load products).

The expression between the outputs and the 6 force- and moment components can be written

$$u_i = \sum_{n=1}^{6} k_{in} x_n + \sum_{n=1}^{6} \cdot \sum_{n=m}^{6} k_{inm} x_n x_m \qquad (20)$$

$1 \leq i \leq 6$

6 Normally it is not necessary to use 6 x 27 coefficients. Are the nominal values of the components reasonably similar, then the non-linear interactions are usually small and may be neglected. Are one or more components small compared to the others, then the former will be marred with non-linear interactions but not the later.

EXTERNAL BALANCES

External wind tunnel balances are used for full-model testing in low speed tunnels and for half-model testing in both low- and high-speed tunnels. External balances are also used for testing of non-aeronautical objects in low speed tunnels. To measure the complete force system on a full-model a six component balance must be used. An external balance positioned in the turn table and provided with an angle-of-attack mechanism measures in a wind-related system. Fig. 8a shows an example of such a balance.

A balance has a metric and a non-metric part. The metric part consists of the model struts and an internal floating frame. The non-metric part consists of the strut windshields (eventually) and a tunnel-fixed structure. Between the floating frame and the non-metric structure are 6 transducers, which react the loads P_1 to P_6 (see Fig. 8b). The forces P_1 to P_6 are transmitted to the transducers through links which are stiff in their primary force directions but weak in the other directions. The system of equations in Fig. 8b illustrates that the transducer outputs are proportional to more than one component in the primary force system. The necessary recalculation can be involved in a system of 6 equations for the evaluation of measured components. The interaction terms must also be included in this system of equations.

If unprotected, the aerodynamic forces on the struts will be included in the outputs. The struts will further interfere on the flow field around the model. An experimental deduction of strut corrections is difficult. Windshields reduce the measured strut forces but increase the interferences.

The non-linear interactions of an external balance are small due to the expanded geometry of the balance, and they can normally be neglected. However, the deflections of the struts must be considered, either in the form of non-linear interactions or in the form of a transformation of the force system. The deflections can be expressed as functions of the forces.

External balances are large and heavy and may have an unconveniently large tare in relation to its nominal force system. It is consequently necessary to be observant on the model weight.

Internally a half-model balance can in principle be built analogous to an external full-model balance. However, half-model-testing does not need as many components, because the side force and eventually the yaw moment have no meaning at such tests. Side and yaw forces are however present and if they produce interactions on the remaining components, they must also be measured.

At transonic and supersonic tests it is usual to provide the half-model-support with a boundary layer plate.

INTERNAL BALANCES

The use of internal wind tunnel balances in supersonic wind tunnels started as a consequence of the large interference effects, which the support and windshield systems of external balances produced. Today internal balances are used with advantage in all types of wind tunnels at full-model-testing. Internal balances always measure in a model fixed force system.

As an example we will consider a balance of the bending-beam type similar to the balance illustrated in Fig. 9. The bending beam principle means that the balance is made in one piece often by use of electric-discharge-milling and that the transducer elements are positioned in series. All six elements must both carry the complete force system of the model and measure its components. The model is mounted on the balance´s left taper and the right taper is carried by the sting.

The normal force and the pitch moment are deduced from measurements of the bending moment in two sections along the balance , positioned symmetrically around the origo of the force system (see Fig. 10). The bending stiffness in the two sections is determined from

$$B = \frac{N \cdot a + m}{\varepsilon \cdot E} \qquad (21)$$

where B = the bending stiffness required
N and m = nominal loads

2a = distance between the transducer elements, which if possible is chosen to allow the two components to cause the same level of strain in the two transducer elements.

ε = strain level corresponding to nominal loads

If the two bridges are wired according to Fig. 10b, then by use of (13) we may write

$$\left.\begin{aligned} N &= \frac{EB}{2a}(\varepsilon_2 - \varepsilon_1) \\ m &= \frac{EB}{2}(\varepsilon_2 + \varepsilon_1) \end{aligned}\right\} \qquad (22)$$

Neglecting interactions (22) and (2) gives

$$\left.\begin{aligned} N &= \frac{EB}{2ag\ V}(\Delta V_2 - \Delta V_1) \\ m &= \frac{EB}{2g\ V}(\Delta V_2 + \Delta V_1) \end{aligned}\right\} \qquad (23)$$

The wiring in Fig. 10c gives alternatively

$$\left.\begin{aligned} N &= \frac{EB}{2ag} \cdot \frac{\Delta V_3}{V} \\ m &= \frac{EB}{2g} \cdot \frac{\Delta V_4}{V} \end{aligned}\right\} \qquad (24)$$

In the latter case, when each bridge consists of one half bridge from the fore and one half bridge from the aft measuring section, then the half bridges must be temperature compensated individually.

The conclusions and equations above are also valid for the components C and n in Fig. 9.

The axial force component T passes the balance as a drag-thrust force. Due to its small size in comparison with the rest of the components the T-element can, however, not be designed as a drag-thrust element, which both produces a reasonable output and which can carry the rest of the components.

A parallelogram spring of the type illustrated in Fig. 9 and 11 a can be used as an axial force transducer. The strain sensing struts in the centre of the parallelogram are calculated from the condition that the sum of all horizontal forces which load the vertical struts must be equal to the T-component.

$$T = \frac{n_2\,\varepsilon_2\,E}{r} \cdot \frac{b_2\,h_2^2}{6}\left[\frac{n_1}{n_2} \cdot \frac{b_1}{b_2} \cdot \left(\frac{h_1}{h_2}\right)^3 + 1\right] \qquad (25)$$

where
- E = Youngs modulus
- $2r$ = the length of the vertical struts
- n_1 = the number of external struts. For the configuration chosen this number can never be less than 4
- b_1 = the width of the external struts
- h_1 = the thickness of the external struts
- n_2 = the number of the centre struts. For the configuration chosen this number can never be less than 2
- b_2 = the width of the centre struts
- h_2 = the thickness of the centre struts
- ε_2 = the maximum strain in the centre struts due to T

An SG senses the middle value of the strain along its length. In the current case when the strain changes from ε to $-\varepsilon$ (see 11a) along the centre struts it is advantageous from the output point of view to use short SGs and to position them as close to the fixed ends as possible. However, from an interaction point of view the symmetry of the installation must be very good. Fig. 11b shows two different wirings which can be used. The only difference is that alternative I produces a larger output than II. Both alternatives have the ability to suppress the influences from the remaining components reasonably well.

The roll moment twists the balance around its longitudinal axis. Fig. 6 and 12 show that it is possible to use principal strains in the way indicated in (15). Is a rectangular section used, then the torsional stiffness may approximately be expressed as

$$W = (8a^2b^2)/(3a + 1,8b) \tag{26}$$

where 2a = the height of the rectangle (a>b)
2b = the width of the rectangle

The wiring in Fig. 6 and 12 suppresses the influence of a bending moment. Is the ℓ-component to small to give the required output level, then it is necessary to use a more complicated transducer geometry with a larger relation between bending stiffness and torsion stiffness.

The maximum load which a balance - sting combination can carry depends at a given strain on the diameter. Fig. 13 shows a limit for bending beam balances, which can be used to estimate necessary balance diameters at different normal forces. The horizontal line reflects the fact, that the bending stresses in a model - balance - sting combination are independent of the model - scale at constant presssure in the wind-tunnel.

The interference from the sting on the base pressure of a model or on the flow around a model is small, if the model is not provided with a marked boat tail. The base pressure will be slightly wrong, but normally this will be measured and subtracted from the axial force measured.

In view of flow turbulence, tunnel vibration and starting loads, the sting must not be too weak. Without causing too much model interference on base pressure and static or dynamic stability a sting with a diameter equal to 75 % of the base diameter may have a parallel length of 2 à 3 bodydiameters followed by a flare with a half cone angle of 4^O.

The model - balance - sting is in principle an undamped spring with a fundamental frequency of

$$\omega = \sqrt{\frac{\kappa}{m}} \tag{27}$$

where ω = the fundamental frequency
κ = the spring constant of the sting
m = the oscillation mass

To get a fundamental frequency as high as possible (27) indicates that the sting should be as stiff as possible and the model weight as small as possible. This also minimizes the parasitic loading of a balance.

The electric connection with the balance is through a cable routed through the sting. For a 6-component balance this must at least consist of 14 to 24 individual leads depending on the number of individually powered bridges. A base pressure tube should also be located in the sting.

The internal balance is model fixed. The angle of attack (α) and angle of yaw (β) attitudes must be set with the sting. This may be done by use of a bent sting (β) which angle of attack is set in the vertical plane or, as illustrated in Fig. 14, by a rotation of the sting (φ) and the same change of angle (θ) in the vertical plane.

Internal balances can be designed for a smaller number of components than 6. It is, however, necessary to know that the interactions from the missing components on the measured components can be neglected.

4-component sting balances can be used as external half-model-balances.

SUB-ASSEMBLY BALANCES

Sometimes it is necessary to measure local forces and moments on a model. Measurements of hinge-moments, panel-loads and after-body forces require often balances which are integral parts of a model structure. The design principles and the bridge wirings are, however, the same as already illustrated.

Fig. 15 shows an example where a 3-component balance and a canard surface are manufactured in one piece. 24 SGs wired into 3 bridges measure the normal force and its position.

DYNAMIC BALANCES

Aerodynamic forces and moments which arise on a flying vehicle during a manoeuvre or a stability disturbance can be determined from measurements in wind tunnels on models, which perform certain elementary motions. It is possible to show that current flight conditions can be divided into

a) motion with constant α and β combinations
b) oscillation in pitch around the centre of gravity
c) oscillation in yaw around the centre of gravity
d) coning motion, e.g. rotation at constant α and β around the wind-vector

when the force system is model-fixed.

The concept of stability derivatives is connected with the classical form of motion equations, where a small deviation from the state of equilibrium is described by summing up contributions caused by small changes in the attitude parameters and their time dependence.

Static derivatives are in-phase with the motion and dynamic derivatives or damping derivatives are 90° out-of-phase. Damping derivatives are of different kinds and may be divided into

A. Direct derivatives
B. Cross derivatives
C. Cross-coupling derivatives

Free-Oscillation

Damping moments proportional to direct derivatives in tip, yaw or roll can be deduced from free-oscillation tests in corresponding degrees of freedom.

A rig for free oscillation tests consists in principle of a cross-spring, a balancing plate spring with a SG bridge and a trigger mechanism (see

Fig. 16). The cross-spring defines the location of the oscillation axis in the model. The balancing spring represents a restoring moment, and the SG output is calibrated to measure the angle of oscillation. The logaritmic decrement of the oscillations represents the damping moment and is deduced from a recording of the oscillation. The static moment is calculated from a measurement of the frequency of the oscillation. The unknown aerodynamic quantities are obtained from differences between wind-on and wind-off measurements.

The trigger mechanism produces the initial amplitude. It must be possible to manipulate it from outside the tunnel.

To get a certain oscillation frequency the spring constant of the rig must be tuned to the inertia of the model. Experience also shows that the aerodynamic moment around the oscillation axis at zero frequency must be balanced by use of control surfaces on the model. It is also advantageous if the centre of gravity of the model concides with the oscillation axis.

The limited stiffness of the sting causes the oscillation axis of the rig to also perform transverse oscillations. Is the sting provided with a SG bridge behind the model, this output can be calibrated against the sting deflection and used for recalculation of the location of the oscillation axis within the model. A condition is, however, that the fundamental frequency of the sting with the model mounted is sufficiently large (>2x) in comparision with the oscillation frequency. Otherwise a time dependent phase error will occur between the two oscillations.

Due to turbulences and vibrations the initial amplitude of the oscillation should be $\geqq \pm 1^{\circ}$.

Forced Oscillation

Cross and cross-coupling derivatives as well as direct derivatives can be measured by use of a rig (see Fig. 17) consisting of a 5-component SG balance (the axial force omitted), a cross-flexure, a balancing plate spring and a mechanism for driving the oscillation at constant amplitude $\geqq \pm 1^{\circ}$).

The damping forces and moments searched for are only small parts of the total forces and moments which load the balance. To provide such a balance with sufficient sensitivities SSGs are used. The evaluation of the different stability derivatives presupposes that the balance outputs are divided in DC- and AC-parts. The latter must also be broken up in in-phase and out-of-phase components.

The driving mechanism supplies the model with energy to maintain the motion. If the balance spring, which also measures the angle, is tuned to maintain a fundamental frequency, the necessary energy will be a minimum.

The same as has been mentioned in connection with free-oscillations about aerodynamic balancing, correction for sting translations and wind-on wind-off testing is also valid for forced oscillation tests.

6 Coning motion

Certain cross and cross-coupling derivatives can be measured with a rig, which rotates a balance provided model with a constant angle velocity around the wind-vector at fixed α and β attitudes. This balance must also be equipped with SSGs, because the damping forces and moments are small in comparison with the total forces which load the balance. The deflection of the sting influences the position of the centre of rotation in the model.

Magnus-forces on a model may be determined if the model is forced to rotate around a fixed balance-sting. The balance should also in this case be provided with SSGs to get a reasonable measuring accuracy.

BALANCE CALIBRATION

An SG balance is an analog transducer. To interpret the outputs the balance must be calibrated against known forces and moments. Nor are evaluated measurements better than the accuracy of the calibration process. There is further no meaning in having better balance accuracies than the accuracies in the remaining elements, which are included in the wind tunnel measuring process.

As a rule wind tunnel balances are only statically calibrated. This is equivalent with a process in which all environmental parameters are constant and in which we slowly change the components in our force-system. The latter are in turn getting constant values and the corresponding outputs are recorded. Application of a multicomponent force system means that one must know both the sizes of the components and also their direction.

(20) describes the relations between the forces and moments, which load the balance and the corresponding outputs. The task of the calibration process is to apply a sufficient number of independent load-combinations to allow the evaluation of all the calibration coefficients

$$R = \frac{n}{2}(n + 3) \qquad (28)$$

where R = necessary number of independent load cases
n = number of components

The fact, that (20) is of 2:nd degree means that it is necessary to load the balance with both single loads and pairs of loads. The result of a calibration can be summarized in matrix form. (20) will take the form

$$[U] = [L][P] + [G][Q] \qquad (29)$$

where [U] = output matrix
[L] = matrix for linear coefficients
[P] = matrix for linear component terms
[G] = matrix for non-linear coefficients
[Q] = matrix for non-linear component terms

Evaluation of the matrices [L] and [G] means repeated solution of systems with linear equations. The accuracy of a calibration can be checked by applying a few load combinations with all components involved.

In the calibration process of an external balance single components in the force system will load more than one transducer element simultaneously in the balance. The matrix [L] will consequently not have a dominating diagonal.

External balances can often not be moved from their positions in the wind-tunnels. It is consequently necessary to make temporary set-ups when such a balance must be calibrated.

Internal balances are easier to calibrate than external balances. The current force system is oriented to the centre of the balance and can be simulated in a calibration rig (see Fig. 18). With proper bridge wiring an internal balance can have a [L] matrix with a dominating diagonal.

Sub-assembly balances are calibrated by use of temporary set-ups. It may be necessary to perform the calibration when the balance is mounted in the model, because the fixation of the balance in the model may influence the balance characteristics.

Balances for damping derivative measurements are also calibrated statically. This means that a balance together with its model must have its first fundamental frequency sufficiently far away from the current oscillation frequency. Damping moments can be applied by use of an electro-magnetic set up. Today this is used as a checking tool rather than a calibration device.

A mechanical calibration may be simulated with shunt-calibrations. In this case (see Fig. 19) a shunt is determined, which produces the same output as a certain mechanical loading. When the calibration results are transmitted from one recording system to another the following equation may be applied

$$k_2 = k_1 \cdot \frac{A_{s1}}{A_{s2}} \tag{28}$$

where k_1 and k_2 are corresponding calibration coefficients
A_{s1} and A_{s2} are outputs due to the same shunting of the bridge

Sometimes during a calibration process results are obtained which can only be explained with terms proportional to the absolute values of the interfering components. Such interactions are "false" and depend on plays between the balance and the orientation of the calibrating force system or depend on an insufficient knowledge of the direction of a force, when it changes sign.

WIND TUNNEL TESTING

A wind tunnel data acquisition system must have a signal conditioning unit, a computer and auxiliary equipment for data presentation. The layout of the wiring between the wind tunnel and the acquisition system is also critical.

The signal-conditioning unit must have a sufficient number of channels, must power the SG bridges and must condition the outputs to interface the computer. To the latter needs amplifiers, filters, a scanner and an analog to digital converter. When it concerns dynamic measurement a time measuring unit must also be included as well as a doubling of the amplifiers to allow the output to be divided in DC- and AC-parts. Formulation of mean values and dividing of dynamic AC-outputs in in-phase and out-of-phase components can be done in the computer.

Balance tests are evaluated by use of (29). The system of equations is now non-linear, because the force system is the unknown quantity. However (29) may be solved by use of an iteration process. Express (29) as

$$[P] = [L^{-1}][U]-[L^{-1}][G][Q] \quad (31)$$

and designate $[L^{-1}]= [B]$ and $[L^{-1}][G] =[C]$. The result will be

$$[P] = [B][U] - [C][Q] \quad (32)$$

[U] here represents the sum of the tare and the aerodynamic loads.

Is the non-linear elements [Q] small, a first approximation will be

$$[P]_1 = [B][U] \quad (33)$$

(33) and (32) permit a new solution

$$[P]_2 = [P]_1 - [C[[Q]_1 \quad (34)$$

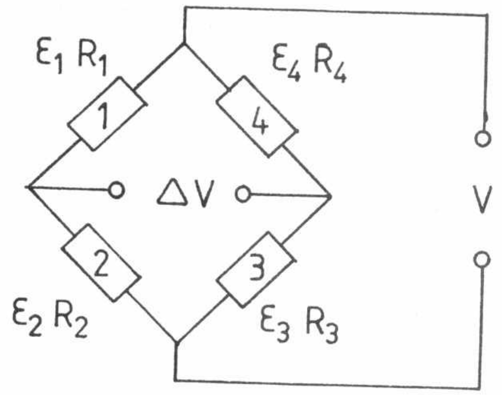
where $[Q]_1$ is calculated by use of (33).

Is the process repeated n times the solution will be

$$[P]_n = [P]_1 - [C][Q]_{n-1} \quad (35)$$

The process can be terminated when the difference between two consecutive solutions is smaller than a given tolerance.

The sting deflections must also be calibrated and be calculated in the data-evaluation process, if true α and β are not measured in the model.



$$R_1=[1+g(\varepsilon_1+\varepsilon_S)+k\cdot\varepsilon_1^2]$$

$$R_2=[1+g(\varepsilon_2+\varepsilon_S)+k\cdot\varepsilon_2^2]$$

$$R_3=[1+g(\varepsilon_3+\varepsilon_S)+k\cdot\varepsilon_3^2]$$

$$R_4=[1+g(\varepsilon_4+\varepsilon_S)+k\cdot\varepsilon_4^2]$$

Fig. 1: Wheatstone bridge consisting of 4 SGs with equal initial resistance.

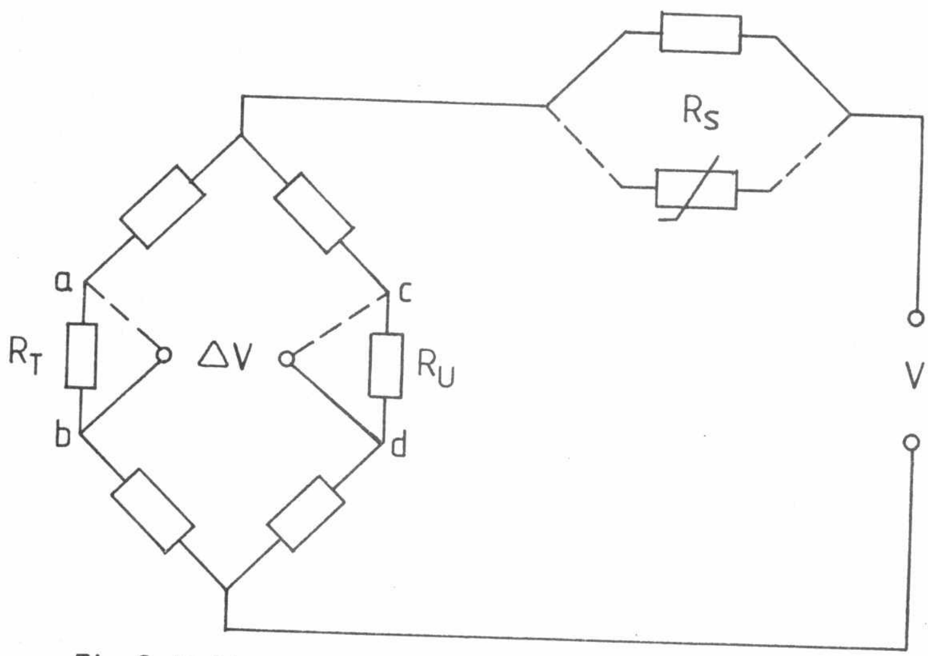


Fig. 2: Bridge with compensating resistances for:
R_T temperature zero drift
R_U initial unbalance
R_S bridge sensitivity

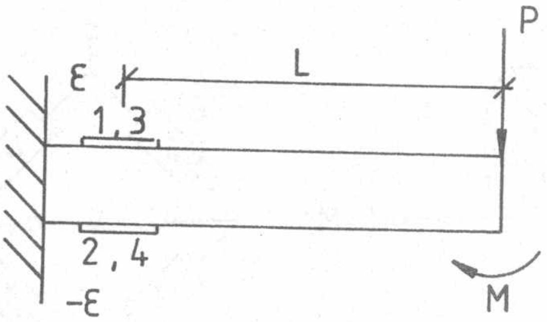


Fig. 3a

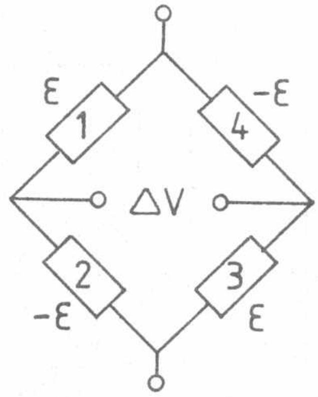


Fig. 3b

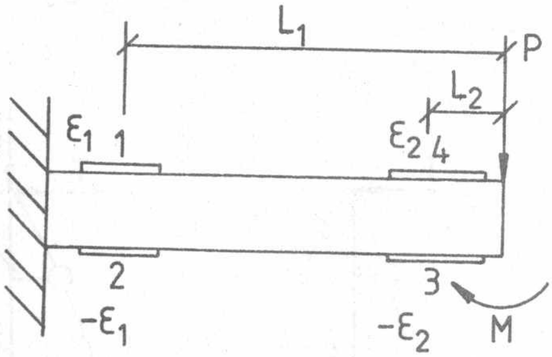


Fig. 4a

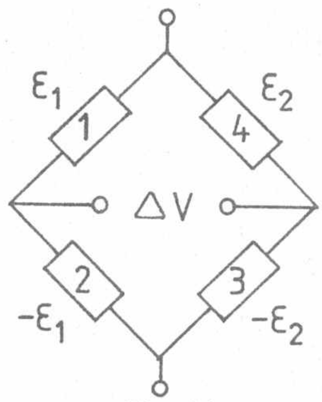
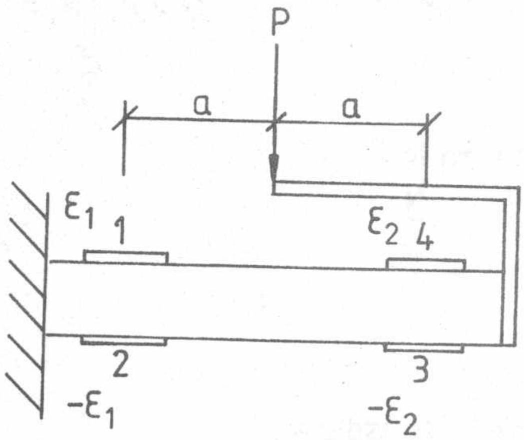


Fig. 4b



$\varepsilon_1 = -\varepsilon_2 = \varepsilon$

Fig. 5a

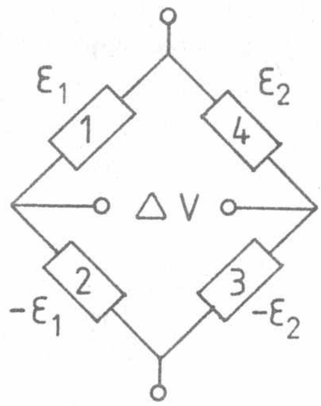
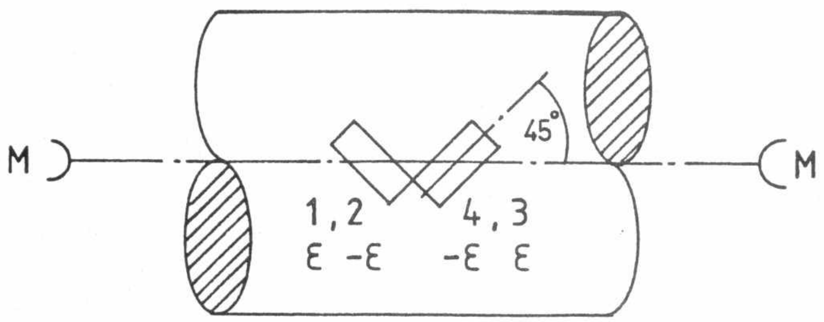


Fig. 5b



Gauges 1 and 4 on the front side
Gauges 2 and 3 on the back side

Fig. 6a

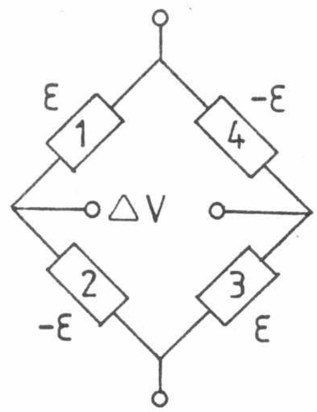
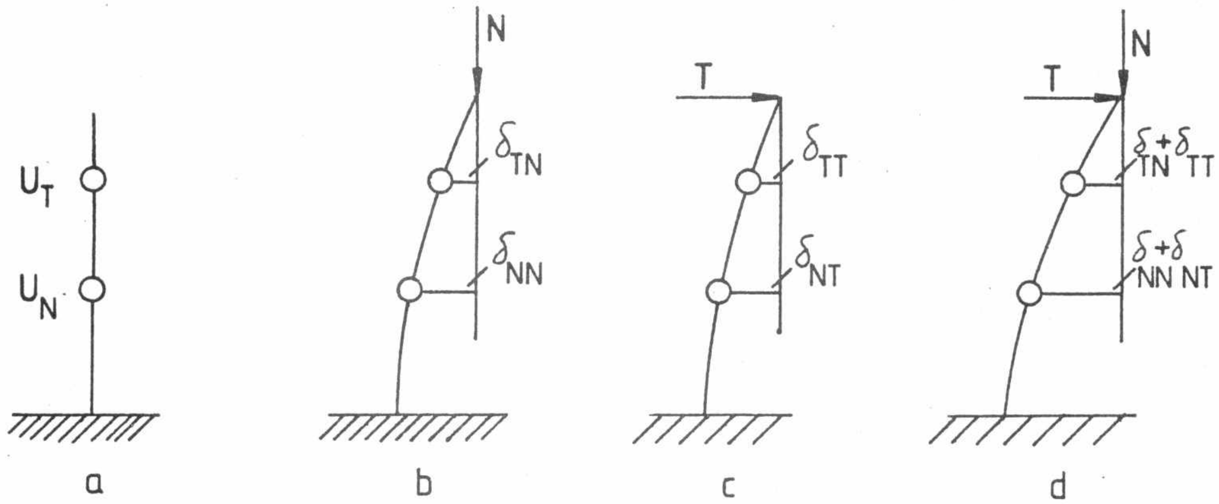


Fig. 6b



δ_{TN} = deflection of the T-element due to N
δ_{NN} = " " " N- " " " N
δ_{TT} = " " " T- " " " T
δ_{NT} = " " " N- " " " T

Fig. 7: Symbolic 2-component transducer.

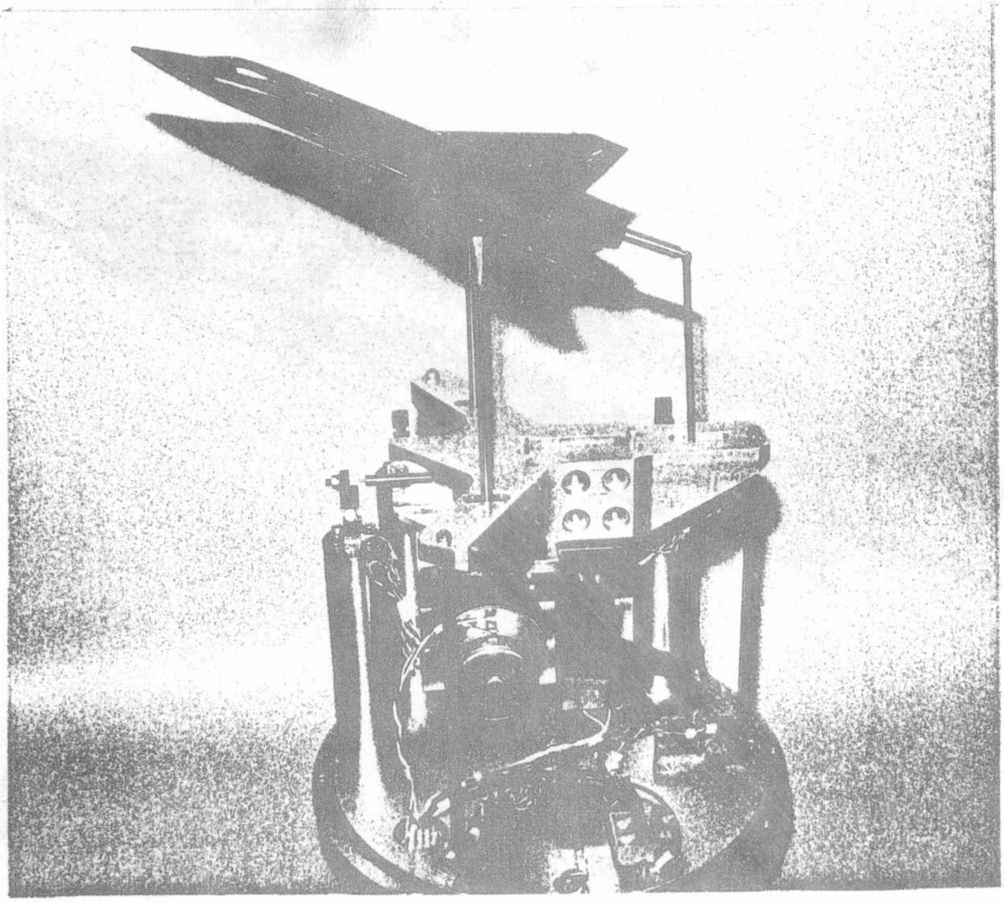
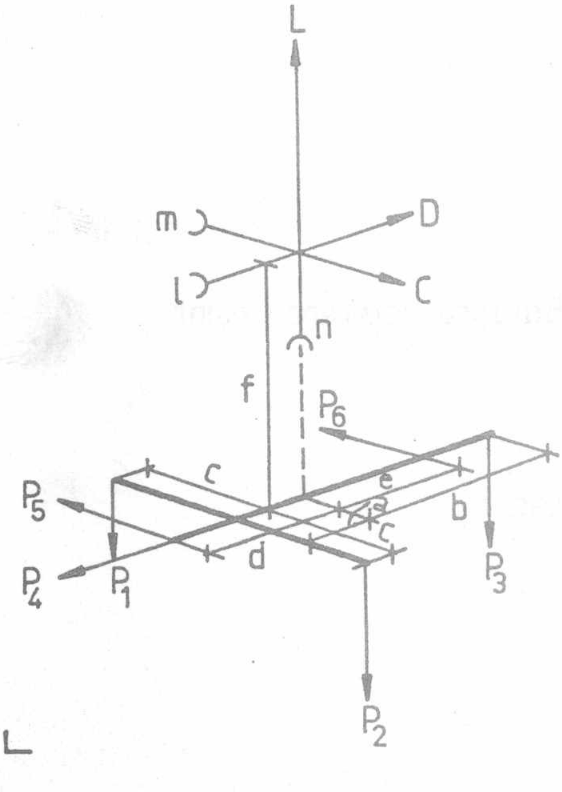


Fig 8a: External 6-component SG-balance.



$$L = P_1 + P_2 + P_3$$

$$D = P_4$$

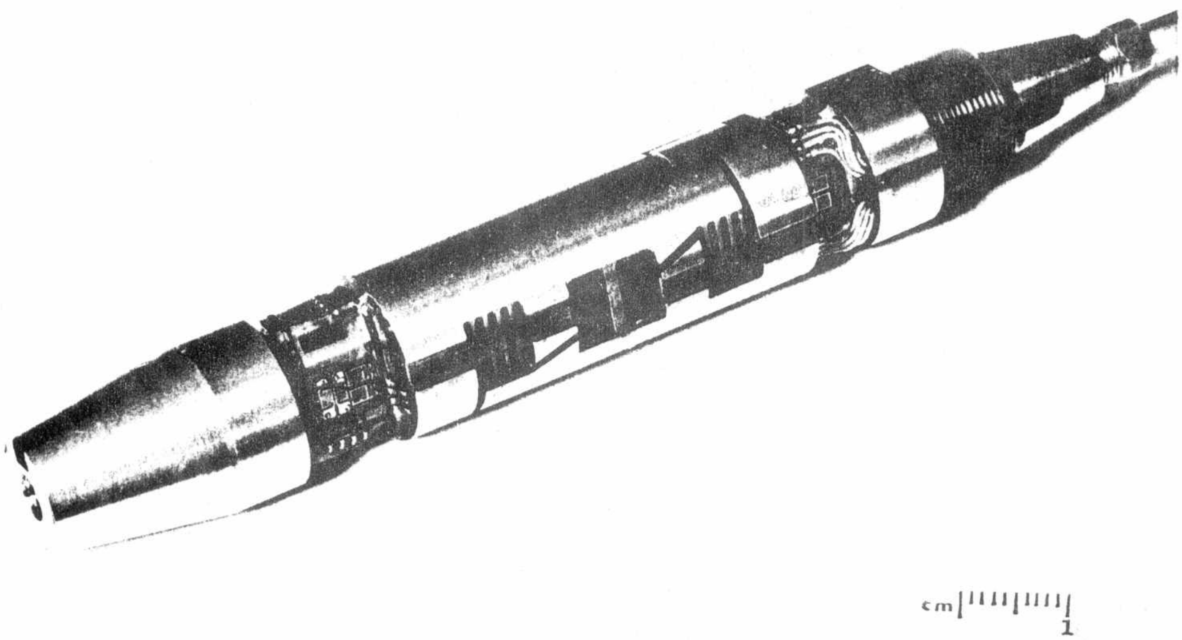
$$C = P_5 + P_6$$

$$m = a(P_1 + P_2) - b \cdot P_3 - f \cdot P_4$$

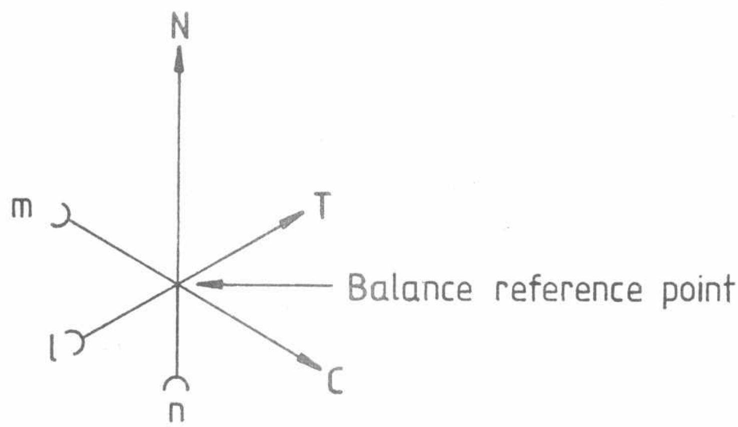
$$n = e \cdot P_6 - d \cdot P_5$$

$$l = c(P_2 - P_1) - f(P_5 + P_6)$$

Fig.8b: Relations between the aerodynamic force system and the balance force system.



Internal SG-balance of bending-beam type.



Force system on internal balances.

Fig. 9

6

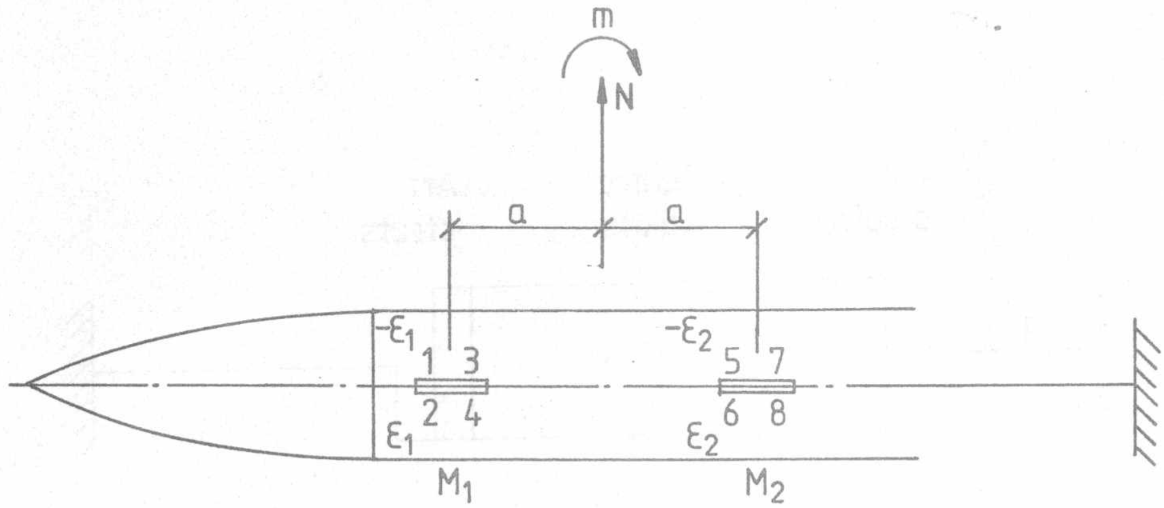


Fig. 10a

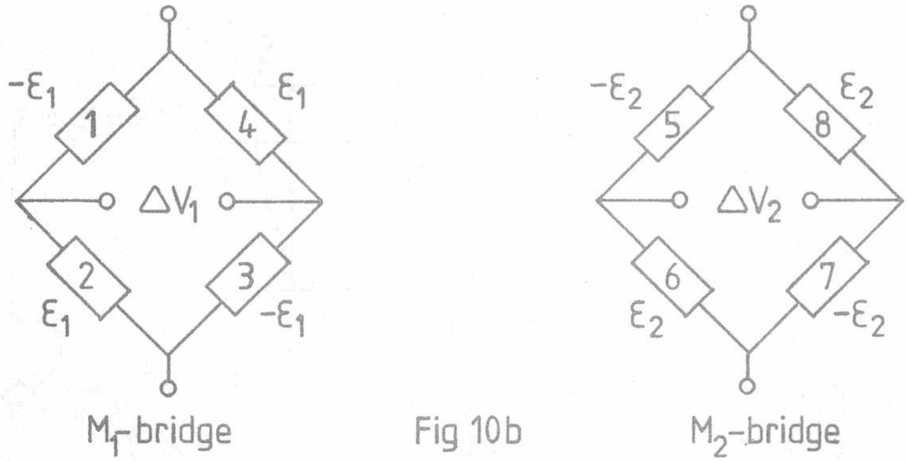


Fig 10b

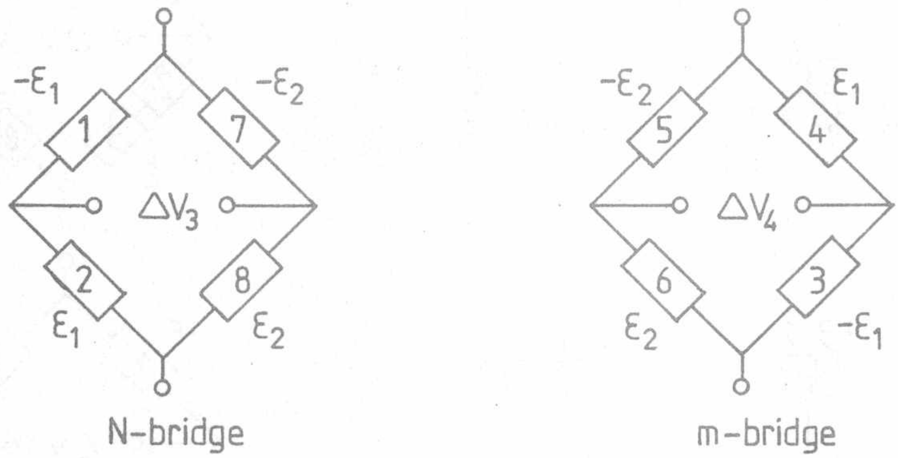


Fig. 10c

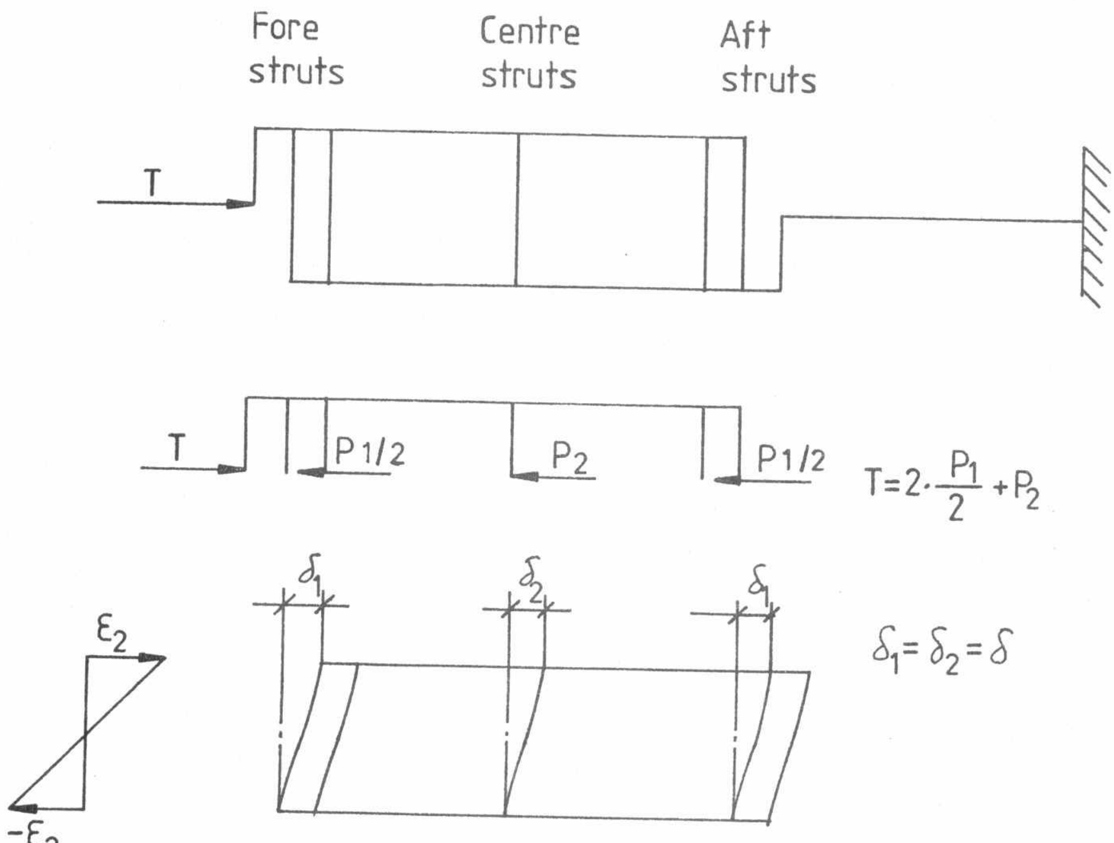


Fig. 11a

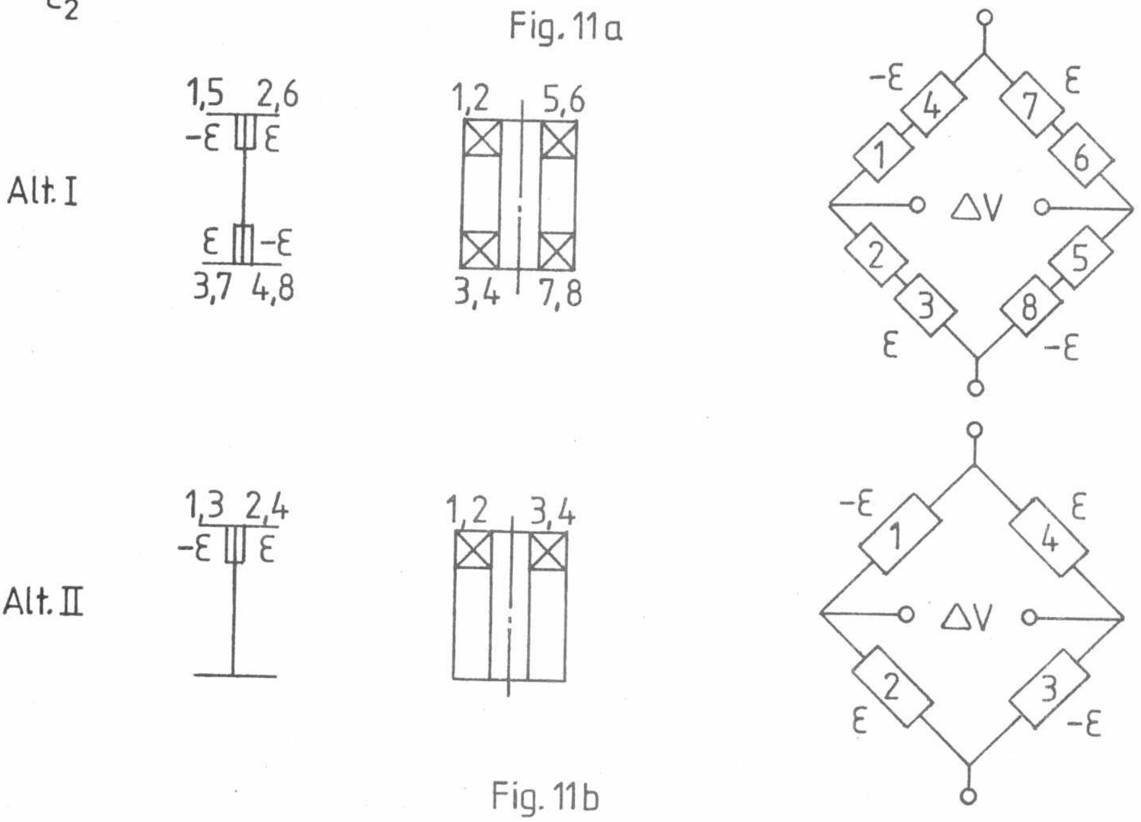


Fig. 11b

6

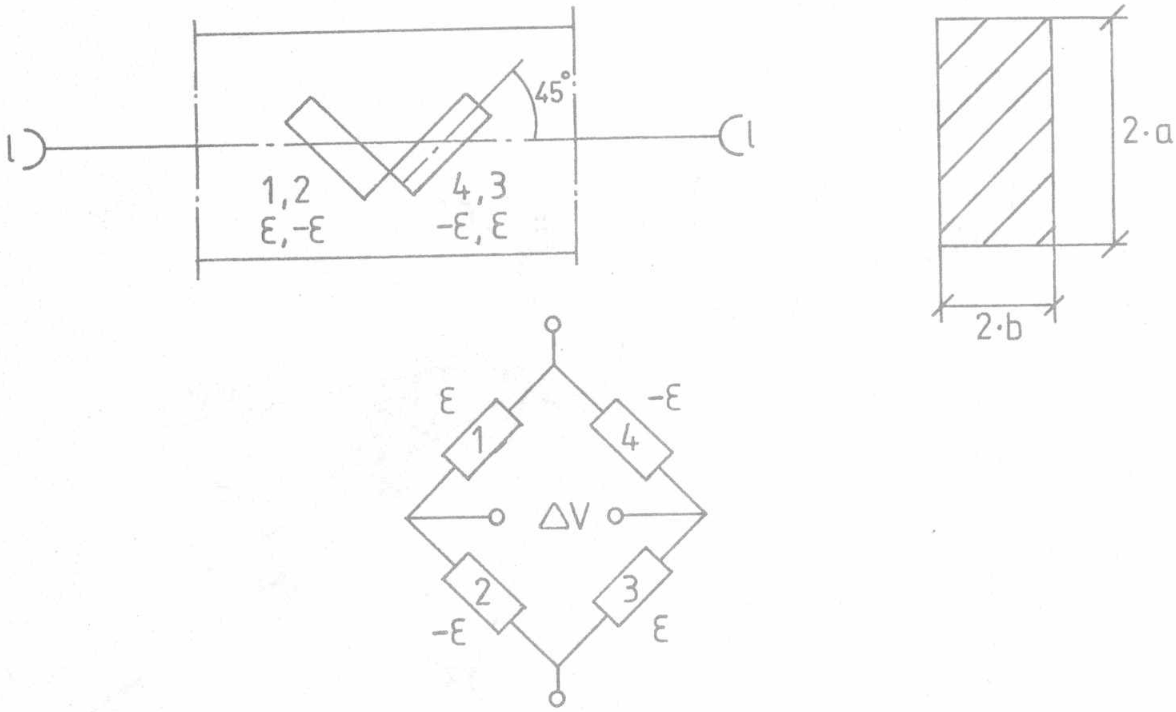


Fig. 12

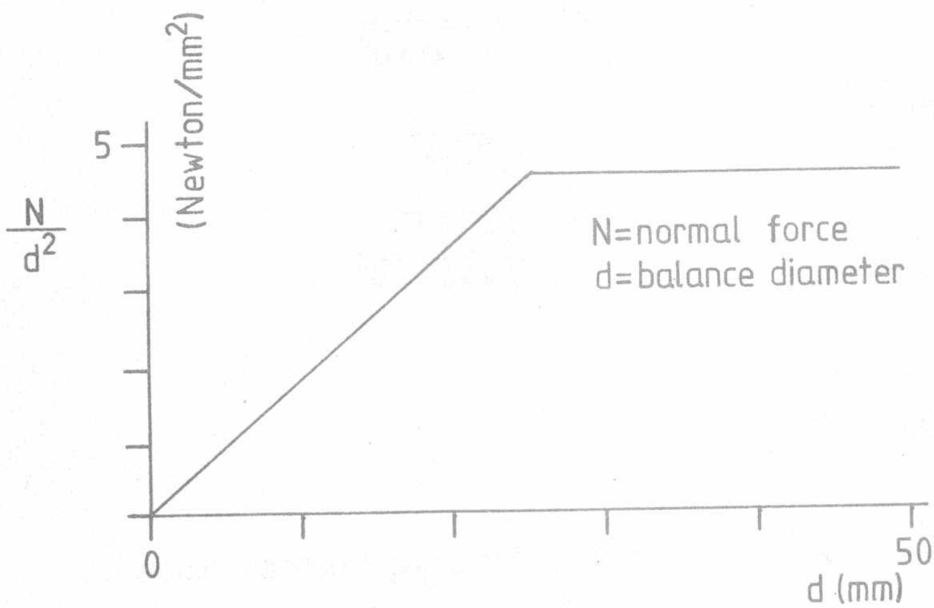
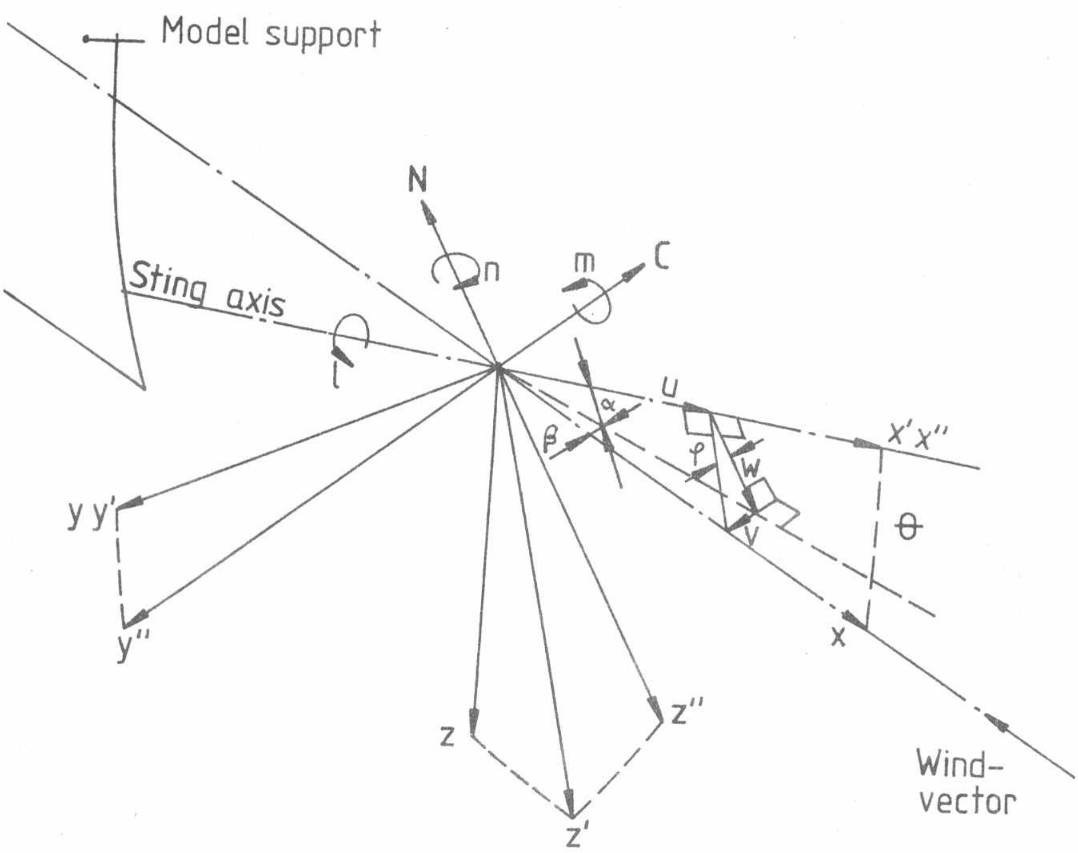


Fig. 13: Approximate limit curve for layout of bending-beam balances.



Geometrical relations between aeronautical angles α, β and wind tunnel angles θ, φ

$$tg\alpha = \frac{w}{u} \qquad \sin\beta = \frac{v}{\sqrt{v^2+w^2+u^2}}$$

$$tg\theta = \frac{\sqrt{v^2+w^2}}{u} \qquad \sin\varphi = \frac{v}{\sqrt{v^2+u^2}}$$

$$\cos\varphi = \frac{w}{\sqrt{v^2+w^2}} \qquad \sin\theta = \frac{\sqrt{v^2+w^2}}{\sqrt{v^2+w^2+u^2}}$$

or

$$\left.\begin{aligned} tg\alpha &= \cos\varphi \cdot tg\,\theta \\ \sin\beta &= \sin\varphi \cdot \sin\theta \end{aligned}\right\}$$

Fig.14: α, β as functions of θ, φ for sting mounted models.

6

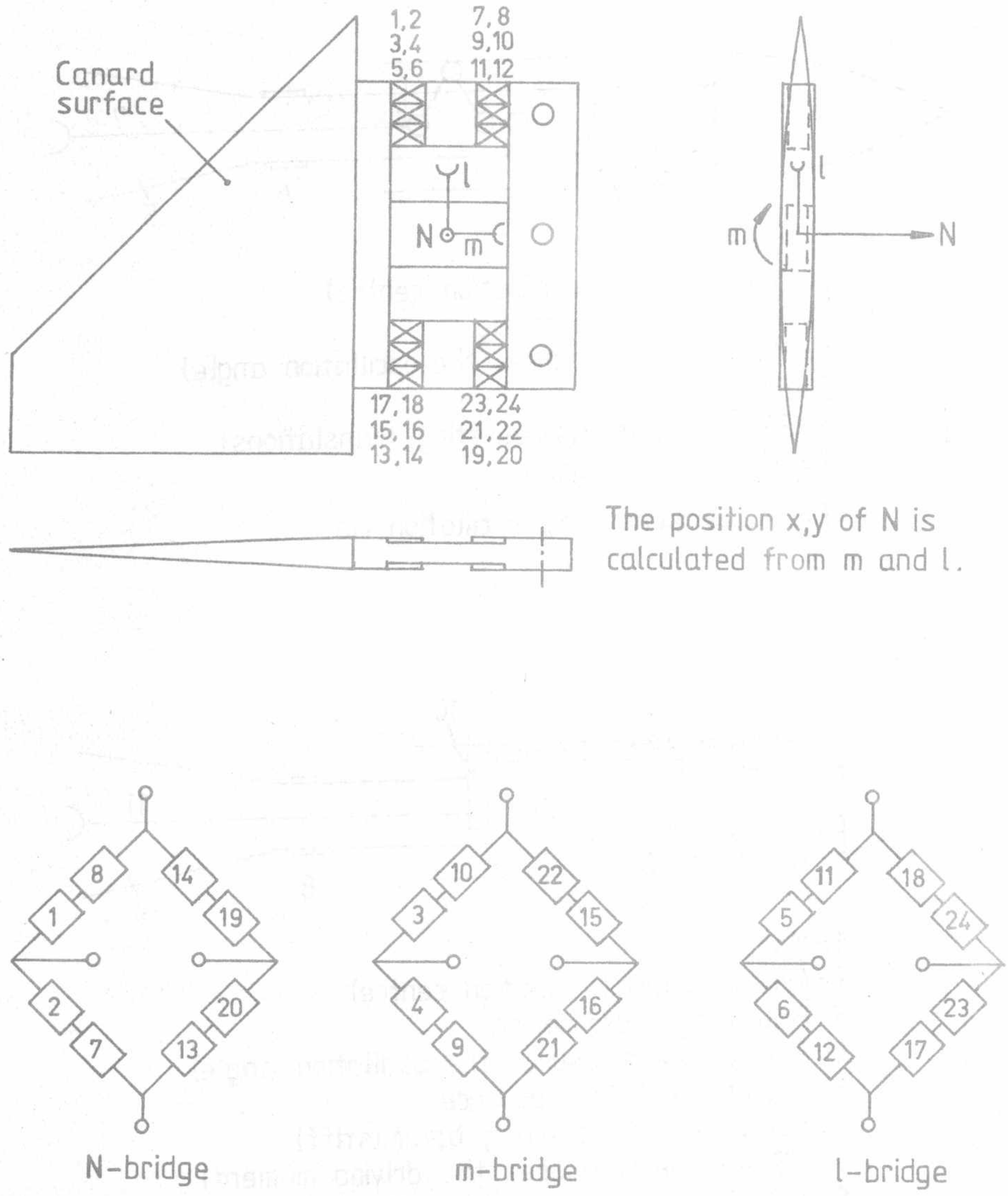
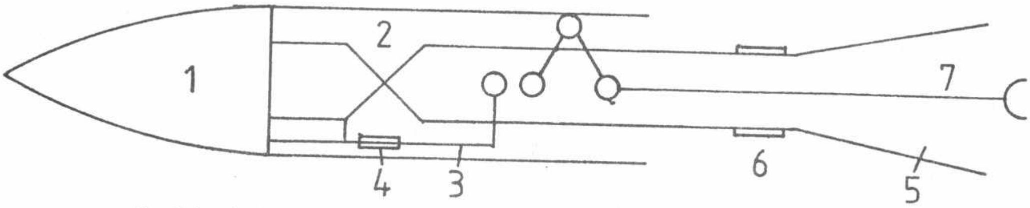
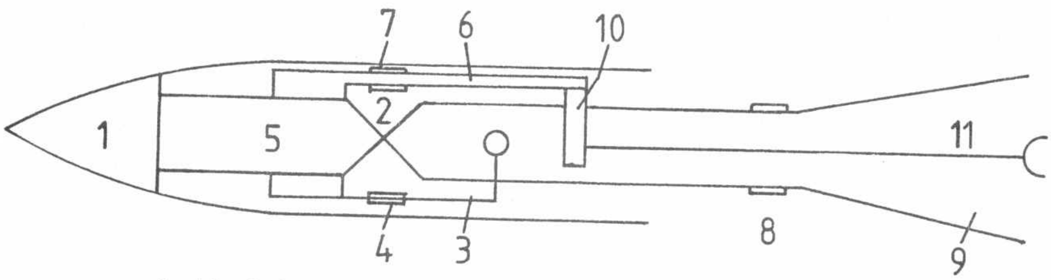


Fig. 15: Example of sub-assembly balance.



1=Model
2=Cross-flexure (oscillation centre)
3=Balancing plate spring
4=FSG-bridge (measures the oscillation angle)
5=Sting
6=SSG-bridge (measures sting translations)
7=Trigger mechanism

Fig 16: Symbolic free-oscillation rig



1=Model
2=Cross-flexure (oscillation centre)
3=Balancing plate spring
4=FSG-bridge (measures the oscillation angle)
5=5-component SSG-balance
6=Oscillation transmitting beam (stiff)
7=SSG-bridge (measures the driving moment)
8=SSG-bridge (measures the sting translations)
9=Sting
10=Rotating eccentric disc (rotation to oscillation transformer)
11=Rotating shaft (constant angular speed)

Fig. 17: Symbolic forced-oscillation rig

6

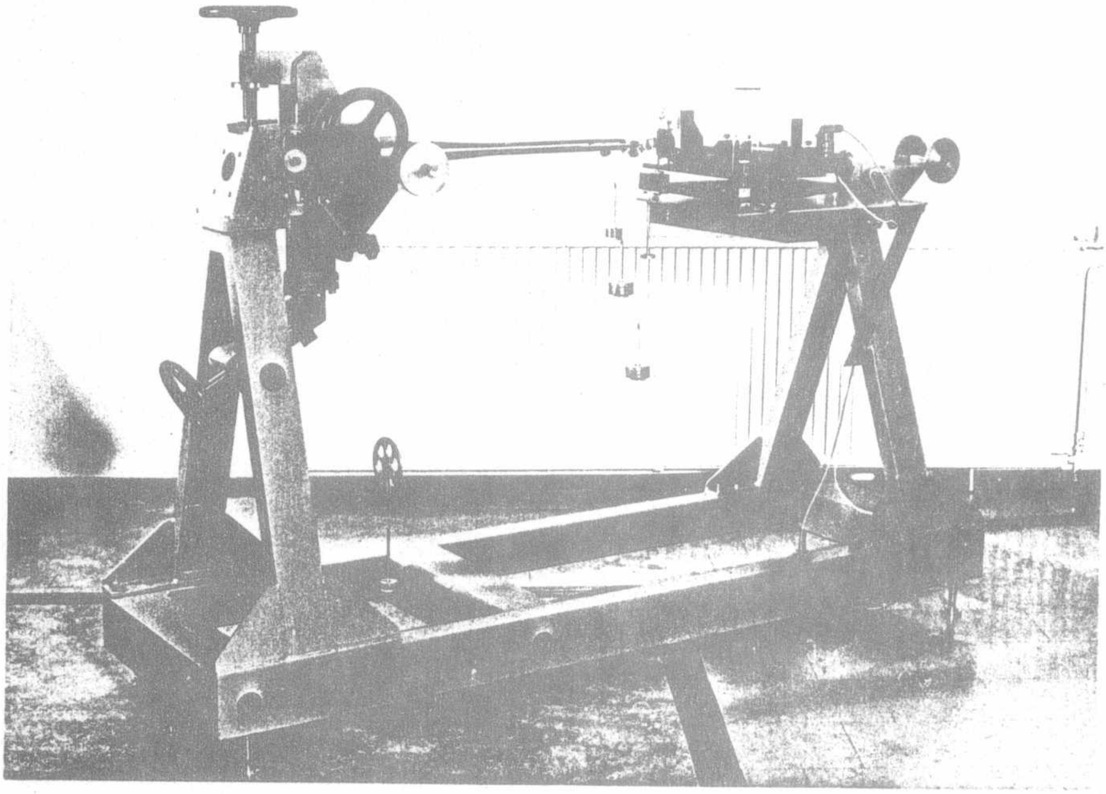
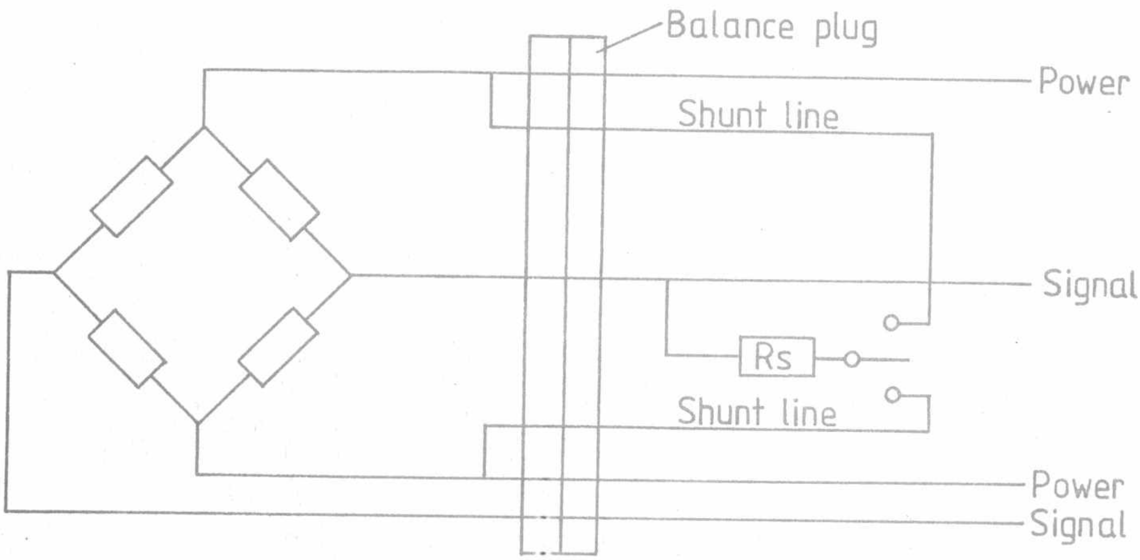


Fig 18: Calibration rig for internal SG-balances. The pitch and roll positions of the dummy-model are optically indicated.



Rs = shunt resistance

Fig. 19: Principle lay-out of a shunt-calibration system for full-bridges