



ALGEBRAIC PREDICTION OF TURBULENT BOUNDARY

LAYERS IN ADVERSE PRESSURE GRADIENT

A.M. EL-KERSH *

ABSTRACT

An attempt has been made to develop a method for predicting the development of turbulent boundary layers in adverse pressure gradient. Simple calculation procedure is presented by incorporating a deceleration rate parameter into boundary layer equations. The compressibility effect of the flow is considered by adopting the Mach number in the momentum and skin friction relations. The predictions of the boundary layers are in good agreement with available experimental results, which validates the calculational technique.

* Lecturer, Mechanical Engineering Dept., Faculty of Engineering and Technology, Minia University, Minia, Egypt.

INTRODUCTION

The behaviour of the turbulent boundary layer in an adverse pressure gradient is probably one of the most important and also one of the most difficult problems in fluid mechanics. Almost all devices involving fluid flow, such as pumps, diffusers, compressors, airfoils and submerged bodies, depend critically upon this behaviour. The effort that has been expended on this one problem gives evidence of the difficulty as well as the importance.

In an adverse pressure gradient the fluid very near the solid surface that has been retarded by viscous forces quickly loses its remaining momentum. When this happens the boundary layer is likely to separate, or stall, and completely change the flow. The loss of momentum by the fluid near the wall, then, is the significant factor in the behaviour of the boundary layer. Because of the mixing motion, the rate of transfer of momentum to the inner layers is much greater in turbulent flow, and separation is usually delayed relative to the laminar case.

A number of schemes for turbulent incompressible, two-dimensional boundary layer predictions have been reviewed by research workers [1-4]. Such prediction methods are classified to differential and integral techniques. The basis of the differential techniques is the solution of the equations of motion at each point in the boundary layer, the distribution of shear stress being obtained through a few semi-empirical relations. The integral techniques form the largest class of the methods which use the momentum integral equation. The boundary layer thicknesses of such equation are usually related to each other by integrating a semi-empirical expression for the velocity profile. The integral techniques are divided into dissipation or moment methods depending on the selection of the second integral equation. Also another method of the integral technique is based on the selection of the entrainment equation.

One of the methods of integral techniques has been extended to account for the flow compressibility on the boundary layer development [5,6]. The extension of such methods to compressible flow is based primarily on the interpretation that turbulence structure is essentially unaffected by compressibility. Also such methods are restricted to compressible adiabatic flow.

The present paper describes a calculation procedure for compressible turbulent boundary layer in adverse pressure gradient. The method adopts the approach of Senoo and Nishi [7] by using a handy parameter which controls the development of the

boundary layer for decelerating flow.

DECELERATION RATE

In steady two-dimensional compressible flow, the momentum integral equation of turbulent boundary layer is classified as

$$\frac{d\theta}{dx} = -(2+H-M^2) \frac{\theta}{U} \frac{dU}{dx} + C_f/2 \quad (1)$$

For small deceleration rate, the increment of momentum thickness is proportional to the length of integration dx .

In decelerating flow, the increment of momentum thickness $d\theta$ is proportional to the change of velocity dU and it is slightly affected by the length dx . Following Senoo and Nishi [7], the number of steps required to integrate equation (1) can be minimized if the increment of the momentum thickness is approximately constant for each step of integration. The step length is chosen so that $-(1/U)(dU/dx)dx = -dU/U$ is constant. Accordingly the distance where the free-stream velocity is decreased by 10 percent is defined by λ , and equation (1) is integrated for the distance $dx = \lambda$. As the mean value of the velocity gradient is expressed as $(1/U)(dU/dx) = -0.1/\lambda$ the above equation is given by

$$\begin{aligned} \frac{\Delta\theta}{\bar{\theta}} &= (0.2 + 0.1\bar{H} - 0.1\bar{M}^2) + 0.5\bar{C}_f \lambda / \bar{\theta} \\ &= \Delta\theta_d / \bar{\theta} + \Delta\theta_f / \bar{\theta} \end{aligned} \quad (2)$$

where $\Delta\theta_d$ and $\Delta\theta_f$ indicate the increments of momentum thicknesses caused by the deceleration and the wall friction force respectively.

The variation of $\Delta\theta_d$ and $\Delta\theta_f$ with respect to the deceleration rate parameter $\bar{\theta}/\lambda$ is shown in Fig. 1. The influence of friction on the increment of momentum thickness is very small for deceleration rate greater than 0.01. Such influence is slightly affected by increasing Mach number.

BASIC EQUATIONS

Momentum Equation

For a large deceleration rate, equation (1) can be integrated over a distance λ neglecting the variation of friction on the shape factor terms. This gives

$$\theta_2/\theta_1 = (U_2/U_1)^{-(2+H_1-M^2)} \quad (3)$$

This equation is corrected to be extended for the cases of

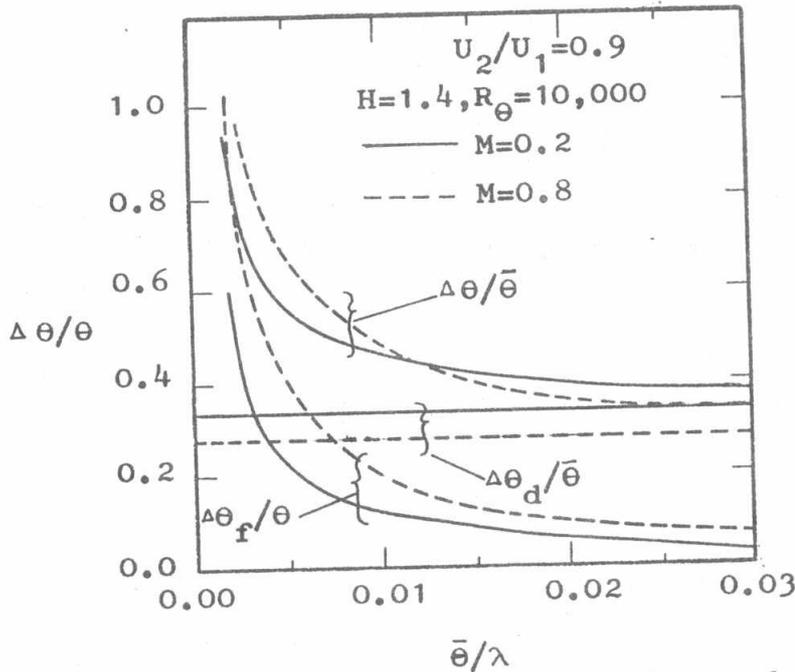


Fig.1 Contribution of friction and of deceleration on increment of momentum thickness.

small deceleration rate which is given by

$$\theta_2/\lambda = (k\theta_1/\lambda)(U_2/U_1)^{-(2+H_1-M^2)} + 0.5C_{f1} \quad (4)$$

The correction coefficient k is chosen to be 1.02 as suggested by Senoo and Nishi [7].

Skin Friction Relation

Skin friction coefficient is determined following East, Smith and Merryman [6]. The skin friction in general flow is related to skin friction on a flat plate by empirical expression

$$((C_f/C_{f0})+0.5)((H^*/H_0^*)-0.4)=0.9 \quad (5)$$

where suffix 0 indicates value in zero pressure gradient. Following Winter and Gaudet [8], the relation of C_{f0} for the compressible flow is given by

$$F_c C_{f0} = \frac{0.01013}{\log_{10}(F_r R_\theta) - 1.02} - 0.00075 \quad (6)$$

where $F_c = (1+0.2 M^2)^{\frac{1}{2}}$

and $F_r = 1+0.056 M^2$

The value of C_{f0} is shown in Fig.2 which is decreases by increasing the Mach number for a certain value of the R_θ

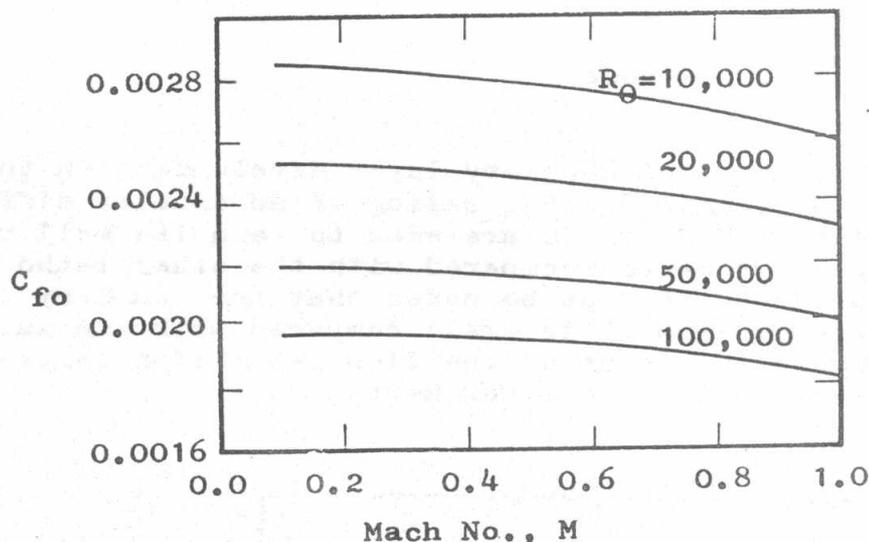


Fig.2 Effect of Mach number on the skin friction coefficient.

To complete the skin friction relation, the shape parameters are given by

$$1 - 1/H_o^* = 6.55(0.5C_{fo}(1 + 0.04M^2))^{\frac{1}{2}} \quad (7)$$

$$1 + H = (1 + H^*) (1 + 0.2rM^2) \quad (8)$$

Equations(5-8) represents a set of relations to calculate skin friction coefficient for compressible flows.

Shape Factor Relation

A relation of shape factor for decelerating incompressible flow is given in Ref. [7]. Such relation can be extended to compressible flow by making assumptions similar to those used in extending Head's method to compressible flow [6]. Thus taking H as the analogue in compressible flows for H at low speeds.

$$\Delta H^* = H_2^* - H_1^* = 0.2H_1^* (1.65 - H_1^*) + (H_1^* - 1.25)^2 \times (\log_{10} \theta_1 / \lambda + 1.9) (\log_{10} R_{\theta 1}) \quad (9)$$

PREDICTION OF BOUNDARY LAYER

Equations(4-9) are used to predict the development of boundary layer along two-dimensional walls. The length of wall is divided into segments, so that the free stream velocity at the downstream end of the segment are used as the values at the upstream end of the next segment, and the calculation is repeated.

To assess the performance of the method, its predictions have been tested against experimental data. The aim is to illustrate the general accuracy of the method both in absolute terms and in

comparison with other methods.

Fig.3 shows predictions of boundary layer development in the experiments by the author [9] on a casing of an annular diffuser. The distributions of H , C_f and θ are seen to be quite well predicted by the present method compared with the other method and the experimental data. It must be noted that the boundary layer thickness in such experiment is small compared with the wall radius. Furthermore the point of the flow separation is predicted in good agreement with the experimental data.

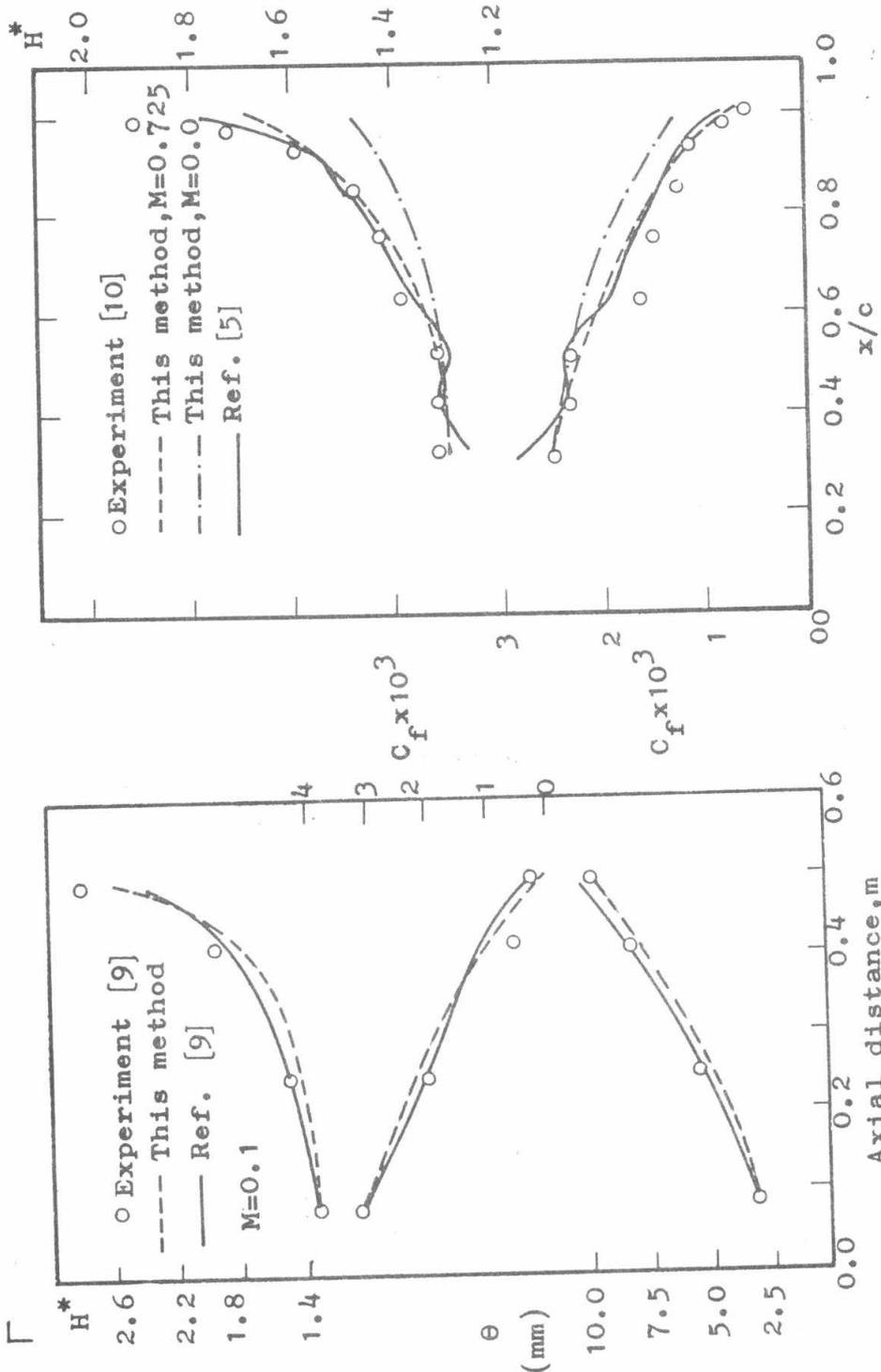


Fig.4 Boundary layer development on a lifting aerofoil.

Fig.3 Boundary layer development on the casing of annular diffuser.

Fig.4 shows prediction of boundary layer development in the experiments by Cook [10] on a lifting aerofoil at high subsonic speed. The calculations made by Green, Weeks and Brooman [5] for the distributions of H and C_f on upper surface of the aerofoil are shown to be in good agreement with predictions of the present method and the experimental data. It is also noted that agreement between the present method and the measurements is provided by allowing for the effect of Mach number.

CONCLUSIONS

Simple calculation procedure is presented for predicting the development of turbulent boundary layers in adverse pressure gradient. The method provides a simple technique for Engineers to use a hand calculator. The prediction method using the decelerating rate parameter is applicable to flow at high subsonic speeds . Also provides good predictions for the flow separation.

NOMENCLATURE

C_f	Skin friction coefficient
H_*	Shape factor, δ^*/θ
H	Velocity profile shape factor
M	Mach number
r	Temperature recovery factor
R_0	Reynolds number, U_0/ν
U	Free stream velocity
x	Distance along wall
Δ_*	Increment which U is reduced to $0.9U$
δ	Displacement thickness
θ	Momentum thickness
λ	Distance where free stream velocity is reduced by 10%
ν	Fluid kinematic viscosity

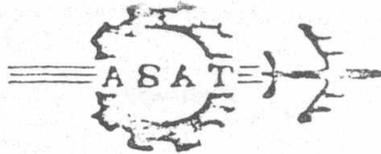
Subscripts

0	Values in zero pressure gradient
1	Upstream end of λ
2	Downstream end of λ

REFERENCES

1. Kline, S., "Proceedings Computation of Turbulent Boundary Layers", 1968 AFOSR-IFP-Stanford conference, Stanford Univ. vol.1 (1968)
2. Wheeler, A.J., and Johnston, J.P., "An Assessment of Three-Dimensional Turbulent Boundary Layer Prediction Methods", J. of Fluids Eng., 415-421 (1973)
3. Kwon, O.K., and Pletcher, R.H., "Prediction of Incompressible Separated Boundary Layers Including Viscous-Inviscid Interactions", J. of Fluid Eng. 101, 466-472 (1979)

4. El-Kersh, A.M., and El-Gammal, A.H., "Boundary layer Calculations in Annular Diffusers With Swirled Flow", The Bulletin of the Faculty of Eng., Alex. Univ. ,XX, 267-295 (1981)
5. Green, J.E., Weeks, D.J., and Brooman, J.W.F., "Prediction of Turbulent Boundary Layers and Wakes in Compressible Flow by a Lag-Entrainment Method", R.&M. No. 3791, London (1977)
6. East, L.F., Smith, P.D., and Merryman, P.J., "Prediction of Development of Separated Turbulent Boundary Layers by the Lag-Entrainment Method", TR 77046, London (1977)
7. Senoo, Y., and Nishi, M., "Deceleration Rate Parameter and Algebraic Prediction of Turbulent Boundary Layer", J. of Fluids Eng. , 390-395 (1977)
8. Winter, K.G., and Gaudet, L., "Turbulent Boundary-Layer Studies at High Reynolds Numbers at Mach Numbers Between 0.2 and 2.8", A.R.C. R.&M. 3653, London (1969)
9. El-Kersh, A.M., "Investigation into Design and Performance of Annular Diffusers", Ph.D. Thesis, Paisley College of Technology, Scotland (1983)
10. Cook, T.A., "Measurements of the Boundary Layer and Wake of Two Aerofoil Sections at High Reynold Numbers and High Subsonic Mach Numbers", A.R.C. R.&M. 3722, London (1971)



A CRITICAL STUDY OF TAYLOR'S PARAMETER OF TRANSITION
OF BOUNDARY LAYERS WITH A TURBULENT FREE STREAM*.

Dr. ABDEL KAREEM , M.S.E.**

Summary

Taylor's parameter for predicting transition is studied .
Following a more recent turbulence analysis, the Taylor's
parameter is modified to be :

$$R_x \text{ (transition)} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{x}{M} \right)^{1/6} \right)$$

It is found that the parameter as such does not correlate
the transition results , although it represents the
correct effect of the constituent variables on transition.
Based on tentative argument, the parameter is reshaped to
be of the form :

$$R_x \text{ (transition)} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{\theta}{L_x} \right)^{1/6} \right)$$

which gives a better collapse of the data . It is noted
that the physical basis of Taylor's parameter has experi-
mentally been proven to be incorrect .

* The reported results is a part of the auther's work
which is reported in his Ph.D. thesis .

**Presently, a lecturer in the Aeronautical department ,
Faculty of engineering , Cairo university .

NOMENCLATURE.

L_x	The stream-wise integral length scale of turbulence .
M	The mesh size of the grid that produces the turbulence .
p	Pressure
R_x	The Reynolds' no. based on X .
R	The Reynolds' no. based on .
t	The plate thickness .
u	The stream-wise velocity fluctuations .
U	The stream-wise mean velocity component .
X	The stream-wise distance over the surface measured from the leading edge .
δ	The boundary layer thickness .
λ_η	The diffusion length scale .
μ	Viscosity .
ν	Kinematic viscosity .
θ	The boundary layer momentum thickness .
ρ	Density .
Λ	The fluctuating pressure gradient parameter .
∞	Refers to the free stream .

INTRODUCTION .

A theory was put forward by G.I. Taylor (1936) which assumed that the finite disturbances in the free stream were the principle factor in causing transition . That is, transition is caused by momentary separation (or points of inflexion in the velocity profiles) in the region of a adverse pressure gradient associated with the turbulent velocity fluctuations at sufficiently large values of the Karman-Pohlhausen parameter $(\frac{\rho^2}{\mu U} \frac{dP}{dx})$. Using this assumption, Taylor showed that the critical Reynolds' no. of transition was a function of $(\frac{\sqrt{u^2}}{U} (\frac{x}{M})^{1/5})$.

Taylor's theory predicted transition on a sphere reasonably well, and also that on elliptic cylinders (Schubauer (1938)). Nevertheless, the theory failed to correlate transition over the flat plate of Hall et al (1938), despite the fact that the theory was basically for flat plates . Yet it was held by some investigators, notably Dryden (1947), to account for transition at high free stream turbulence levels of 0.5 % and over, where the Tollmien-Schlichting stability theory ceased to be valid. In an attempt to improve Taylor's parameter correlation of the transition results, Fage et al (1941) suggested the use of the local parameter $(\frac{\theta}{L_x})$ at transition instead of $(\frac{x}{M})$ in Taylor's parameter. But to the best of my knowledge, the usefulness of this suggestion was not tried .

Moreover, Klebanoff et al (1959) found no sign of intermittent separation before transition even at a free stream turbulence level of 0.8% .

Eventually, nearly all the investigators (such as Wells (1967), Hall (1968), Spangler et al (1968), and Michel (1974) and others correlated their transition results

using the free stream turbulence intensity directly and did not use Taylor's parameter at all . While some other investigators (e.g. Hall et al (1972) and McKeough(1976)) reported more scatter when their transition data was correlated with Taylor's parameter rather than the free stream turbulence intensity directly .

The purpose of the present work is to investigate experimentally as well as analytically the validity of Taylor's parameter .

EXPERIMENTAL APPARATUS AND TECHNIQUES.

The start of transition was detected on the surface of a $1\frac{1}{2}$ inch thick smooth flat plate fitted with an elliptic nose 8 inches long . The hot wire anemometer was used to detect transition as the start of amplification of small disturbances or the start of appearance of high frequency bursts in the laminar boundary layer , using a technique described in detail in Abdel-Kareem (1978) .

Also bi-planner grids were used to generate free stream turbulence in the test section of the 3x3 feet wind tunnel used in the experiment .

MODIFICATION OF TAYLOR'S PARAMETER.

Taylor, in his analysis, expressed the mean square pressure fluctuations in a turbulent flow in the form :

$$\left(\overline{\left(\frac{\partial P}{\partial x} \right)^2} \right)^{\frac{1}{2}} = \sqrt{2} \rho \frac{u^2}{\lambda_{\eta}}$$

where (λ_{η}) is the length determined in the diffusion experiments to be of the form :

$$\frac{\lambda}{M} = \left(\frac{\nu}{M u} \right)^{\frac{1}{2}}$$

Using a similar approach to the one used in the Karman-Pohlhausen theory of the boundary layer, Taylor determined the fluctuating pressure gradient parameter $(\hat{\Lambda})$ to be given by :

$$\hat{\Lambda} = - \frac{34 X}{\rho U^2} \left(\frac{\partial p}{\partial x} \right)$$

Then Taylor concluded that

$$R_{xT} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{x}{M} \right)^{1/5} \right)$$

But, more recent experiments (Batchelor, 1953) shows that :

$$\frac{\lambda}{M} \propto \left(\frac{U M}{\nu} \right)^{1/2} \frac{\nu}{u M}$$

So that , following Taylor's analysis, we get that :

At transition

$$R_{xT} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{x}{M} \right)^{1/6} \right)$$

Which suggests that the length scale of turbulence has a slightly weaker effect on transition than suggested by the original Taylor's parameter .

RESULTS AND DISCUSSION .

Figures 1&2 show the intensity $(\sqrt{u^2}/U_\infty)$ versus the Reynolds' no. $(U_\infty x/\nu)$ & $(U_\infty \theta/\nu)$ respectively . Although nearly all the workers in the field calculated (θ_T) from the laminar boundary layer relations, it was always measured in the present work (i.e. calculated from the measured mean velocity profiles at transition). This is because the free stream turbulence induced some variations in the values of the boundary layer integral parameters, and this may be why figure 2 shows a better collapse of the data than figure 1 . The two figures show that $(\sqrt{u^2}/U_\infty)$ represents a major parameter in deciding transition, while the scatter is due to the fact that $(\sqrt{u^2}/U_\infty)$ may not be sufficient to correlate the occurrence of transition .

Figure 3 shows the dependence of $(R_{\theta T})$ on (L_x/t) , where (t) is the plate thickness (which is constant for the present experiment). Also shown in the same fig. is the line formed by the triangular symbols . This line is obtained by plotting the values of $(\sqrt{u^2}/U_\infty)$ belong-

ing to each curve in the figure at the $(R_{\Theta T})$ of the intersection of the lines $L_x / t = \text{constant}$ and the respective curve . The process is repeated for

$$(L_x / t) = 0.5 \ \& \ 0.6$$

We observe the following :

- * The interesting linearity of the curve of transition at constant (L_x) suggests that it is the changes in (L_x) that cause the scatter of the data in fig.2 and possibly cause the relation between $(\sqrt{u^2} / U)$ and R_T to deviate from linearity .

- * It is shown that at constant $(\sqrt{u^2} / U)$

$$R_T \propto L_x / t$$

i.e., the laminar boundary layer is less sensitive to large scale turbulence , which is a reasonable observation because the boundary layer will tend to see large scale turbulence as unsteadiness in the free stream . While at the same $(\sqrt{u^2} / U)$, a large (L_x) will tend to cause less disturbance in the boundary layer . It may also be said that the intensity of the pressure fluctuations associated with grid turbulence increases as (L_x) is decreased at constant $(\sqrt{u^2} / U)$.

Figure 4 shows the ratio (Θ / L_x) at transition as a function of (R_{XT}) with $(\sqrt{u^2} / U)$ as a parameter . The plot shows that in general, at the same intensity, transition is more sensitive to smaller values of (L_x / Θ) and this conclusion is expected to be true until a certain limit at which the Tollmien-Schlichting instability processes become more evident . This observation is in line with fig.3 . Figure 4 also shows that R_{XT} is , in general, less sensitive to changes in (Θ / L_x) at smaller

values of $(\sqrt{u^2}/U)$.

TRANSITION PARAMETER .

We have seen above that :

$$R_T = f(\sqrt{u^2}/U, \theta/L_x)$$

Hence, any parameter correlating the transition with the free stream turbulence should have, besides $(\sqrt{u^2}/U)$, a representative length of the boundary layer in the numerator and another representative length of the free stream turbulence in the denominator. Taylor (1936) deduced such a parameter, which in its modified form has become :

$$R_{xT} = f\left(\frac{\sqrt{u^2}}{U} \left(\frac{x}{M}\right)^{1/6}\right)$$

Figure 5 shows the data of figure 1 plotted according to the new parameter. We notice that such presentation did not improve the collapse of the data. In fact, the scatter is greater in fig.5 and therefore, Taylor's parameter as such does not correlate the present transition data. Hislop (1940) and Hall et al (1938) found the same result where the scatter in their plots eventually increased when they were plotted using Taylor's parameter. Most of the later workers did not bother to pursue the issue any further and simply correlated their results using the free stream turbulence intensity $(\sqrt{u^2}/U)$ only. Because, using the grid dimensions and the distance until transition imply assumptions about the nature of the free stream turbulence at transition and the way the laminar boundary layer develops until transition beneath a free stream turbulence that may not be strictly true (see Kareem (1978)), perhaps if Taylor's parameter is expressed in terms of local quantities, a better collapse of the data may be obtained.

From the above discussion and Abdel-Kareem (1978) , (θ) may replace (x) and (L_x) may replace (M) , and we may write that :

$$R_{xT} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{\theta}{L_x} \right)^{1/6} \right)$$

Figure 6 shows the same data plotted accordingly . We see that in general the scatter is much less than that in figs. 1 , 2 & 5 (notice the change of the scale of the ordinate) .

CONCLUSION .

We conclude that the occurrence of transition of the laminar boundary layer over a flat plate is reasonably correlated with the flow parameters based on local estimates of such quantities as follows :

At transition :

$$R_{xT} = f \left(\frac{\sqrt{u^2}}{U} \left(\frac{\theta}{L_x} \right)^{1/6} \right)$$

In fact, this conclusion gives more weight to the argument put forward by Abdel-Kareem (1978) that the free stream turbulence affects the laminar boundary layer stability while the layer is still very thin . Since the turbulence entrained by the layer will cause it to develop in a way different from that in a smooth flow and will cause (θ) to be different at transition from that predicted by the laminar boundary layer theory . This change in (θ) is gradual rather than local as seen from figs. 6.14 & 6.15a in Abdel-Kareem (1978) . Hence, the use of $\left(\frac{\theta}{L_x} \right)$ rather than $\left(\frac{x}{M} \right)$ in the parameter, although being a local parameter at transition, yet allows for the observation that the development of the laminar boundary layer beneath a turbulent free stream is different or in other words, is an expression of the history of the laminar boundary layer .

This argument and the results of Klebanoff (1959) draw the attention to the fact that the basic physical assumption of Taylor that at transition , local separation occurs due to the pressure fluctuations associated with the free stream turbulence, is not correct .

REFERENCES

- 1 - ABDEL-KAREEM M.S.E. (1978)
"Effect of free stream turbulence on boundary layer transition."
Ph.D. thesis, London University .
- 2 - BATCHELOR, G.K. (1953)
"The theory of homogeneous turbulence."
Cambridge University press.
- 3 - BATCHELOR, G.K. (1960)
"Taylor scientific papers, Vol.2, paper 27, pp 288 ."
Cambridge University press .
- 4 - DRYDEN, H.L. (1947)
"Some recent contributions to the study of transition and turbulent boundary layers."
NACA TN no. 1168 .
- 5 - FAGE, A. and PRESTON, J.H. (1941)
"On the transition from laminar to turbulent flow in the boundary layer ."
Proc. R. Soc., A., Vol. 178, pp. 201 .
- 6 - HALL, D.J. (1968)
"Boundary layer transition."
Ph.D. thesis, University of Liverpool .
- 7 - HALL, D.J. and GIBBINGS (1972)
"Influence of free stream turbulence and pressure gradient upon boundary layer transition."
J. of Mech. Eng. Sc., Vol. 14, No. 2, pp. 134
- 8 - HALL, A.A. and HISLOP, G.S. (1938)
"Experiments on the transition of the laminar boundary layer on a flat plate ."
ARC R&M , No. 1843 .

- 9 - HISLOP, G.S. (1940)
"The transition of a laminar boundary layer in a wind tunnel."
Ph.D. thesis, Cambridge University .
- 10 - KLEBANOFF, P.S. and TIDSTROM, K.D. (1959)
"Evolution of amplified waves leading to transition in a boundary layer with zero pressure gradient."
NASA TN D-195 .
- 11 - MICHEL, R. (1974)
"Effect of flow turbulence and noise on aerodynamic phenomena and wind tunnel results."
AGARD R - 615 .
- 12 - McKEOGH, P. (1976)
"Effect of turbulence on airfoils at high incidence."
Ph.D. thesis, I.C. Aero. Dept., London Univ.
- 13 - SCHUBAUER, G.B. (1938)
"The effect of turbulence on transition of the boundary layer of an elliptic cylinder."
Proc. 5th. Int. Cong. App. Mech. (John Wesley & son, Chichester) .
- 14 - TAYLOR, G.I. (1936)
"Statistical theory of turbulence, Part v :
'Effect of turbulence on boundary layer .
Theoretical discussion of the relationship between scale of turbulence and critical resistance of spheres .'"
Proc. Roy. Soc. (London), A, Vol. 150, pp307
- 15 - SPANGLER, J.G. and WELLS, C.S. (1968)
"Effects of free stream turbulence on boundary layer transition."
AIAA J., Vol. 6, No. 3, pp. 543 - 545 .
- 16 - WELLS, C.S. (1967)
"Effects of free stream turbulence on boundary layer transition."
AIAA J., Vol. 5, No. 1, pp. 172 - 174 .

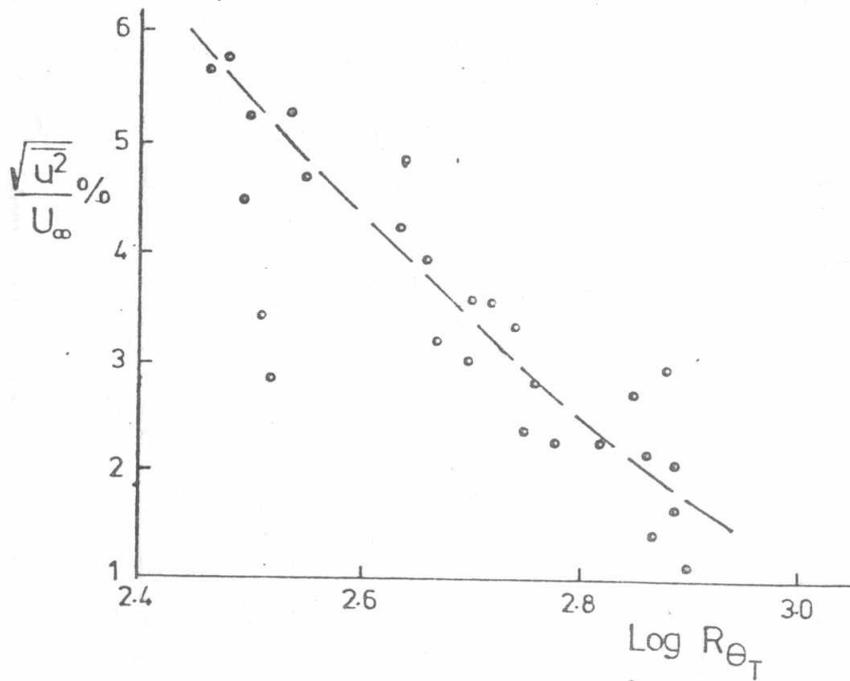
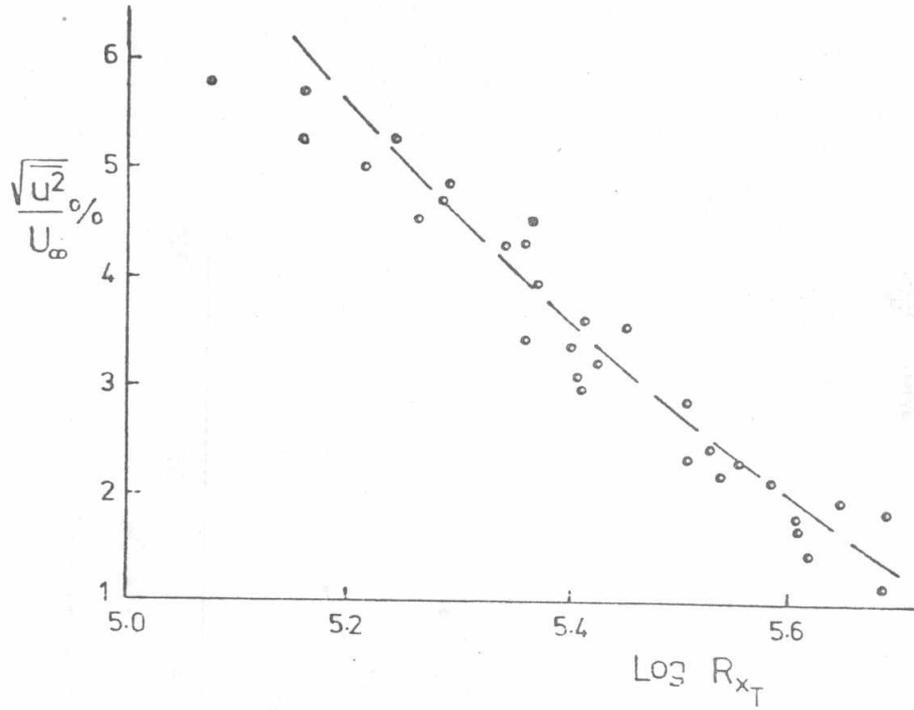


Fig. 1&2 Transition Reynolds no. as a fn. of $\frac{\sqrt{u^2}}{U_\infty}$

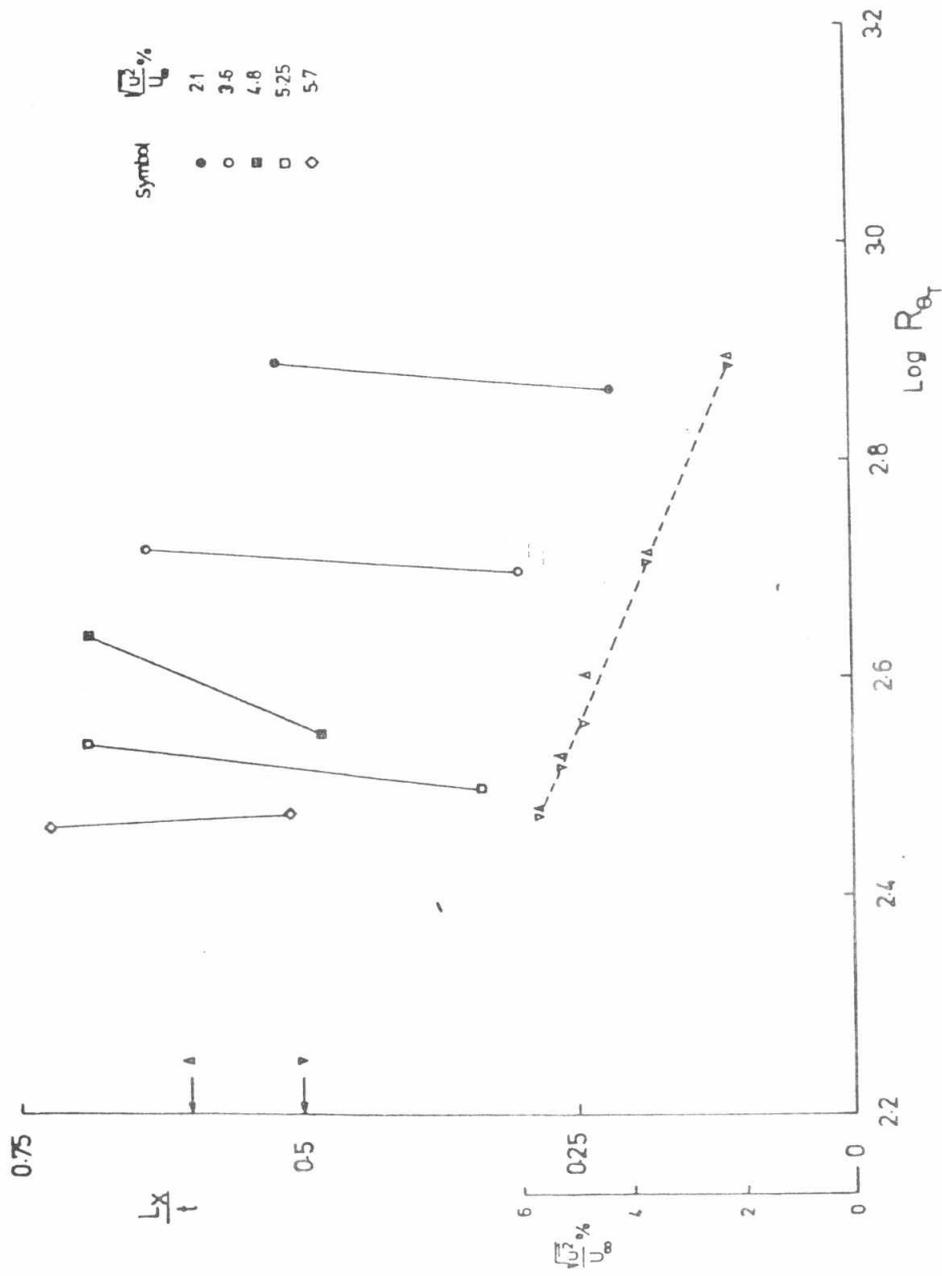


Fig. 3 $\frac{Lx}{t}$ at transition for constant $\frac{U^2}{U_0^2}$ & Transition at constant $\frac{Lx}{t}$

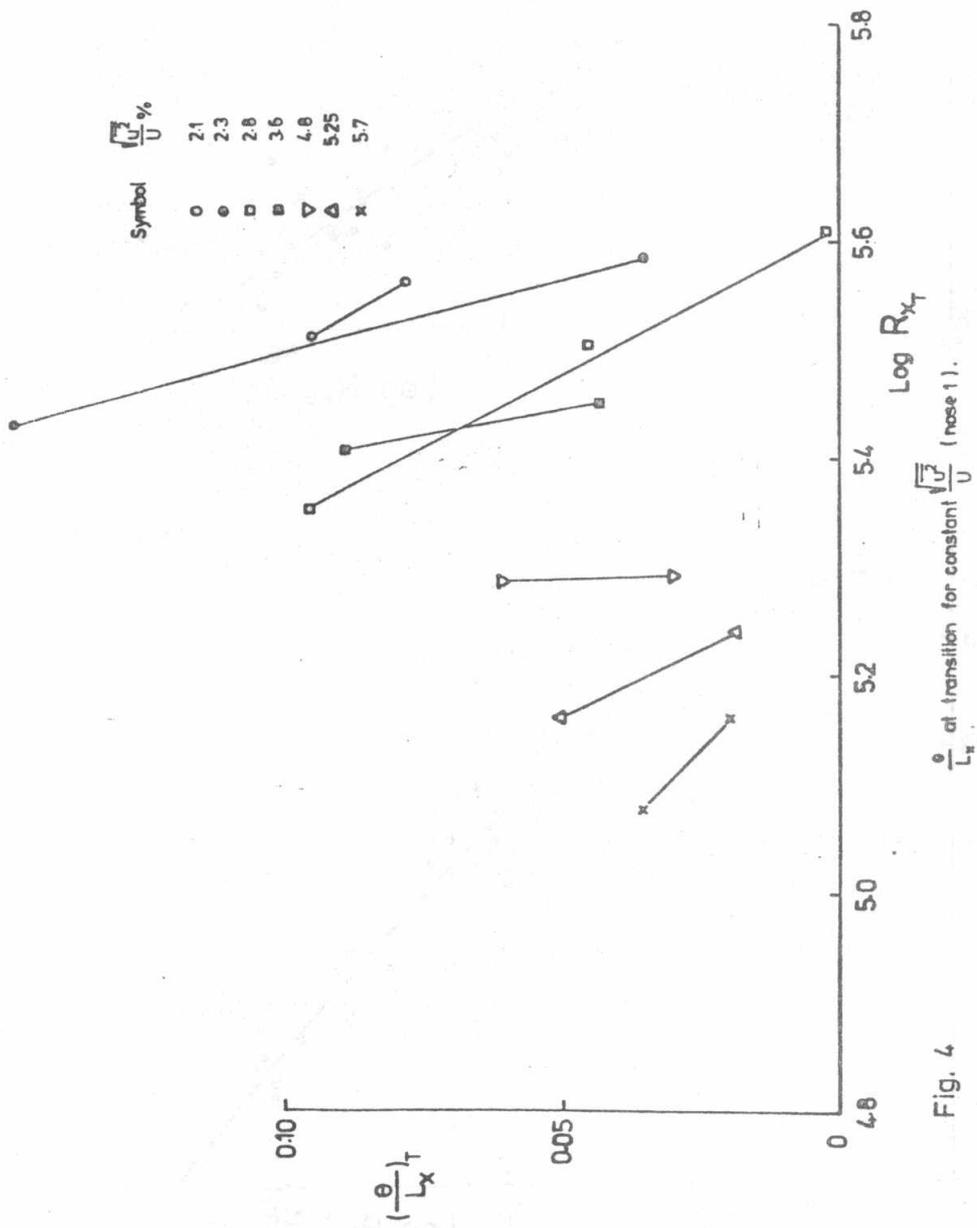
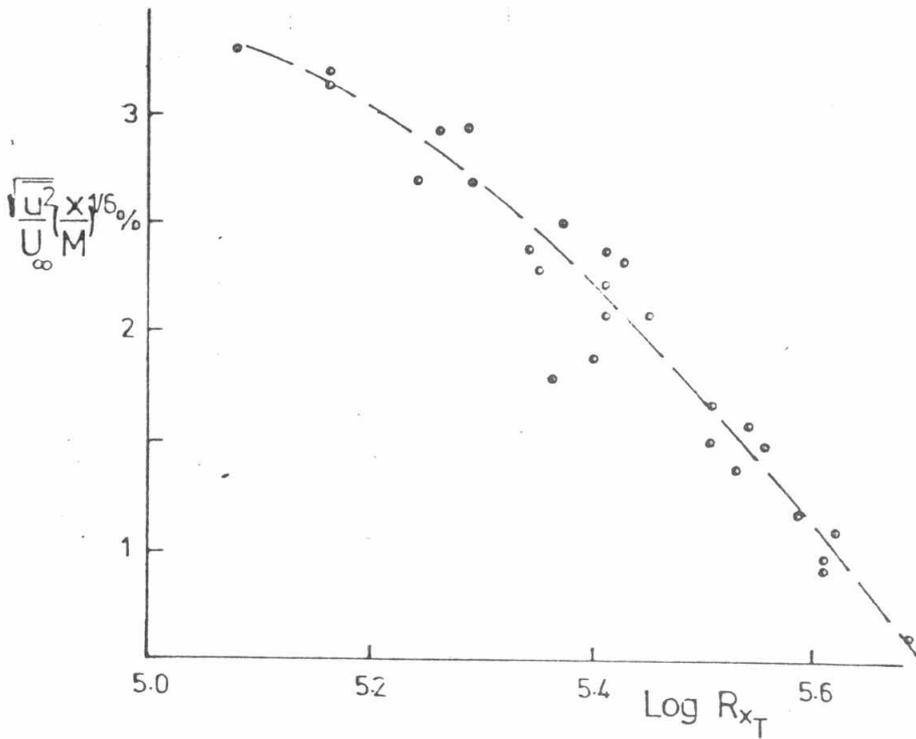
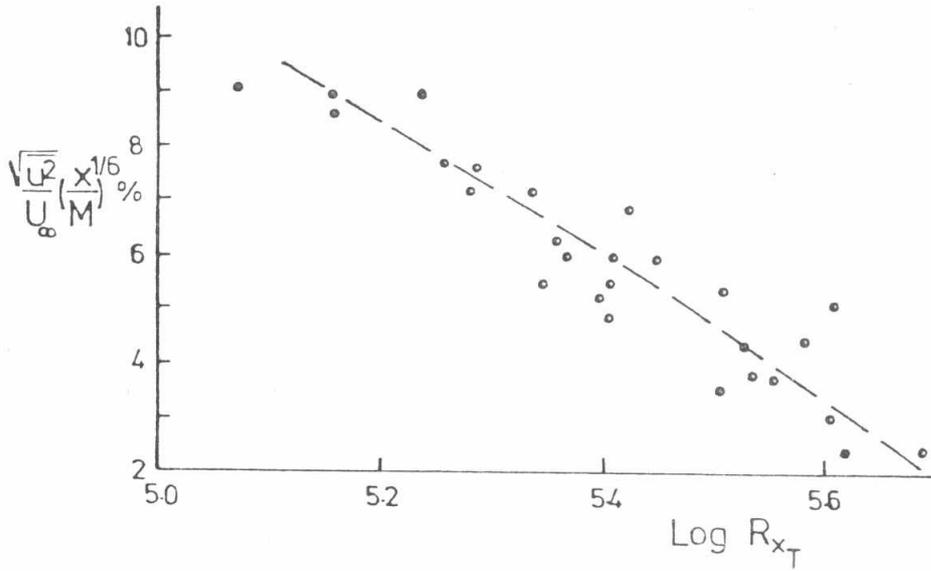


Fig. 4 $\frac{\theta}{L_x}$ at transition for constant $\frac{\sqrt{u'}}{U}$ (nose 1).



Figs. 5&6 Transition as a fn. of the modified Taylor's parameter.