



STATISTICAL DESIGN OF RECURSIVE DIGITAL FILTERS
USING ITERATIVE TECHNIQUES

F. IBRAHIM*(MSc) A.HANAFY**(Ph.D), S. GHONIEMY(Ph.D)*

ABSTRACT

In this paper, the linearization iterative method in frequency domain for the design of recursive digital filters having a desired spectral density characteristic is proposed. For getting the initial solution, the separation of spectra algorithm is employed. This initial solution affects hardly the rate of convergence of the already existing (steepest descent) algorithm. However a care has been made to the choice of the initial condition which has consequently lead to a better performance than that obtained using nonoptimal initial solution for the same number of iterations.

For the same near optimal initial solution the linearization algorithm has been applied. This algorithm has lead finally to a design procedure by which the filter parameters best approximate the desired specifications with smaller number of iterations.

The extensive study of the practical results of the proposed and already existing algorithms has shown that, with the same initial solution the linearization results in a better performance with less computation time.

* Radar & Guidance Chairs, M.T.C., Cairo, Egypt.

1. INTRODUCTION

The problem of design digital filters is involved mainly in the extraction of filter's coefficients such that a specific aspect of the filter response (impulse response, frequency response, or spectral density characteristic) approximates a desired behaviour. Accordingly, the filter design problem is basically a mathematical approximation problem and consequently is strictly treated from a mathematician's point of view. A recursive filter having p -poles and q -zeros designed by approximating a desired spectral density characteristic is called an ARMA (p,q) (Autoregressive Moving Average)-Filter, and the methods performing this design are named "statistical methods" /1/-/3/.

As the non-iterative algorithms give a rather good, but suboptimal solution, the steepest descent gradient iterative method has been proposed in /4/ , for improving the initial filter's chars. The steepest descent method is relatively simple to implement, since it requires only the calculation of the first derivatives required to determine the gradient vectors. Unfortunately, it is practically proved that it converges in a most disappointingly slow fashion. For this reason a powerful algorithmic procedure must be implemented.

In this paper, the linearization algorithm has been proposed and applied for designing recursive digital filters knowing their spectral density characteristics. In Sec. 2, the separation of spectra algorithm /3/, which is used to obtain the initial coefficients and the spectrum of the filter, is introduced. Sec.3 gives a review of the steepest descent algorithm /4/. In Sec.4, the analysis of the proposed approach with the flow scheme of the FORTRAN program are given. The experimental results and the comparison with the method of /4/, will be given in Sec.5, demonstrating that, with the same initial solution the linearization results in better performance with less computation time.

2. INITIAL DESIGN OF THE DIGITAL FILTER

The separation of spectra algorithm proposed by A. Hanafy /3/ is implemented to calculate the different parameters of the digital filter which are then used as an initial estimate for subsequent design techniques. These parameters include the order of the desired filter, pole-and zero-coefficients so that the resulting spectrum is as close as possible to the specified one, and the spectrum of the resulting filter. In this algorithm the pole-and zero-coefficients are calculated using Levinson algorithm /5/. The flow chart of this design procedure is given in Figure 1.

3. STEEPEST DESCENT METHOD IN FREQUENCY DOMAIN

The complete analysis of this method is presented in /4/. It is based on the well-known fact that a function locally decreases most rapidly in the direction of its negative gradient. The error function to be minimized is given by:-

$$\varepsilon(a,b) = \sum_{n=1}^M (g R(e^{jnw}) - R^d(e^{jnw}))^2 \quad (1)$$

where g is the gain-square of the filter, $R^d(e^{jnw})$ is the desired spectral density characteristic and M is the range of optimization and $R(e^{jnw})$ is given by:

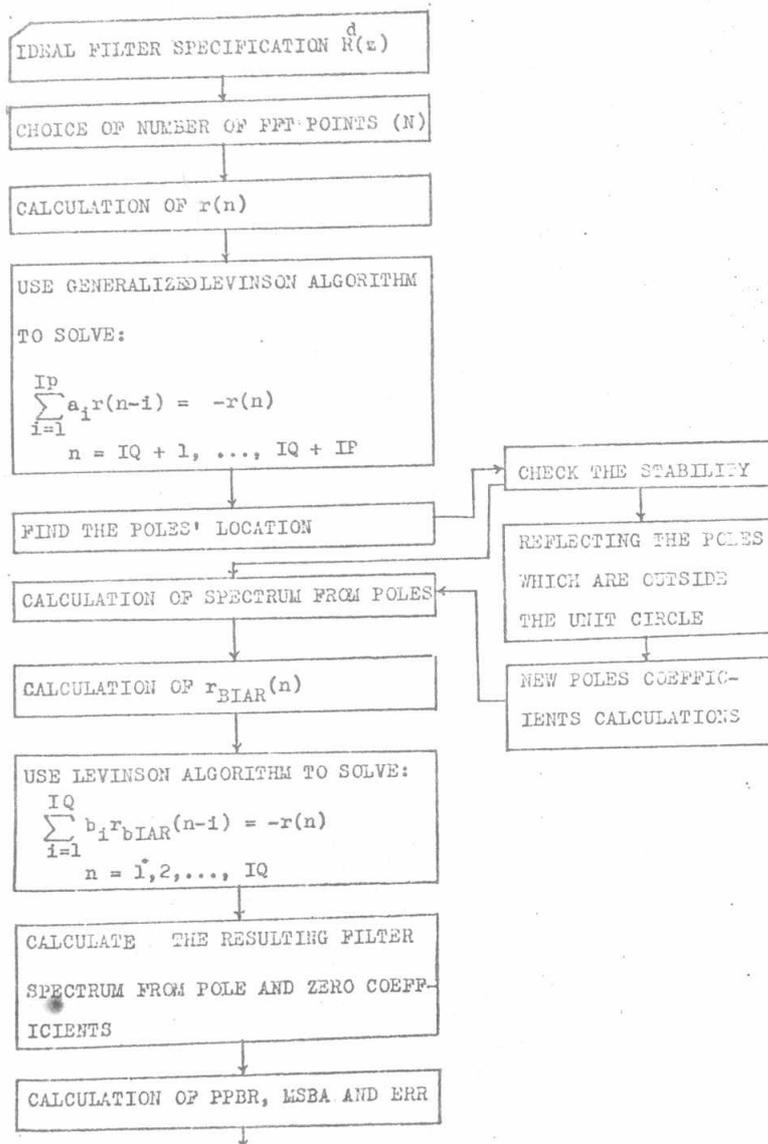


Fig. 1 Flow chart of separation of spectra algorithm

$$R(e^{j\omega}) = \frac{\left| \sum_{n=1}^q b_n e^{-j\omega n} \right|^2}{\left| \sum_{n=1}^p a_n e^{-j\omega n} \right|^2} \quad (2)$$

Where a_i and b_i are the pole-and zero-coefficients of the filter respectively.

The iterative relationships used in this approach are of the form:

$$a^{m+1} = a^m + \alpha \Delta a \quad (3)$$

$$b^{m+1} = b^m + \alpha \Delta b \quad (4)$$

where a^m and b^m denote the values of coefficient vectors at the m th iteration and Δa and Δb are suitably chosen such that

$$\Delta a = -\sqrt{\epsilon} \nabla_a (a^m, b^m) \quad (5)$$

$$\Delta b = - \vec{\nabla} \epsilon_b (a^m, b^m) \quad (6)$$

The flow chart of this algorithm is given in Fig.2.

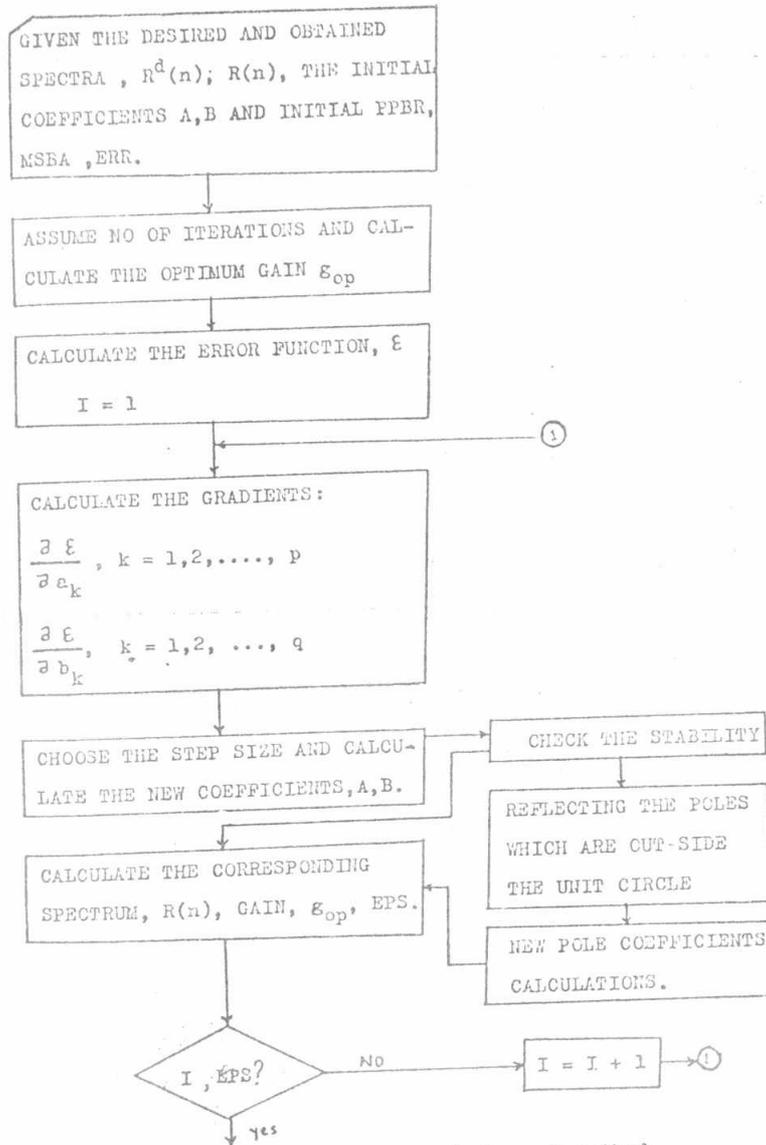


Fig. 2 Flow scheme of steepest descent method

4- LINEARIZATION METHOD IN FREQUENCY DOMAIN

In the linearization algorithm, the error function to be minimized is given by (1) and a quadratic approximation to the functional $\epsilon(a,b)$ is made which is dependent upon. Linearizing the desired response about the prevailing coefficient vectors as follows.

$$R_{op}(a + \Delta a, b + \Delta b) \approx R_{op}(a,b) + T(a,b) \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} \quad (7)$$

where R_{op} is defined by:

$$R_{op}(z_n) = g R(z_n) \tag{8}$$

where $z_n = e^{jnw}$ and the $M \times (p+q)$ matrix $T(a,b)$ is found to be composed of column vectors which are equal to the recursive system's first-order sensitivity vectors, that is :

$$T(a,b) = [\partial R_{op} / \partial a_1, \dots, \partial R_{op} / \partial a_p, \partial R_{op} / \partial b_0, \dots, \partial R_{op} / \partial b_q] \tag{9}$$

To get $\partial R_{op}(z_n) / \partial a_k$ and $\partial R_{op} / \partial b_k$, differentiating (8) with respect to a_k and b_k -coefficients which gives:

$$\frac{\partial R_{op}(z_n)}{\partial a_k} = -2 R_{op}(z_n) \cdot R_{opa}(z_n) \cdot \sum_{i=0}^p a_i \cos \frac{2\pi n}{N}(i-k) \tag{10}$$

and

$$\frac{\partial R_{op}(z_n)}{\partial b_k} = 2g R_{opa}(z_n) \cdot \sum_{i=0}^p b_i \cos \frac{2\pi n}{N}(i-k) \tag{11}$$

where N is the number of DFT points used in calculation and $R_{opa}(z_n)$ is given by:

$$R_{opa}(z_n) = \frac{1}{|\sum_{i=0}^p a_i z_n^{-i}|^2} \tag{12}$$

By substituting (7) into (1) and using standard gradient techniques the set of perturbation vectors which minimize (1) is readily found to be :

$$T(a,b)^t T(a,b) \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} = \begin{bmatrix} \nabla \epsilon_a(a,b) \\ \nabla \epsilon_b(a,b) \end{bmatrix} \tag{13}$$

where

$\frac{\partial \epsilon(a,b)}{\partial a_k}$ and $\frac{\partial \epsilon(a,b)}{\partial b_k}$ are the gradients of the error function with respect to the a - and b -coefficients respectively. These gradients can be obtained by differentiating (1) which leads to :

$$\frac{\partial \epsilon(a,b)}{\partial a_k} = -4 \left(\sum_{n=1}^M (R_{op}(z_n) - R^d(z_n)) \cdot R_{op}(z_n) \cdot R_{opa}(z_n) \cdot \sum_{i=1}^p a_i \cos \frac{2\pi n}{N}(i-k), k=1,2, \dots, p \right) \tag{14}$$

and

$$\frac{\partial \epsilon(a,b)}{\partial b_k} = 4g \left(\sum_{n=1}^M (R_{op}(z_n) - R^d(z_n)) \cdot R_{opa}(z_n) \cdot \sum_{i=0}^q b_i \cos \frac{2\pi n}{N}(i-k), k=1,2, \dots, q \right) \tag{15}$$

To determine the search direction vectors Δa and Δb , one must solve the system of $(p+q)$ linear equations (13). Knowing the search direction vectors, the a -and b -coefficients can be calculated iteratively using (3) and (4). The iteration step size at step $(m+1)$ is usually chosen as :

(16)

$$\alpha = \frac{c}{m+1}$$

where c is a constant less than one

The schematic representation of this algorithm is shown in Fig.3.

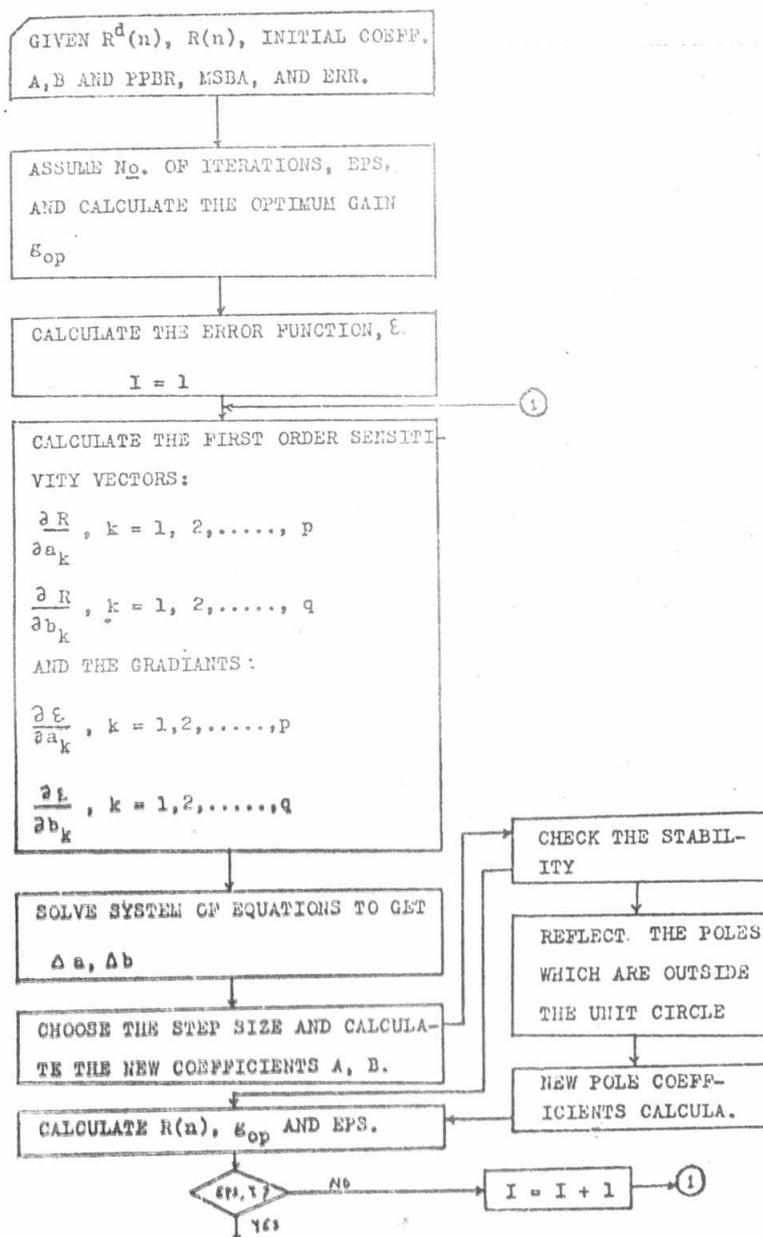


Fig 3. Flow scheme of linearization method

5. PRACTICAL RESULTS AND COMPARISONS

In this section, practical examples for the design of digital filters, employing the above three methods are introduced. The filter with the best performance is that having minimum peak pass-hand ripple (PPBR), maximum stop-band attenuation (MSBA), minimum error between the obtained and desired spectra (ERR), and transition region (TR) as close as possible to the specified one. These parameters will be considered as measures of efficiency of the used techniques.

In each example, the procedure of design is as follows:

1. From the desired specification, the initial parameters of the filter are found using separation of spectra algorithm /3/. These parameters are: pole and zero coefficients ;roots of denominator and numerator polynomials, peak pass band ripple (PPBR), minimum stop-band attenuation (MSBA) and error between the desired and obtained spectra (ERR).
2. By using these parameters as an initial solution, the steepest descent /4/ and the proposed Linearization methods are employed and the same parameters are recalculated.

Analysis of the results and comparison among the three methods are accomplished in the following examples.

Example 1

Fig.4. indicates the characteristic of the desired low pass filter

Comparison between the specifications of the resulting ARMA (11,9) from the design using separation of spectra, steepest descent and Linearization algorithms are shown in table 1, while Fig.5. indicates the behaviour of the designed filter in both pass-and stop-bands.

Table.1. Comparison of the specifications of ARMA (11,9) designed using different techniques.

Method of Design	Specifications			No. of iterations
	PPBR(dB)	MSBA (dB)	ERR	
Separation of Spectra.	0.313461	49.534	0.0636	-
Steepest descent.	0.118187	49.754	0.0048	300
Linearization	0.0587	49.808	0.0012	100

From Fig.5. and table 1. it is clear that the linearization is more efficient than the steepest descent method. The improvements observed in pass-band ripple (PBR), stop band ripple (SBR) and the error (ERR) with respect to that obtained by application separation of spectral algorithm is given in table.2.

Table .2. Comparison of the improvements in the specifications of ARMA(11,9) when employing iterative techniques.

Method of Design	Improvement in specification		
	PBR (%)	SBR (%)	EBR (times)
Steepest descent	60.30	47.21	15
Linearization	81.46	58.79	50

Examples 2.

Fig.6. Indicates the characteristic of the desired high-pass filter.

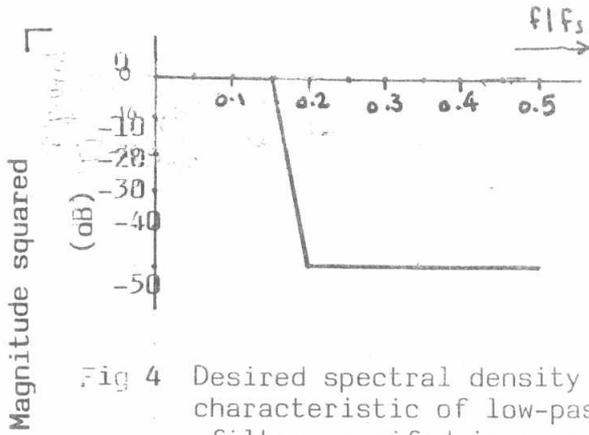


Fig 4 Desired spectral density characteristic of low-pass filter specified in example 1.

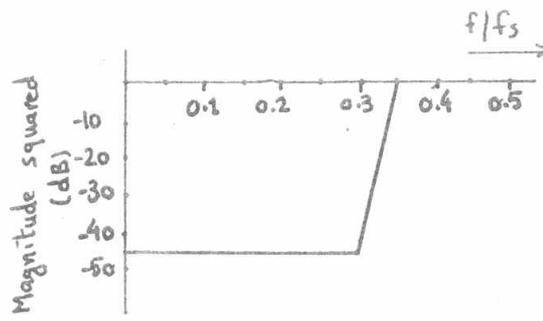


Fig 6. Required spectral density characteristic of high-pass filter specified in example 2.

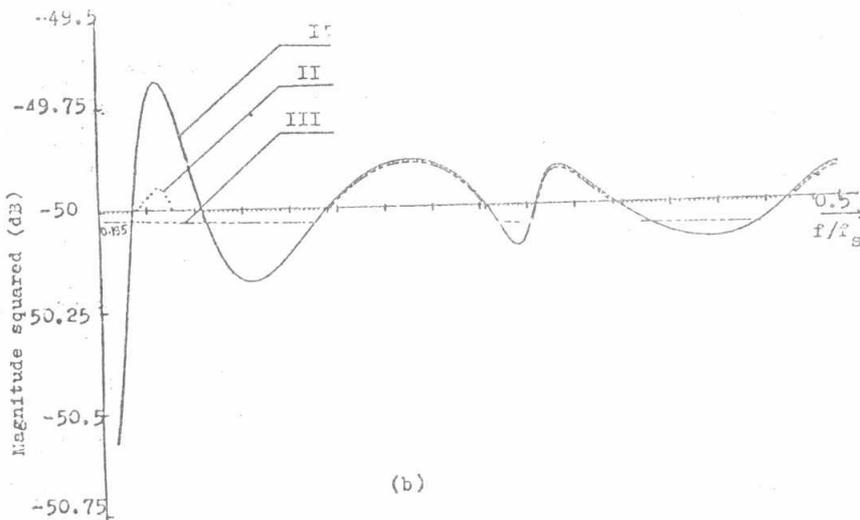
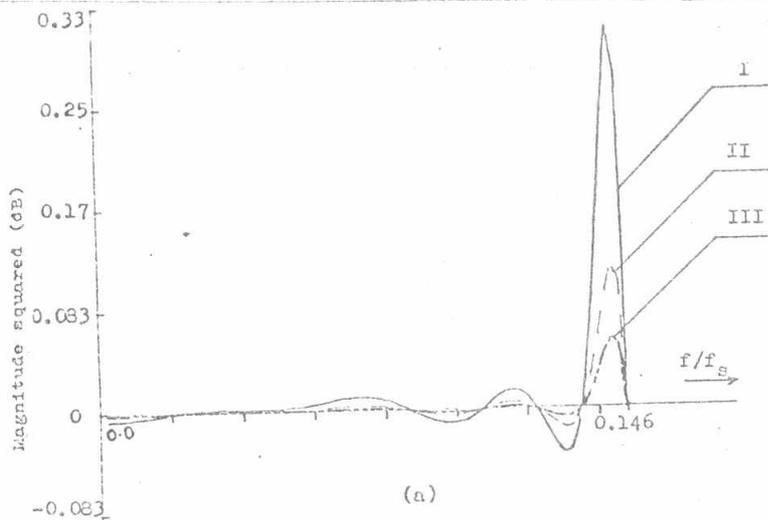


Fig. 5 Behaviour of ARMA (11,9) low-pass filter
(a) in pass-band (b) in stop-band
I. Separation of spectra algorithm.
II. Steepest descent. III. Linearization method

Comparison between the specifications of resulting ARMA(11,11) using different techniques are given in table 3 while the behaviour of the designed filter in both stop-and pass-band is shown in Fig.7.

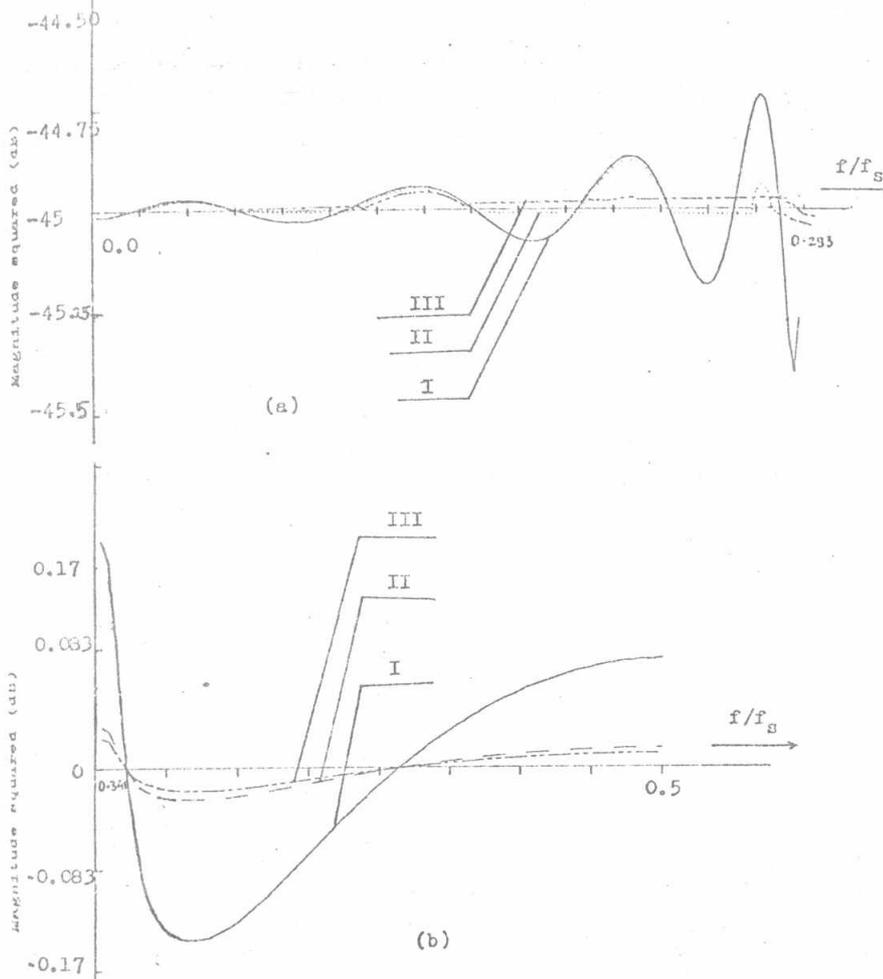


Fig. 7 Behaviour of ARMA (11,11) high-pass digital filter. (a) in stop-band , (b) in pass-band
I. separation of spectra.
II. steepest descent, III. Linearization method.

Table 3. Comparison specifications of ARMA (11,11) designed with different techniques.

Method of Design	Specifications			No of iterations
	PPBR (dB)	MSBA (dB)	ERR	
Separation of spectra	0.188	44.715	0.0956	-
Steepest descent	0.034	44.878	0.0021	300
Linearization	0.024	44.921	0.0011	100

From Fig.7. and table 3 it is clear that the improvements in the resulting filter's performance using linearization method is better than that when

applying the steepest descent method. The values of these improvements are given in table 4.

Table 4. Improvements occurred in the specifications of ARMA (11,11) using the iterative techniques.

Used Method	Improvements in the specifications		
	PBR (%)	SBR(%)	ERR (times)
Stcepest descent	81.91	57.19	45.5
Linearization	87.23	71.92	86.4

6. CONCLUSION

Since the initial solution affects hardly the rate of convergence of the iterative techniques, a care has been made to the choice of this initial solution which has consequently lead to a better performance of the filter designed using steepest descent than that obtained using nonoptimal initial solution for the same number of iterations.

The extensive study of the practical results of the proposed (linearization) and already existing algorithms has shown that, the proposed algorithm has lead finally to a design procedure by which the filter parameter best approximate the desired specifications with smaller number of iterations. So with the same initial solution the linearization results a better performance with less computation time.

REFERENCES

1. C.T. MULLIS, and R.A. ROBERTS, " The use of Second Order Information in the approximation of Discrete-Time LInear Systems", IEEE Trans. A coust., Speech, and Signal Processing, Vol. ASSP-24, No. 3, PP. 226-238, June, 1976.
2. A.A.BEEX, and L.L. SCHAPF, " Recursive Digital Filter Design Via Covariance Sequence Approximation" IEEE Trans. Acoust., Speech, and Signal Processing , Vol. ASSP-29, No. 1, PP.51-57, February, 1981.
3. A.HANAFY, J.LE ROUX, and J. PRADO," ARMA Digital Filter Design Technique Through AR-Processes", European Conf. On Circuit Theory and Design, PP. 1008-1012, The Hague, 1981.
4. A.HANAFY, J. LE ROUX, and J. PRADO," Iterative and Non Iterative Techniques For The Design of Recursive Digital Filters", ICASSP,Paris, 1982.
5. S.ZOHAR, "The Solution of a Toeplizy Set of LInear Equations", J. of the Association for computing Machinery , Vol. 21, No.2, PP. 272-276, April, 1974.
6. F. IBRAHIM , " Proposed Approaches to Digital Filter Design Using Iterative Techniques" , Thesis of M.Sc., MTC, Cairo, 1984.



NAVSTAR/GPS System and DIFF. GPS System Tropospheric
Range Correction Modelling

by

Sayed Yousef and M. Samy Abo El Soud
Avionics Dept., Military
Technical College

ABSTRACT

The NAVSTAR/GPS System is the satellite navigation system which is being developed since early 1973 and is expected to be in full operational phase from 1987/88. This system is based on range determination corresponding to a one-trip time delay of the received signals from the satellite transmitters which use atomic frequency standards. There are several sources of error in determination of range between a satellite and the user. The dry component represents larger than 90% of the total tropospheric range error. Balloon data have been collected and analysed at different sites world-wide over several years. World-wide models are given that enable calculation of the correction at a specific location based on surface meteorological parameters at this location. Also, algorithms are available to find the value of the correction at an arbitrary elevation angle ($>5^{\circ}$). Here, it is given how to extend locally valid models to world-wide applicable models using average parameters and a residual of random nature. The variance of the residual is evaluated as function of the source temperature. Also, an algorithm to evaluate the value of this variance at an arbitrary angle ($>3^{\circ}$) is provided. Further, a

refractivity statistics at an Egyptian site (Mersa Matrouh) is used to study the application of this model when using the NAVSTAR/GPS System in Egypt.

1. Introduction

The satellite navigation system NAVSTAR/GPS System is based on range determination via a one-trip delay time measurement. This system uses a constellation of 18 satellites, (24 in a later phase of system operation), to provide a worldwide navigation coverage. On each satellite, a transmitter with an atomic frequency standard transmits clock correction parameters to the user in the navigation signal. The requirement for the user to have precision clock synchronized to the GPS time is eliminated when range measurements from at least four satellites are used for position fixing. The four unknowns computed here are the user three coordinates in earth centered earth-fixed (ECEF) coordinate system, plus the user clock offset bias.

2. Sources of error in GPS System

The errors in the GPS System are due to several factors. The ionosphere dual frequency error is modeled as a Markov process and is of the order of 3.0 m. The tropospheric error is of the order of 1.0 m and is modeled as white noise. The range mechanization error is of the order of 1.0 m and is modeled as white noise. Range quantization error is of the order of 0.25 m and is modeled as white noise. Satellite ephemeris error is of the order of 1.8 m and is modeled as bias. Pseudo range measurement error is of the order of 1.2 m and is modeled as Markov process.

3. Differential GPS System

The differential GPS System implies in addition to the normal GPS System, the range measurement between same satellites used by the user and an additional base point (reference) receiver. Based on the number of satellites used, i.e. either three or four, is determined the system precision and the number of unknowns that can be determined. When three satellites are used, five unknowns can be found, these are the three

coordinates of the system relative to the base point (reference), the user clock offset bias, and the base receiver clock offset bias. On the other hand, when four satellites are used, three more unknowns can be found. These are the three absolute coordinates of the user measured in ECEF coordinate system. The inherent accuracy enhancement of the differential GPS System is due to the fact that the user and reference receivers are measuring range to the same satellite at the same time. Therefore, the range errors due to satellite ephemeris errors and clock ephemeris errors can be cancelled out to a very high degree of precision. Moreover, when the user and reference receivers are relatively near to each other then the range errors due to the atmospheric effects at both sites are largely correlated together and can be cancelled out to a high degree of precision, the higher the nearer the two points to each other.

4. Tropospheric range error

The tropospheric range correction is given by

$$\begin{aligned} \Delta R &= \int (n-1) ds \quad \text{along the signal path} \quad (1) \\ &= \int 10^{-6} N ds \end{aligned}$$

where

$$N = 10^6 (n-1) \quad (2)$$

is the refractivity and n is the refractive index of the air. Hopfield, [8]-[10] has used the model of Smith and Weintraub, [11] to derive a model for the dry component of the tropospheric correction as function of surface meteorological data and the satellite elevation angle E . Considering the dry component for which an accurate model is elaborated by Hopfield that has a prediction error less than 0.08%. That is

$$N_d(h) = N_{ds} \frac{(h_d - h)^4}{h^4} \quad (3)$$

where

$$h_d = h_{od} + a_d T_c \quad (4)$$

and N_{ds} is the surface refractivity, while T_c is the temperature.

latter case is different from the former case by the value σ_{av} . The dependence of this value on temperature is drawn in Fig.1. From this figure it is evident that the value of σ_{av} is less than 8 mm for cold regions such as Europe and the USA, and less than 5 mm for tropical regions such as Egypt. σ_{av} is defined as

$$\sigma_{av}^2(T) = \frac{1}{M} \sum_{i=1}^M \sigma_{1i}^2 + \frac{1}{M} \sum_{i=1}^M \sigma_{2i}^2(T) \quad (10)$$

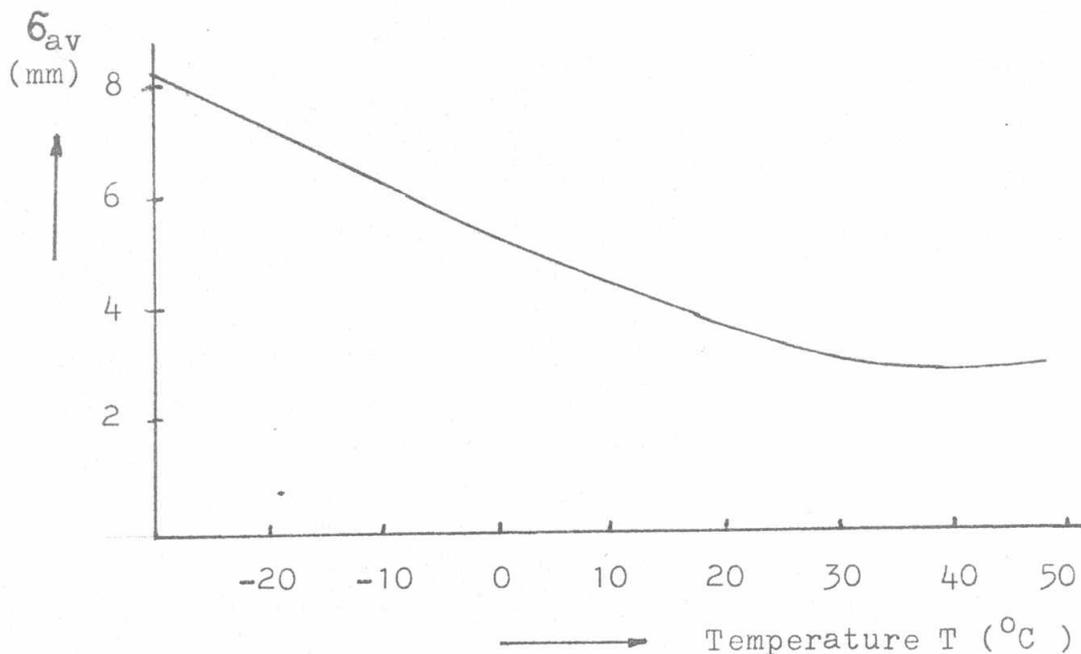


Fig. 1 Dependence of Tropospheric prediction error σ_{av} on temperature

and it is the average prediction error of tropospheric range correction in meters averaged over one year of observation. As shown in [13] this definition accounts for the fact that the model can be used at a temperature different than the temperature of balloon measurements providing the data for model parameters determination. Further, the dependence of $\sigma_{\Delta R_d}$ on elevation angle is drawn in Fig.2. This is the dependence of the random residual of the tropospheric range correction on elevation angle E (assuming $E > 2.5^\circ$).

6. Tropospheric range error at an Egyptian site

A program has been initiated in Alexandria University to study the statistics of the surface refractivity N_S over one year of observations using the formula

$$N = \frac{77.6}{T} \left(p + 4810 \frac{e}{T} \right) \quad (11)$$

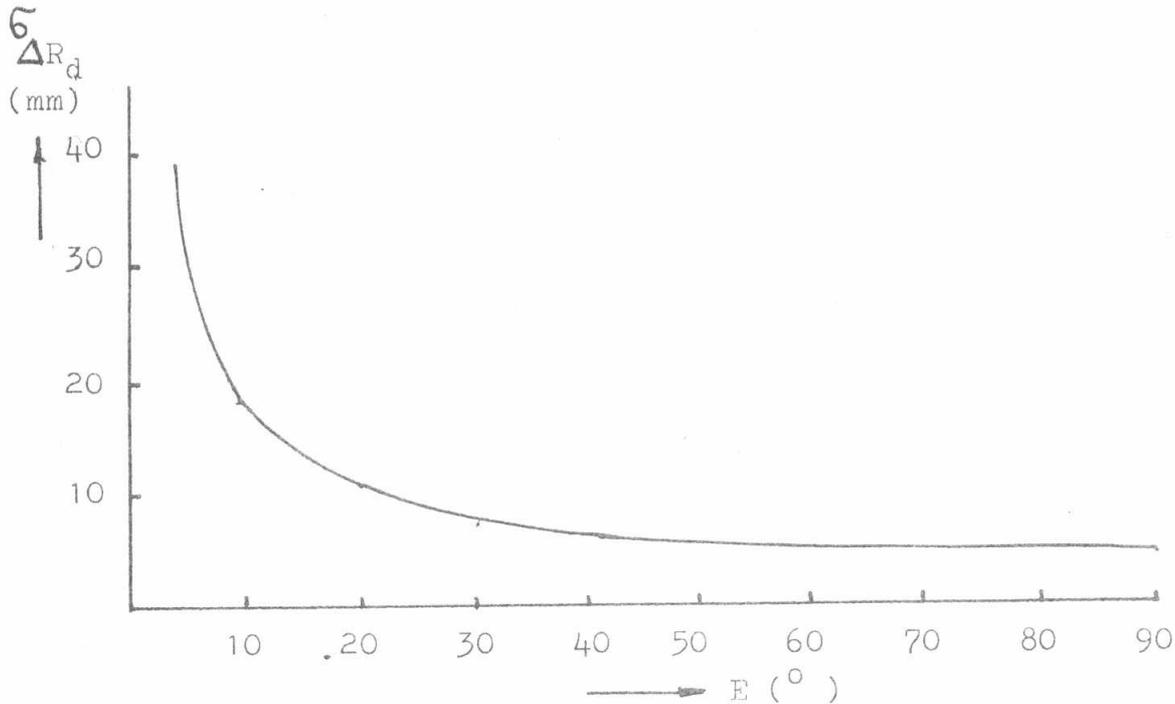


Fig.2 Dependence of the rms of the residual R_d on the elevation angle E

The data used in this program are collected by a weather station in Mersa Matrouh, 28 m above mean sea level. Instrumented weather balloons are released in the atmosphere twice daily at 1200 and 2400 hours approximately to obtain vertical profiles of basic meteorological data. The balloon instruments measure pressure, temperature, and relative humidity at specified pressure levels and transmits the data to the weather station.

The statistical behaviour of the surface refractivity N_S is described in Fig.3 and Fig.4. In Fig.3 the monthly behaviour of mean surface refractivity with one standard deviation indicated. In Fig.4 is drawn the monthly behaviour of the mean

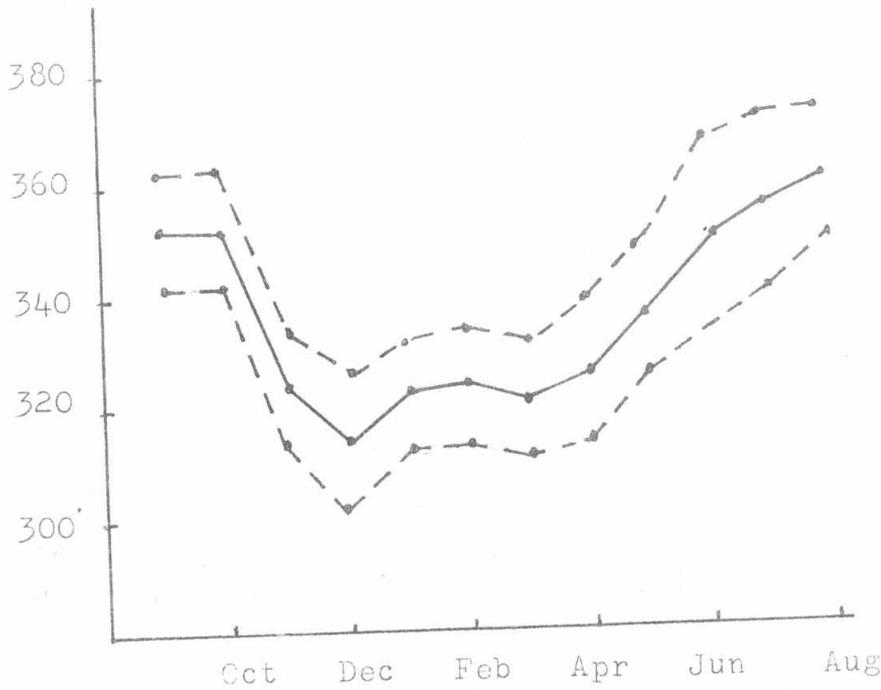


Fig.3 Monthly behaviour of the mean surface refractivity N_s with one standard deviation indicated.

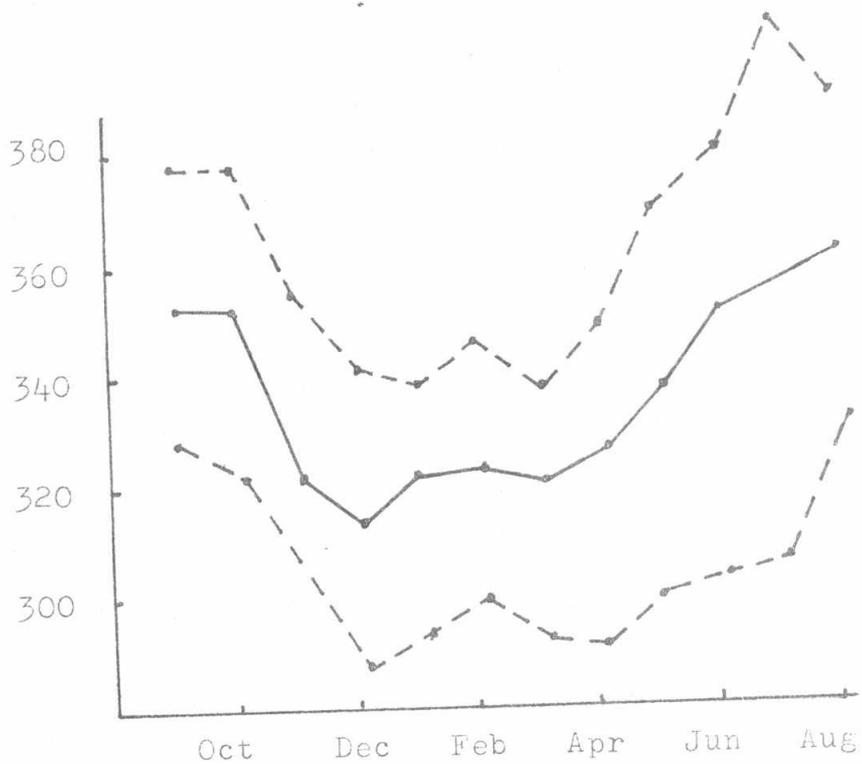


Fig.4 Monthly behaviour of the mean surface refractivity N_s with maxima and minima indicated.

14-16 May 1985, Cairo

surface refractivity with corresponding maxima and minima indicated.

References

1. Milliken, R.J. and C.J. Zoller 'Principle of Operation of NAVSTAR and System Characteristics' NAVIGATION, J. of ION Vol 25, No 2, pp 3-14, Summer 1978
2. Spilker, J.J. 'Signal Structure and Performance Characteristics' _____, pp 29-54
3. Bartholomew, C.A. 'Satellite Frequency Standards' _____, pp 21-28
4. Martin, R.H. 'GPS User Equipment Error Model' _____, pp. 109-118
5. Beser, J. and B.W. Parkinson 'The Application of NAVSTAR DIFFERENTIAL GPS in the Civilian Community' NAVIGATION, J. of ION, Vol 29, No 2, Summer 1982, pp 107-136
6. Kalafus, R. M. (et al) 'Differential Operation of NAVSTAR GPS' NAVIGATION J. of ION, Vol 30 No 3, Fall 1983, pp 187-204
7. Jorgensen, P.S. 'Navigating Low Altitude Satellites Using the Current Four NAVSTAR/GPS Satellites' _____, pp 234-243
8. Hopfield, H.S. 'Two Quartic Tropospheric Refractivity Profile for Correcting Satellite Data' J. Geophys. Res. 74, 4487-4499, 1969
9. Hopfield, H.S. 'Tropospheric Effect on Electromagnetically Measured Range' Radio Science 6, 357-367, 1971
10. Hopfield, H.S. 'Tropospheric Refraction Effects on Satellite Range Measurements' APL, April 1972, 11-19
11. Smith, E.K. and S. Weintraub 'The Constants in the equation of atmospheric refraction index at radio frequencies' Proc. IRE 41, 1035-1037, 1953
12. Black, H.D. 'An Easily Implemented Algorithm for the Tropospheric Range Correction' J. of Geophys. Res. 83, 1825-1828, 1978
13. Sayed Yousef 'A Program to Calculate The Tropospheric Range Correction and Residuals For Satellite Navigation' NAECOM' 83 Dayton, Ohio, May 1983, pp 276-278
14. Kheirallah, H.N. and A. Shaalan 'Refractive Index Statistics For Mersa Matrouh' Second National Radio Science Symposium, Cairo, March 1984, pp 64-70