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THE TRACKING OF HIGHLY MANEUVERING TARGETS BY KALAMAN FILTERING TECHNIQUES

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ABSTRACT

Tracking of maneuvering targets represents one of the basic tasks to be carried out by radar and weapon systems. Usually the radar system is connected with a computer. The main problem involved in this case is the proper description of the target maneuver random process. Different models are used to represent the target maneuvering process. Simplest models should be used so as to reduce the burden on computer and to decrease the computation time to the minimum value permitted by the weapon system involved. Here is assumed a radar system with range and bearing measurements. The target is assumed to have constant velocity plus a random component that accounts for maneuvers. Different targets of different maneuverability are assumed. Augmented Kalman filter is assumed. The steady state behaviour of the system is obtained through studing the error covariance matrix in an off-line analysis. Convergence is also analyzed.

Different radars detecting individual targets are considered and the performance of each radar is analysed to find the best performance conditions required such as the sampling time T.

I. INTRODUCTION

The problem of tracking a maneuvering target is of great importance to military and Civilian applications. In military it is involved in fire control systems, in missile guidance systems, in automatic traffic control systems and in interception systems. The Kalman filtering technique has been used in this concern as early as the work of Singer, [1]. The target modelling has been always one of the major sources of difficulty for the problem solution. The filter structure is dependent upon the target model adopted. Several target models have been proposed, [2] - [3]. These models affect the filter tracking performance and the filter complexity as well. Generally, the target is either non-maneuvering i.e. moving at constant velocity straight line trajectory or excercising a maneuver. When the filter is designed to track non-maneuvering targets, or similarly targets with very weak maneuvers, its performance will be degraded when the target tracked will excercise a maneuver. In severe conditions it can even loose track of the maneuvering target. On the other hand, when the filter is designed to track a maneuvering target its performance will be degraded even from that of simpler filters when it will track non-maneuvering targets.

Several techniques are described that provide a compromise solution for these criteria. Adaptive Kalman filtering is the main of these techniques, [4] - [8]. In most of these cases the best state estimate is a weighted sum of Kalman filters each conditions on a particular maneuver value. Another technique, [9], is based on a maneuver detector and a least-squares estimator that yields an estimate of the acceleration input vector. The result is used in conjunction with a standard Kalman filter to estimate the state of the target. This is done by removing the filter bias caused by the target deviating from the assumed constant velocity straight line motion.

A variable dimension filter is described, [10], that does not rely on a statistical description of the maneuver as a random process. Instead, if a maneuver is detected at time k, the filter assumes that the target had a constant acceleration starting at k-m-1, where m is the effective window length. The state estimates within the window are then modified by introducing extra state components.

II. THE APPLICATION

The Kalman filter considered here is as follows :

Firstly, the augmented model of the tracker is given by:

$$x(k+1) = \emptyset\, x(k) + W(k)$$

where

$$x(k) = \left[r(k) \;\; \dot{r}(k) \;\; \ddot{r}(k) \;\; \theta(k) \;\; \dot{\theta}(k) \;\; \ddot{\theta}(k) \right]^T \text{ is the state vector}$$

$$= x_k$$

with $r(k)$, $\dot{r}(k)$ and $\ddot{r}(k)$ being the range, the radial velocity and the radial acceleration respectively. Similar definition for the bearing quantities are assumed.

$$\emptyset = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}$$

$$W(k) = \left[0 \;\; 0 \;\; W_1(k) \;\; 0 \;\; 0 \;\; W_2(k) \right]^T$$

$W_1(k)$, $W_2(k)$ are zero- mean uncorrelated white noise processes corresponding to zero-mean ρ- correlated maneuvering process given in Fig. 1. ($\rho = e^{-\alpha T}$, $\alpha > 0$). Thus the correlation matrix Q is given by :

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{M1}^2 (1-\rho^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{M_2}^2 (1-\rho^2) \end{bmatrix}$$

Secondly, the observation (in range and bearing) equation is :

$$Z(k) = H\,X(k) + V(k)$$
$$= Z_k$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and $V(k) = \left[V_1(k) \quad V_2(k) \right]^T$

$V_1(k)$ and $V_2(k)$ are zero-mean uncorrelated measurement (observation) noises having the following correlation matrix R

$$R = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

with σ_R^2 and σ_θ^2 being the variances in range and bearing respectively . Finally , the Kalman algorithm realizing the minimum mean square error estimate of the state vector $\hat{X}(k)$ has :

i- $\hat{x}_k = \hat{x}_k^- + K_k\,(z - H\,\hat{x}_k^-)$

$\hat{x}_k^-$ is the prior estimate of x_k

ii- $K_k = P_k^-\,H^T\,(H\,P_k^-\,H^T + R)^{-1}$

P_k^- is the prior error covariance matrix, i.e

$$P_k^- = E\left[(x_k - \hat{x}_k^-)\,(x_k - \hat{x}_k^-)\right]^T$$

iii- $P_k = (I - K_k\,H)P_k^-$

P_k being the error covariance matrix.

iv- As to the project - a head values :

$$\hat{x}_{k+1}^- = \emptyset\,\hat{x}_k$$

and

$$P^{-}_{k+1} = \emptyset P_k \emptyset^T + Q$$

This is depicted in Fig. 2.

III. CONVERGENCE OF THE KALMAN TRACKER TO ITS STEADY STATE.

In order to evaluate the performance of the Kalman tracker under consideration we are going to examine its dynamic behaviour before reaching the steady state if exists.

3.1. Existance and Uniqueness of the tracker Steady State.

The key equations to analyze this problem are

$$K_k = P_k H^T R^{-1}$$

$$P_k = (I - K_k H) P^{-}_k$$

$$P^{-}_{k+1} = \emptyset P_k \emptyset^T + Q \qquad (1.a)$$

Only for sake of mathematical convenience $M_k = P^{-}_k$

Thus $P_k = (I - K_k H) M_k$

and $M_{k+1} = \emptyset P_k \emptyset^T + Q \qquad (1.b)$

These are the famous Discrete-time matrix Riccatti equations.
These eqns are to have steady state if $\emptyset$, H, R, Q and not functions of time and $\emptyset$ is stable [11] . The stability condition on $\emptyset$ is derived using various approaches as seen in [12,13].

In our problem , to check the stability of $\emptyset$ (since other conditions are satisfied) we should ensure that eigen values λ_i of $\emptyset$ are such that:

$$|\lambda_i| < 1 \qquad \forall i$$

<u>Eigen values of $\emptyset$</u> :

It can be shown that:

$\lambda_i = 1$ of multiplicity 4 and ρ of multip. 2. from which we see that the stability of $\emptyset$ is not guaranteed.
A more weakened conditions are given in [14] to ensure the existance and

uniqueness of the steady state. The conditions are :

i- The initialization matrix M_o is positive semidefin. and R is positive definite

ii- $(\emptyset^T, H^T)$ is stabilizable.

iii- $(C, \emptyset^T)$ is observable i.e $(\emptyset, C^T)$ controllable.

with C being such that : $Q = C^T C$

The first condition is satisfied as will be seen later for M_o.
As to condition (ii), the definition of the stabilizability is : If $\exists$ a matrix S such that $\emptyset^T + H^T S$ is stable then $(\emptyset^T, H^T)$ is stabilizable.

For the problem under consideration, it can be seen that a matrix

$$S = \begin{bmatrix} -1 & -\frac{1}{3T} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -\frac{1}{3T} \end{bmatrix}$$

Proves to ensure the stability of $\emptyset^T + H^T S$
As to the third condition, let

$$C = \begin{bmatrix} 0 & 0 & \beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_2 \end{bmatrix} \qquad \begin{aligned} \beta_1 &= \sigma_{M1} \sqrt{1-\rho^2} \\ \beta_2 &= \sigma_{M2} \sqrt{1-\rho^2} \end{aligned}$$

To prove the observability, i.e $(\emptyset, C^T)$ controllable, we use the conditions entailing that every eigen value λ_i of $\emptyset$ is controllable [14] and then prove this property for the case under consideration.

The eigen values are : ρ multip. 2 and 1 multip. 4
then

$$\Psi_1 = \emptyset - \rho I = \begin{bmatrix} (1-\rho) & T & 0 & 0 & 0 & 0 \\ 0 & (1-\rho) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-\rho) & T & 0 \\ 0 & 0 & 0 & 0 & (1-\rho) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank of $\left[C^T, \ \psi_1 C^T, \ \psi_1^2 C^T, \ \psi_1^3 C^T, \ \psi_1^4 C^T, \ \psi_1^5 C^T \right]$

was proved to be 6

Similarly for

$$\psi_2 = \emptyset - 1.I = \begin{bmatrix} 0 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & (\rho-1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & (\rho-1) \end{bmatrix}$$

that (ψ_2, C^T) is controllable was proved. Proofs are omitted for the sake of brevity. With this proof, we guarantee that equations (1)

have unique steady state solution.

When one is interested only in computing steady state solution, he needs to solve eqn (1) for $M_k = M_{k+1} = D$. i.e

$$D = \emptyset D \emptyset^T - \emptyset D H^T (H D H^T + R)^{-1} H D \emptyset^T + Q \qquad (2)$$

Techniques for solution of equation (2) are numerous including numerical computation, recursive methods and eigen-value eigen vector method (see e.g. [12, 14]) . However, since we are interested in the solution of (1) and system is of low order ($n = 6$) we shall solve eqn (1) whose solution converges to the unique solution of (2).

3.2. Steady state solution through forward solution of the filter.

Equations (1) are rewritten in the form:

$$M_{k+1} = \emptyset M_k \emptyset^T - \emptyset M_k H^T (H M_k H^T + R)^{-1} H M_k \emptyset^T + Q \qquad (3)$$

Using the matrices $\emptyset$, H, R, Q (being sparse) equation (3) was splitted into 12 algebraic equations profitting the character symmetric positive semidef-inite of M_k.

These equations are omitted for the sake of brevity.

These equations were programmed on the computer , using as initialization the following positive semidefinite matrix M_o

$$M_o = \begin{bmatrix} 1 & \frac{1}{T} & 0 & 0 & 0 & 0 \\ \frac{1}{T} & \left(\frac{\sigma_{M1}^2}{\sigma_R^2} + \frac{2}{T^2}\right) & \rho . \frac{\sigma_{M1}^2}{\sigma_R^2} & 0 & 0 & 0 \\ 0 & \rho . \frac{\sigma_{M1}^2}{\sigma_R^2} & \frac{\sigma_{M1}^2}{\sigma_R^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sigma_\theta^2}{\sigma_R^2} & \frac{\sigma_\theta^2}{\sigma_R^2 . T} & 0 \\ 0 & 0 & 0 & \frac{\sigma_\theta^2}{\sigma_R^2 . T} & \left(\frac{\sigma_{M2}^2}{\sigma_R^2} + \frac{2}{T^2}\right) & \frac{\sigma_{M2}^2}{\sigma_R^2} . \rho \\ 0 & 0 & 0 & 0 & \frac{\sigma_{M2}^2}{\sigma_R^2} . \rho & \frac{\sigma_{M2}^2}{\sigma_R^2} \end{bmatrix}$$

Convergence curves are shown in Fig. 3 , where diagonal elements of the error covariance matrix are displayed for a number of iterations of about 10 000. We can see some sort of threshold behaviour and the asymptotes determine a <u>no</u> ranging from 100 up 150 iterations for attaining practically the steady state. Hence the steady state curves displayed later are the results for 120 iterations.

The corresponding Kalman gain matrix is given by :

$$K^T = \begin{bmatrix} \frac{P_{11}}{\sigma_R^2} & \frac{P_{21}}{\sigma_R^2} & \frac{P_{31}}{\sigma_R^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{P_{44}}{\sigma_\theta^2} & \frac{P_{54}}{\sigma_\theta^2} & \frac{P_{64}}{\sigma_\theta^2} \end{bmatrix}$$

Variations of K with the sampling period is displayed only for $K_{11} = \frac{P_{11}}{\sigma_R^2}$

and $K_{42} = P_{44}/\sigma_\theta^2$

from these curve we see that filtering gain amounting up to 20 for T= 0.01 sec and $(\sigma_{M_1}/\sigma_R)^2 = 10^{-4}$ is attainable.

IV. STEADY STATE BEHAVIOUR OF THE KALMAN TRACKER

In order to know the sensitivity of the tracker to the sampling period T, the maneuver statsitics (σ_M^2, ρ) and the measurement noise variances σ_R^2 and σ_θ^2, a versatile program based on the solution of the matrix equations depicted in Fig.2. was developped. Diagonal elements of the P_k matrix are drawn parametrically as function of T after 120 iteration. These parametric curves can be utilized to predict the tracker performance for various configurations of targets and radars (see Fig. 4.).

Examination of these curves leads to the following conclusions:

1. The Kalman filter is capable of reduction of the range error variance by a factor ranging from 1 up to about 20 depending upon the sampling rate, maneuver statistics, and measurement noise variances. Similar conclusions for the bearing error variance can be stated.
2. The parametric curves shows that the filtering error variances increase with the sampling period, and the maneuver error variance. However, as far as P_{11} and P_{44} are concerned some kind of saturation w.r.t. sampling period is observed specially for larger σ_M^2 in the span of the considered sampling periods.
3. The effect of the correlation time constant α^{-1} on the parametric curves is less significant, and the behaviour of these curves remains the same and that is why the parametric curves are plotted only for one value of α, that is $\alpha = 0{,}01$.

V. CONCLUSION

The problem of maneuvering target tracking using Kalman filtering techniques is studied. Several available techniques are analysed. The range-bearing case is considered using Singer's model. The steadystate behaviour and the convergence criteria are analysed for this typical case. The number of iterations to attain nearby steady - state is determined. The behaviour of the Kalman tracker is analysed accordingly. Sensitivity of the Kalman tracker to sampling period and target model parameters are discussed.

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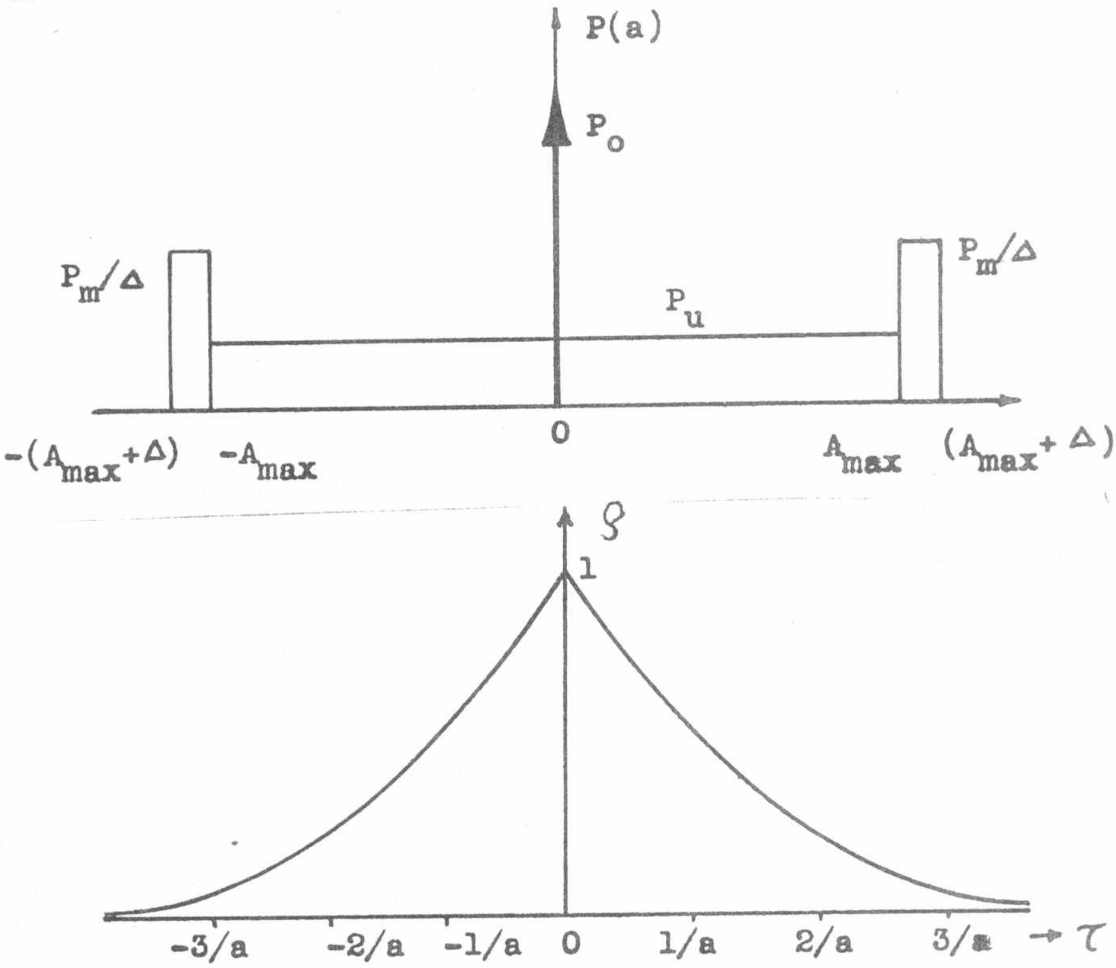

Fig.1.Maneuver model: probability density and correlation functions.

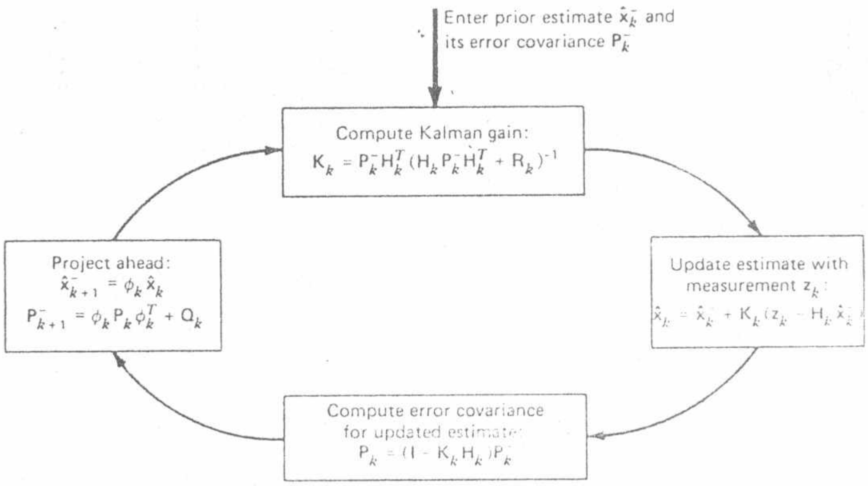

Fig.2.The Kalman filter loop.

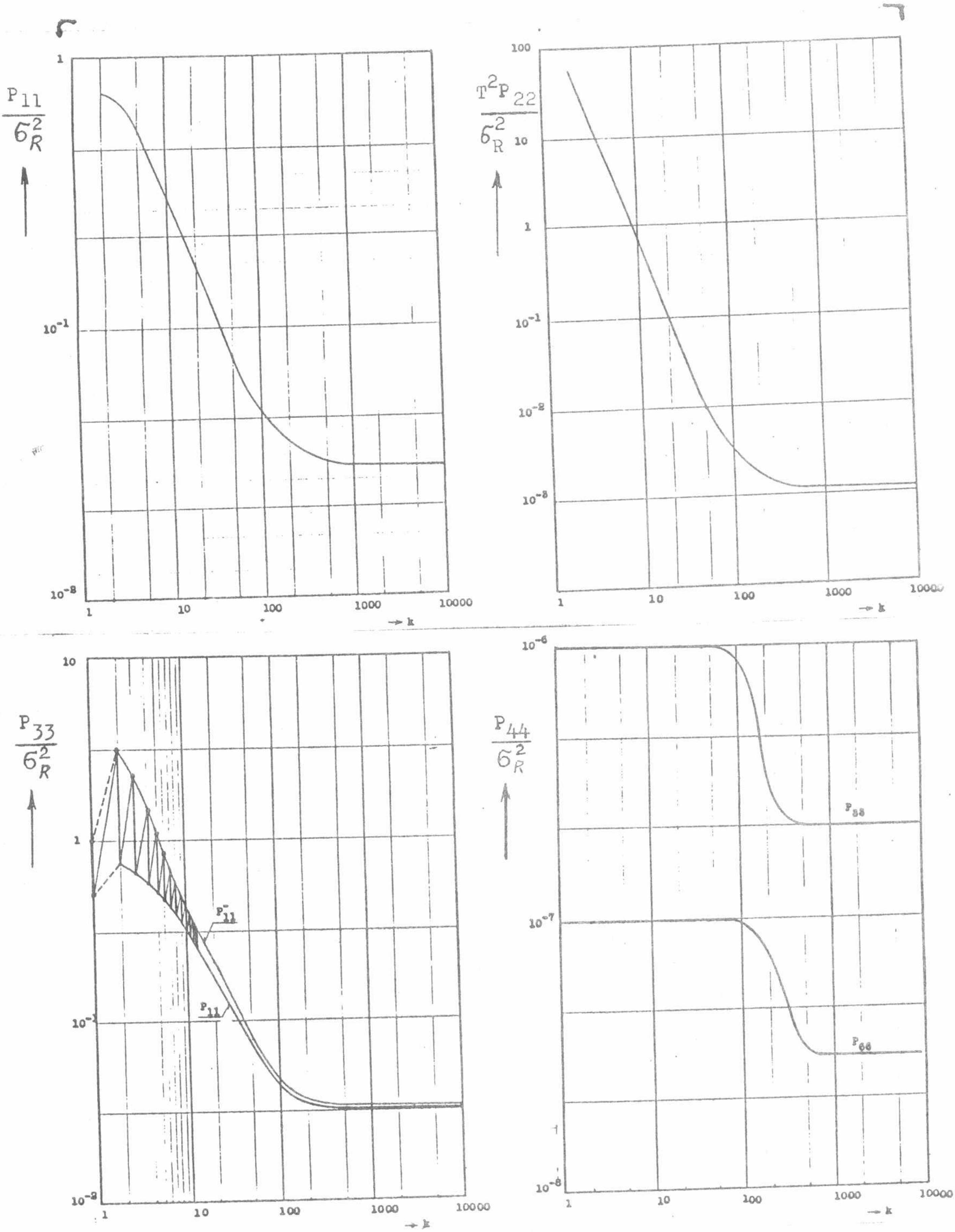

Fig.3. Typical convergence curves.

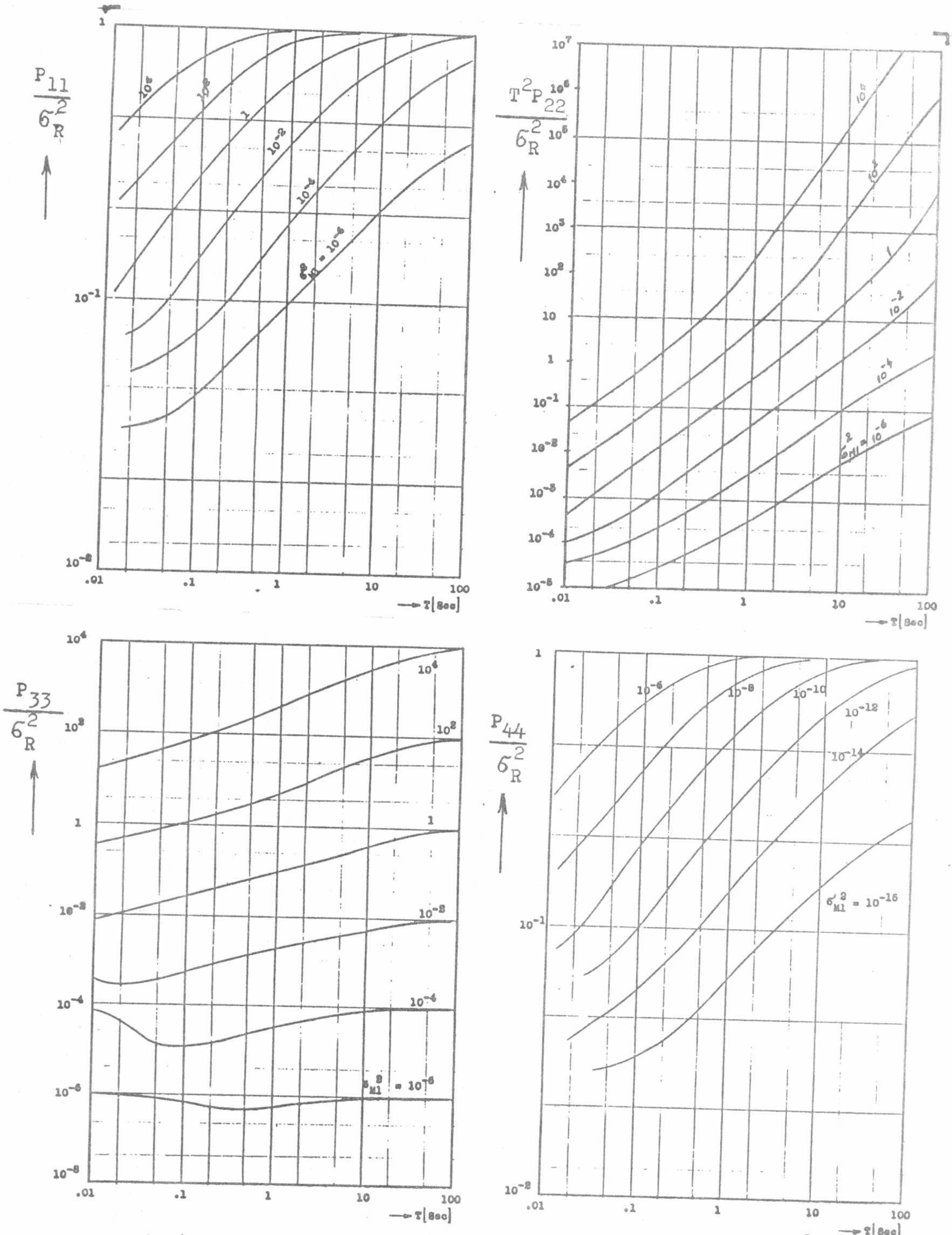

Fig.4. Steady-state parametric curves of the Kalman filter.