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ERROR CONTROL IN DIGITAL SPACE COMMUNICATION SYSTEMS

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ABSTRACT

In this paper we introduce the use of replication decoding with linear block codes as a mean for improving the performance and reducing the probability of error in digital space communication systems. Replication decoding is a symbol-by-symbol maximum likelihood decoding based on soft decision. The space communication channels are among the few physical communication channels that can be modeled by an additive Gaussian noise channel model.

Replication decoding will be described, and an upper bound on the decoding error probability for Gaussian noise channels will be derived. The performance of the proposed replication technique, when sequentially implemented, over additive Gaussian noise channel is evaluated by computer simulation. We will show that the replication decoding scheme improves the performance considerably even for poor channels, i.e., low signal to noise ratio channels. Cases of contiuous a priori algebraic values (which corresponds to soft decision) will be considered for performance evaluation.

The Hamming (15,11) code, which is a single error correcting linear cyclic block code, is the code used in the simulation. Probability curves and tables of the simulation results will be presented to demonestrate the performance of the proposed method quantitatively.

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INTRODUCTION

Since Shannon demonestrated in his "noisy coding theorem" in 1948 [1] that errors in the data transmitted over a noisy channel can be reduced to any desirable level, several developments in the construction of error detecting and correcting codes have been made [2] and several efficient decoding techniques have been proposed [3,4,5]. Error control codes can be classified in general into: block codes and convolutional codes. For convolutional codes, the optimum decoding scheme is generally known as the Viterbi algorithm. The complexity of Viterbi decoding increases for large constraint length and another approach, known as sequential decoding, becomes more attractive. For block codes, several decoding procedures have been introduced such as the Berlekamp-Massey algebraic method. Some of the decoding algorithms are well suited for both block and convolutional codes, such as majority decoding, aposteriori probability decoding, and threshold decoding. One of the recently proposed decoding techniques, that suits both block and convolutional codes, is the Replication Decoding[6] . Replication decoding is a symbol-by-symbol decoding based on a maximum likelihood soft decision with a compromise between the optimality and the complexity of the decoder.

In this paper we introduce the use of replication decoding with linear block codes as a mean for improving the performance and reducing the probability of error in digital space communication systems. Space channels are among the few physical communication channels in which errors do not tend to cluster together into bursts, and which can be modeled as an additive Gaussian noise channel.

A brief introduction to the replication decoding method will be given in the next section of the paper, then the problem will be mathematically formulated with an upper bound on the decoding error probability for Gaussian channels in section three. The performance of the replication decoding technique, when sequentially implemented, over additive Gaussian noise channels is evaluated by computer simulation. The results of the simulation will be presented in the last section. The Hamming (15,11) code which is a linear, cyclic, block code is used in the simulation. We will show that the replication decoding method when used,even with this simple and single error correcting code, the performance improves considerably. Even for poor channels, i.e., low signal to noise ratio, an error probability improvement occurs and the improvement increases as the S/N increases.

REPLICATION DECODING

When a message is encoded by a redundant code,i.e., in which constraints are introduced between the transmitted symbols, the availability of only parts of this message enables its full reconstruction, and several alternative expressions of a given symbol, to be referred to as its replicas, can be computed in terms of other symbols. Thus decoding decoding a particular symbol becomes a diversity reception problem, given the set of its received replicas, to be referred to as replication decoding [6,7] . Replication decoding is an interpretative concept rather than a single algorithm. An advantage of it may be its intuitive appeal, interpreting the correcting power of codes in terms of diversity reception or computational diversity. Moreover it provides a common framework to diversity reception and coding.

The communication system considered, shown in Fig.1, is modelled as follows: a q-ary source where the symbols are chosen independently of each other with equal probability feeds a redundant encoder. The symbols it delivers independantly modulate a carrier. The channel is assumed memoryless, which is the case of the space communication channels. We shall assume that the following conditional probabilities are available in the demodulator, which is a reasonable assumption,

$$p_{ij} \triangleq P_r\,(c_i = j/z_i) \qquad j = 0,1,\ldots,q-1$$

where c_i is the symbol transmitted at instant i, and z_i is the corresponding received signal. Thus the demodulator output will consist of vector

$$\underline{p}_i = [p_{io}\ p_{i1}\ \cdots\ p_{i(q-1)}]$$

i.e., soft decision instead of a mere hard decision. The symbol -by-symbol decoder is intended to determine the a posteriori probability vector, say

$$\underline{P}_i = [P_{io}\ P_{i1}\ \cdots\ P_{i(q-1)}]$$

where P_{ij} is the probability that the ith symbol actually transmitted is j when the code constraints, expressed in terms of a number of received signals $z0$, $z1,\ldots$, are taken into account. The maximum likelihood decision is then $D(P_i)=m$ where m is the second subscript of the largest among the components P_{ij} .

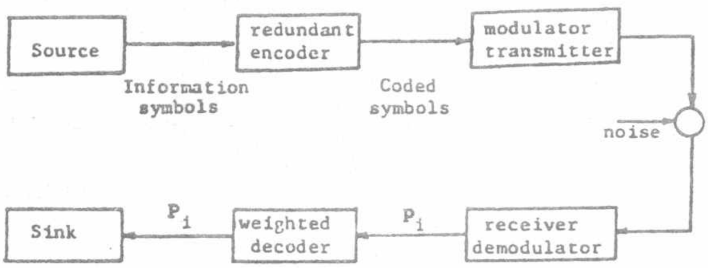

Fig.1 The Communication System Considered

For the binary case, which is practically the most important, instead of vectors p_i or P_i we shall associate a single real number with each decoder input or output. We shall use in the following the a priori algebraic value of a binary symbol b_i

$$a_i \triangleq \log\,(p_{io}/p_{i1}) = \log\,(1 - p_{i1}) - \log p_{i1} \qquad (1)$$

when $b_i = c_i + e_i$, where c_i is the transmitted symbol and e_i is independant of c_i and takes the value 1 with probability $p_i < 1/2$, we can write

$$a_i = S_i \log [(1 - P_i)/P_i] = S_i W_i \quad (2)$$

where the sign represents c_i according to the following rule

$$S_i = +1 \quad \text{if } c_i=0 \qquad (3)$$
$$\quad\; -1 \quad \text{if } c_i=1$$

and W_i is the log-likelihood of e_i, to be referred to as the weighting factor. Similarly defining the a posteriori algebraic value of a decoding decision by

$$A_i = \log (P_{i0} / P_{i1}) \quad (4)$$

We can express the decision rule on the ith symbol as

$$A_i = F(a_0, a_1, a_2, \ldots) \quad (5)$$

As a single real number, the algebraic value thus represents the most likely binary value and the error probability of a random binary variable.

Let C(n,k) be a linear block code with the parity check matrix H. We can select a number r of algebraic replicas for each symbol, where the maximum value of r is r=n-k which is called the exhaustive set of replicas. The only necessary and sufficient condition to formulate the decision rule is to choose the r replicas linearly independent. The aposteriori algebraic value of a decision on a set of statistically independent replicas is the sum of the algebraic values of these replicas [6]. The complexity of the implementation is measured by q^r for q-ary case. More detailed information the replication decoding, the decision rule, and on implementation can be found in [6,7,8] .

ANALYTICAL FORMULATION

As we mentioned before, the space channels are among the few practical communication channels in which errors do not group together in bursts and which can be modeled as an additive Gaussian noise channel. The following analytical formulation, deduced from [6,8] , will help us in simulating the effect of the channel and the demodulator, and in finding a theoretical bound for the decoding error probability with which we will compere our simulation results.

Since the a priori algebraic value depends on the received signal, which is random, so an a priori algebraic value is a random variable. Assuming that $|a| = v$ is given, then (2) results in

$$P_r (\text{sign}(a)=s /|a|=v) = 1-P_i$$

$$P_r (\text{sign}(a)=-s/|a|=v) = P_i$$

Knowing that $|a|= \log(1-P_i/P_i)$, therefore

$$P_r(\text{sign}(a)=s/|a|=v) = e^v P_r(\text{sign}(a)=-s/|a|=v)$$

If a is a continuous random variable having probability density $P_a(a)$, we thus have the following constraint

$$P_a(a) = P_a(-a)\ e^a \qquad (6)$$

In the case when the a priori algebraic values have continuous Gaussian distribution (which is our case), constraint(6) directly results in a variance which equals two times the mean

$$\sigma^2 = 2|m| \qquad (7)$$

Therefore $P_a(a)$ in this case only depends on a single parameter and thus take the form

$$P_a(a) = \left[1 \Big/ 2\sqrt{\pi\ |m|} \right] \exp\left[- (a - m)^2 / 4\ |m| \right] \qquad (8)$$

m can aesly be proved to be

$$m = E(a) = 4S\ E/N_o \qquad (9)$$

Similar results hold for a posteriori algebraic values. The mean a periori error probability P_e results from integrating $P_a(a)$ in the half-axis

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\frac{\sqrt{|m|}}{2} \right) \qquad (10)$$

Substituting equ.(9) in (10), we can write

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{(E/N_o)}$$

Which we will call the apriori error probability or channel error prob. For the decoding error, Battail in 6 approximately evaluated the mean error probability of decoding decision to be

$$P_r \simeq \frac{1}{2} \operatorname{erfc} (\sqrt{M}/2)$$

where M is the mean of the aposteriori values, which was found to have the following lower bound $M \geqslant dm - (k-1) \log 2$ where d is the minimum distance of the used code. Thus, the upper bound on P_r will be

$$P_r \leqslant \frac{1}{2} \operatorname{erfc} \left(\frac{1}{2} \sqrt{dm - (K-1) \log 2} \right) \qquad (11)$$

SIMULATION RESULTS

The performance of the replication decoding algorithm for a linear cyclic block code over additive Gaussian noise channel has been evaluated by computer simulation. We have chosen the Hamming (15,11) code, which is a single error correcting code with minimum distance = 3, to illustrate that the replication decoding-even with such a simple unsphisticated code-will improve the performance of the channel.

The following case has been simulated: 668 words of the H(15,11) code, i.e., 10020 symbols, are transmitted over additive Gaussian noise channel. Each of the received code words, subject to a different noise pattern, is decoded using replication decoding technique. The decoding error rate is calculated, and the channel error rate before decoding is calculated. We also compute the mean and the quadratic mean of the a posteriori algebraic values to see whether relation (7)will be satisfied or not. The above process is repeated for different signal to noise ratios (S/N), and the error probability curves are drawn. The theoretical upper bound of the decoding error probability, and the theoretical channel error probabilities for the different S/N are computed by substituting in the previous section's equations, to be compared with those obtained by simulation.

Table I shows the computed theoretical error probabilities, while table II summariezes the simulation results.

Table I. Theoretical Error Probabilities

S/N	1	1.5	1.7	2	2.5
10 log S/N (dB)	0	1.76091	2.30449	3.0103	3.9794
Channel Error Probability	78.6496×10^{-3}	41.604×10^{-3}	33.101×10^{-3}	22.693×10^{-3}	12.726×10^{-3}
Decoding Error Probability Upper Bound	55.647×10^{-3}	9.3227×10^{-3}	4.7290×10^{-3}	1.7482×10^{-3}	3.4159×10^{-4}

Table II. Simulation Results for The Gaussian Noise Channel Case

S/N	1	1.5	1.7	2	2.5
10 log S/N (in dB)	0	1.76091	2.30449	3.0103	3.9794
The a posteriori Mean M	5.9164928	10.728227	12.773247	15.886452	21.1367
The a posteriori quadratic mean M_q	47.357929	136.09953	187.32985	281.10199	483.1475
Number of Channel Errors	761	412	331	229	117
Number of Decoding Errors	458	87	46	13	2
Channel Error Probability P_e	75.9481×10^{-3}	41.1177×10^{-3}	33.0339×10^{-3}	22.8542×10^{-3}	11.6766×10^{-3}
Decoding Error Probability P_r	45.7085×10^{-3}	8.68263×10^{-3}	4.59081×10^{-3}	1.2974×10^{-3}	1.996×10^{-3}

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From table I and II we can see that the theoretical channel error probabilities are nearly the same as those obtained from the simulation. From table II it is easy to check that the mean and the quadratic mean of the a posteriori algebraic values satisfy equation (7), i.e., $M_q = M^2 + 2M$, as theoretically assumed.

Fig. 2 illustrates the gain obtained by using the replication decoding method. The upper curve is the channel error probability, and the lower curve is the decoding error probability. It is obvious from the curves that the decoding improves the performance, and as the S/N increases the performance increases.

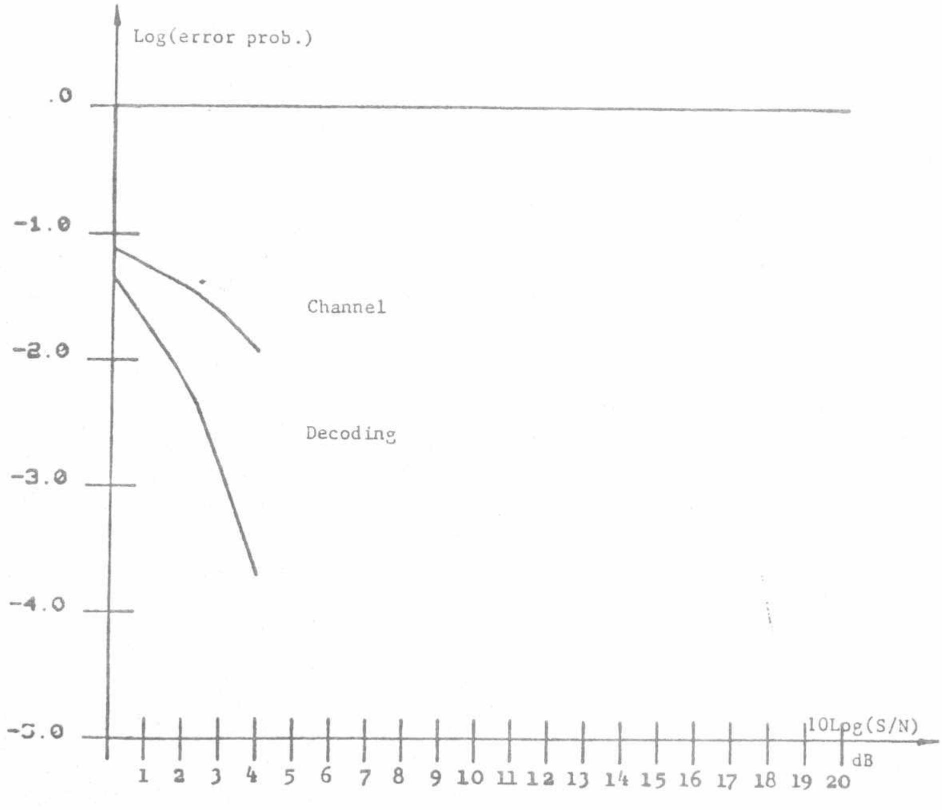

Fig.2 Channel and Decoding Error Prob. Curves

Fig. 3 shows the simulation decoding error probabilities, of table II, and their theoretical upper bound, of table I,drawn versus the signal to noise ratio. From the figure and the tables we can see that all the decoding error probabilities, at the different S/N, satisfy the theoretical upper bound.

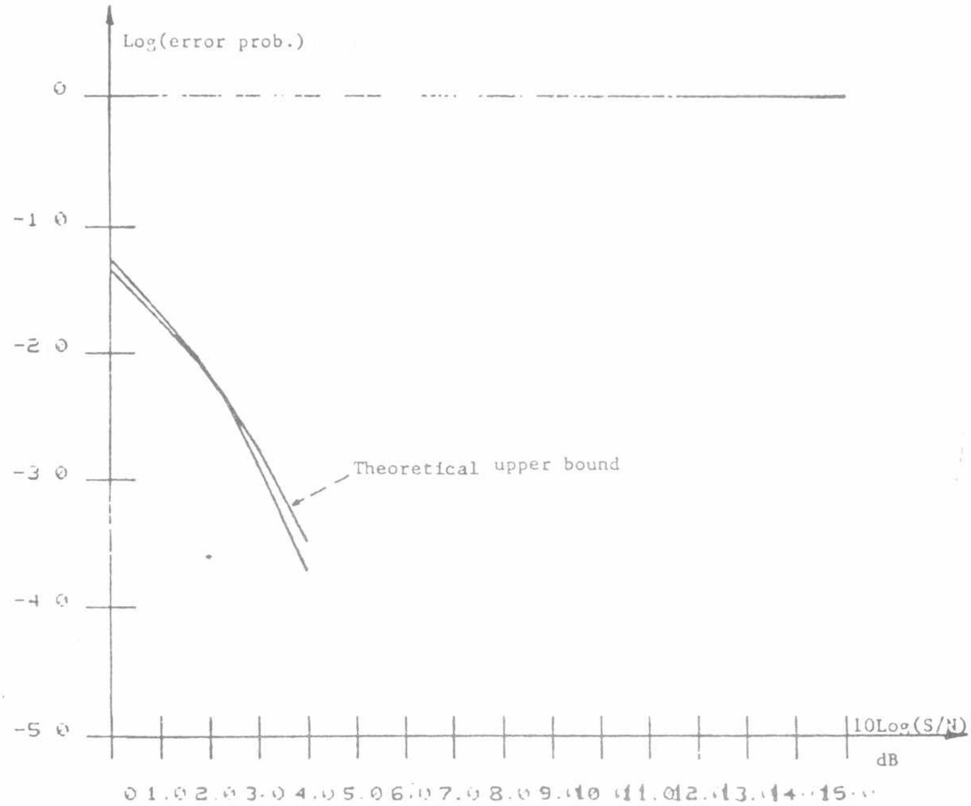

Fig.3 Decoding Error Probability Curve (Simulation) and The Theoretical Upper Bound

All the previous simulation results cnfirm that block coding and replication decoding method improve the performance of the communication system when an additive Gaussian noise channel is assumed. The results support the theory and check the correctness of the formulas for performance prediction.

More ellaborate testing of the decoding method and its physical implementation complexity should be carried out before recommending the method for actual use. But the replication method seems promising, and the preliminary results are encouraging.

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