1 PERFORMANCE ANALYSIS OF 16-ary-OFFSET QAM SIGNALS THROUGH SATELLITE NONLINEAR CHANNELS

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4 ABSTRACT

An expression is derived for numerical computation of the symbol error probability of 16-ary offset quadrature amplitude modulation with sinusoidal shaping (16-ary offset-QAM). The channel considered here is a nonlinear satellite channel with additive Gaussian noise (AGN) in both up-link and down-link.
The main source of the nonlinear distortion in two-link satellite channels is the travelling wave tube (TWT) power amplifier on-board of the satellite. The transponder nonlinearity considered in this paper is of the band-pass type (BPNL), which introduces a nonlinear input-ampliutde to output-amplitude (AM-to-AM) distortion and nonlinear input-amplitude to output-phase (AM-to-PM) distortion . Finally, a comparative study of 16-ary QAM and 16-ary offset QAM with sinusoidal shaping is performed with the BPNL operated at various levels of back-off from saturation.

I. INTRODUCTION

16-state quadrature amplitude modulation (16-QAM) is expected to find increased applications in future communication by satellites due to its spectrum efficiency, its relative simplicity of implementation and good error performance through linear Gaussian channels [1]. However, 16-QAM signals are sensitive to nonlinearities [2-4] . Nonlinearities are encountered in both transmitting earth station and the satellite repeater power amplifier. MORAIS and FEHER[5]discussed an approach in which the 16-QAM is generated in Earth station by summming two QPSK signals (16-QAM/QPSK). Since the QPSK signals has a constant envelope, each signal can be amplified by nonlinear amplifier with a minimum back-off and a nonlineary amplified 16-QAM signal is generated in earth stations (NLA-16-QAM/QPSK).
In this case, however, the transponder nonlinearity has the same effect on the NLA-16-QAM/QPSK signal as that for conventional 16-QAM signal. The same advantage can be offered when QPSK submdulators are replaced by MSK submodulators. Moreover the nonlineary amplified 16-ary-offset QAM generated by two MSK submodulators (NLA-16-ary offset QAM/MSK) is

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expected to perform better than NLA-16-ary QAM/QPSK signal in nonlinear satellite channel due to the one-half symbol interval overlaping between I-and Q-channels and the pulse shapping. In this paper, the methods of EKANAYKA [6], and AGHWAMI [4] are extended to analyze the performance of NLA-16-ary-offset QAM signal (generated by two MSK submodulators) transmitted through nonlinear satellite channel in the presence of additive Gaussian noise (AGN) preceeding (up-link) and following (down-Link) transponder nonlinearity. The transponder nonlinearity, considered in this paper is of the bandpass type, which exhibits AM-to-AM and AM-to-PM effects. Section II is concerned with error probability analysis for the NLA-16-ary offset QAM signal generated by two MSK submodulators transmitted through two-link nonlinear satellite channel. Section III presents computations, results and discussion of results. Section IV contains the conclusion.

II- ERROR PROBABILITY ANALYSIS

A satellite communication system under consideration is modeled as in figure (1). The MSK digital modulated signals can be expressed as [6-9]:-

$$S'(t) = \sum_{K even} a_k\, p(t-KT) \cos w_o t - \sum_{Kodd} a_k\, p(t-KT) \sin w_o t \qquad (1)$$

$$S''(t) = \sum_{K even} b_k p(t-KT) \cos w_o t - \sum_{Kodd} b_k p(t-KT) \sin w_o t \qquad (2)$$

The binary data $a_k(b_k)$ are assumed to be independent and identically distributed. Also, the $a_k(b_k)$ assume the values +1 or -1 with equal probability.

The binary data rate at the input of each MSK is 1/T. The pulse shape p(t) is defined for MSK as [6]

$$p(t) = \begin{cases} A \cos \pi t/2T & -T < t < T \\ 0 & \text{elsewhere} \end{cases} \qquad (3)$$

The transmitted signal s(t) which is the sum of the two signals s'(t) and s"(t), that are 6-dB different in power, takes the form 16-ary offset QAM with sinusoidal shaping [8-9] . It can be expressed as :-

$$s(t) = \sum_{K even} d_k\, p(t-KT) \cos w_o t - \sum_{K\ odd} p(t-KT) \sin w_o t \qquad (4)$$

where d_k is a random variable result from the addition of the two r.v($a_k \mp 2b_k$), that assumes the values $\{\pm 1 , \pm 3\}$ with equal probabilities = $\frac{1}{4}$ [13].

The signal s(t) is bandlimited by a filter whose impulse response is given by :-

$$H(t) = 2h(t) \cos w_o t \qquad (5)$$

where h(t) is the impulse response of the equivalent basehand filter. The response of this filter due to excitition signal s(t) is given by : -

$$S_1(t) = \sum_{K\ even} d_k\, q(t-KT) \cos w_o t - \sum_{K\ odd} d_k\, q(t-KT) \sin w_o t \qquad (6)$$

where :-

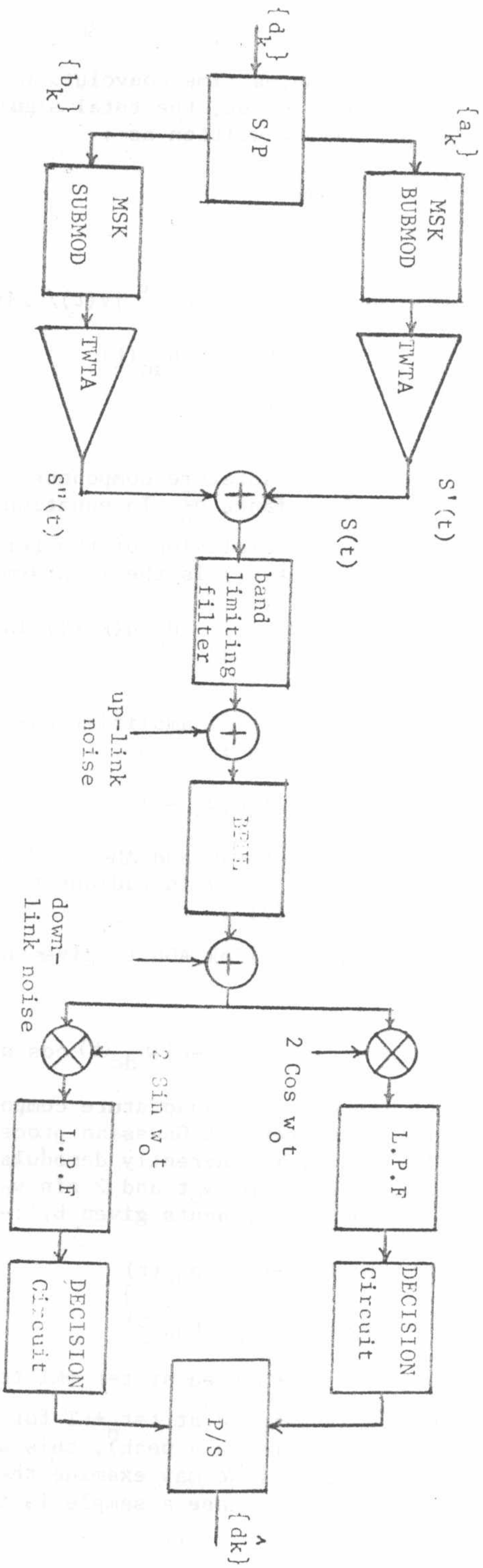

Fig.1. System model.

$$q(t) = p(t)*h(t) \tag{7}$$

In equation (7), the asterisk denotes time convolution. After corruption with up-link narrow-band Gaussian noise, the total signal at the input to the bandpass nonlinearity (BPNL) may be written as :-

$$S_2(t) = R(t) \cos \left[w_o t + \Phi(t)\right] \tag{8}$$

where :-

$$R^2(t) = x^2(t) + y^2(t) \quad \text{and} \quad \Phi(t) = \tan^{-1} |y(t)/ x(t)|$$

$$x(t) = d_o q(t) + \sum_{K even}' d_k\, q(t-KT) + n_{uc}(t) \tag{9}$$

$$y(t) = \sum_{Kodd} d_k\, q(t-KT) + n_{us}(t)$$

n_{uc} and n_{us} are the in-phase and quadrature components of the narrowband up-link AGN with zero mean and variance σ_u^2 In equation (9), the notation Σ' is used in order to indicate the exclusion of the term K=0.
It is to be noted that $\sum'_{Keven} d_k q(t-KT)$ is the intersymbol interence (ISI) terms in the in-phase channel, and $\sum_{Kodd} d_k\, q(k-KT)$ is the ISI terms in the quadrature channel.

The signal $S_2(t)$ is amplified by the TWT amplifier on-board of the satellite, and the output can be written as :-

$$S_3(t) = f(R) \cos\left[w_o t + \Phi(t) + \psi(R) - \varepsilon\right] \tag{10}$$

where f(.) and (.) denotes the AM-to-AM and AM-to-PM conversion respectively of the BPNL and ε is a phase delay in radians to account for the AM-to-PM compensation of the BPNL [7] .

$S_3(t)$ is now corrupted with the downlink AGN to give the input to the coherent receiver as :-

$$S_4(t) = f(R) \cos |w_o t + \Phi(t) + \psi(R) - \varepsilon| + n_{dc}(t) \cos w_o t - n_{ds}(t) \sin w_o t \tag{11}$$

where n_{dc} and n_{ds} are the in-phase and quadrature components of the down-link Gaussian noise that are independent Gaussian processes with zero mean and variance σ_d^2. The receiver coherently demodulates the input signal $S_4(t)$ with the reference carrier 2 cos $w_o t$ and 2 sin $w_o t$ to give the in-phase and quadrature baseband components given by :-

$$\left.\begin{aligned} x_4(t) &= f(R) \cos[\Phi(t) + \psi(R) - \varepsilon] + n_{dc}(t) \\ y_4(t) &= f(R) \sin[\Phi(t) + \psi(R) - \varepsilon] + n_{ds}(t) \end{aligned}\right\} \tag{12}$$

The in-phase baseband waveform is sampled at $t=t_o+KT$ for K even and the quadrature baseband waveform is sampled at $t=t_o+KT$ for K odd (t_o is the instant at which the pulse q(t) attains a peak), this assumes perfect carrier and symbol synchronization . We may examine the signal during any data interval, say $-T< t< T$, in this case a sample is taken at $t=t_o$ from the in-phase baseband component and a decision is made to determine whether (d_o) is one event from the set {$\pm$ 1, $\pm$3}. The value of the in-phase baseband sample at $t=t_o$ may be written as :-

$$x_4(t_o) = f(R) \cos[\phi(t_o) + \psi(R) - \varepsilon] + n_{dc}(t_o) \tag{13}$$

Since the symbol transmitted (d_o) can take a value from the set {$\pm$1,$\pm$ 3} with equal probabilities, the average symbol error probability denoted by P_e is given by [4]

$$P_e = \frac{1}{4} \sum_{i=1}^{4} P_{e, \Delta_i} \tag{14}$$

where $P_{e,\Delta i}$ denotes the error probability assuming the symbol transmitted at $t=t_o$ (k=o) takes a value d_{o_i} (with i=1 to 4) and Δ_i indicates the corresponding received symbol. The conditional probability density functions corresponding to the four input (transmitted) symbols could be then derived. Since the symbols are equiprobable, the optimum threshold leves are assumed between the values of the maximum levels at each symbols, it is assumed at d',0,-d'. Omitting the time variable t_o in equation (13) for notation conveniences then ;

$$x_4 = f(R) \cos\theta + n_{dc} \tag{15}$$

where $\theta = \phi + \psi(R) - \varepsilon$

It is shown that [7] :-

$$P_{e, \Delta_1} = P_{e, \Delta_3} \text{ and } P_{e, \Delta_2} = P_{e, \Delta_4} \tag{16a}$$

From equation (14) thus :

$$P_e = \frac{1}{2} (P_{e, \Delta_1} + P_{e, \Delta_2}) \tag{16b}$$

where

$$P_{e,\Delta_1} = 1 - \int_o^{d'} P_{\Delta_1}(x_4)\, dx_4 \tag{17a}$$

$$P_{e,\Delta_2} = 1 - \int_{d'}^{\infty} P_{\Delta_2}(x_4)\, dx_4 \tag{17b}$$

$P_{\Delta_1}(x_4)$ [$P_{\Delta_2}(x_4)$] denotes probability density function(Pdf) of x_4 assuming Δ_1 [Δ_2] is received. Equation (17a) can be written as :-

$$P_{e, \Delta_1} = 1 - \int_o^{d'} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x_4/x,y)\, P_{\Delta_1}(x,y)\, dxdy \right] dx_4 \tag{18}$$

where $p(x_4/x,y)$ denotes (Pdf) of x_4 conditional on n_{uc} and n_{us} and ISI random variables i.e conditioned on x and y given in equation (9).

$P_{\Delta_1}(x,y)$ is the joint pdf of x and y assuming Δ_1 is received i.e d_o=1 .

In equation (15) x_4 may be regarded as Gaussian random variable with mean= f(R) cos θ and variance σ_d^2 , then :-

$$P(x_4/x,y) = \frac{1}{2\ \sigma_d^2} \quad \exp\left[-\left(\frac{x_4 - f(R)\ \cos}{\sqrt{2\ \sigma_d^2}}\right)^2\right] \qquad (19)$$

Substituting (19) into (18), we obtain

$$P_{e,\Delta_1} = 1- \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\frac{1}{\sqrt{2\ \sigma_d^2}} \int_0^{d'} \exp\left[-\left(\frac{x_4 - f(R)\cos\theta}{\sqrt{2\ \sigma_d^2}}\right)^2\right]dx_4\right\}$$

$$P_{\Delta_1}(x,y)\ dxdy$$

$$= 1- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{erf}\left(\frac{f(R)\ \cos\ \theta}{\sqrt{2\ \sigma_d^2}}\right) + \text{erf}\left(\frac{d' - f(R)\cos\theta}{\sqrt{2\ \sigma_d^2}}\right)$$

$$\times\ P_{\Delta_1}(x,y)\ dxdy \qquad (20)$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-u^2)\ du \quad \& \quad \text{erf}(-x) = -\text{erf}(x)$$

In the same way P_{e,Δ_2} in equation (17b) can be written as :-

$$P_{e,\ \Delta_2} = 1- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[1- \text{erf}\left(\frac{d' - f(R)\cos}{\sqrt{2\ \sigma_d^2}}\right)\right] P_{\Delta_2}(x,y)dxdy \qquad (21)$$

In wide-hand satellite channel i.e. without ISI terms in equation (9), the random variables x and y which can be written as

$$x = d_o q_o + n_{uc} \quad \text{and} \quad y = n_{us}$$

are independent Gaussian random variables with mean $d_o q_o$ and zero respectively and variance σ_u^2 . Hence the joint pdf of x and y $P_{\Delta_1}(x,y)$ can be written as :-

$$P_{\Delta_1}(x,y) = \frac{1}{2\pi\ \sigma_u^2} \exp\left[-\left(\frac{x-q_o}{\sqrt{2\ \sigma_u^2}}\right)^2\right] \exp\left[-\left(\frac{y}{\sqrt{2\ \sigma_u^2}}\right)^2\right] \qquad (22a)$$

$$P_{\Delta_2}(x,y) = \frac{1}{2\pi\ \sigma_u^2} \exp\left[-\left(\frac{x-3q_o}{\sqrt{2\ \sigma_u^2}}\right)^2\right]\exp\left[-\left(\frac{y}{\sqrt{2\ \sigma_u^2}}\right)^2\right] \qquad (22b)$$

Substituting equations (22a) and (22b) into equation (20) and (21) , we obtain :-

$$P_{e,\Delta_1} = 1 - \frac{1}{4\pi\,\sigma_u^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\mathrm{erf}\left(\frac{f(R)\cos}{\sqrt{2\,\sigma_d^2}}\right) + \mathrm{erf}\left(\frac{d' - f(R)\cos}{\sqrt{2\,\sigma_d^2}}\right)\right]$$

$$\exp\left[-\left(\frac{x-q_o}{\sqrt{2\,\sigma_d^2}}\right)^2\right] \exp\left[-\left(\frac{y}{\sqrt{2\,\sigma_d^2}}\right)^2\right] dxdy \qquad (23a)$$

$$P_{e,\Delta_2} = 1 - \frac{1}{4\pi\,\sigma_u^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[1 - \mathrm{erf}\left(\frac{d' - f(R)\cos}{\sqrt{2\,\sigma_d^2}}\right)\right] \exp\left[-\left(\frac{x-\beta q_o}{\sqrt{2\,\sigma_d^2}}\right)^2\right]$$

$$\times \exp\left[-\left(\frac{y}{\sqrt{2\,\sigma_d^2}}\right)^2\right] dxdy \qquad (23b)$$

III.COMPUTATION-RESULTS AND COMMENTS

The symbol error rate (SER) for a wideband satellite channel is obtained by substituting equations (23a) and (23b) into equation (16b). The expressions for P_{e,Δ_1} and P_{e,Δ_2} are readily evaluated using the Cartesian products of Gauss-Hermite formulas [12] . The computation requires knowledge of the nonlinear AM-to-AM and AM-to-PM functions ($f(.)$ and $\psi(.)$ respectively. Furthermore, the quantities q_o , ε, d', σ_u and σ_d have to be specified.

The following analytical expressions for $f(.)$ and $\Psi(.)$ represent the amplitude and phase nonlinearities modul [2],[11]

$$F(R) = \begin{cases} 10^{(\alpha[\cos\{\log_{10}(R/\hat{R})/\beta\}-1])} & , R > \tilde{R} \\ R & , R < \tilde{R} \end{cases} \qquad (24)$$

$$\psi(R) = K_1\{1 - \exp(-K_2 R^2)\} + K_3 R^2$$

where $\alpha, \beta, \tilde{R}, K_1, K_2$ and K_3 are constants chosen to fit measured data and $\hat{R}$ is the amplitude which causes saturation. A TRW DSCS II Satellite TWT is assumed in all our computation and for this tube Thomas et al . [10] give the following values : $\alpha=0.394, \beta=0.475$, $\hat{R} = 2.317$, $\tilde{R} = 0.355$ $K_1=0.602$, $K_2=0.66$, and $K_3= 1/102.4$

The parameter q_o for wideband channel is the maximum amplitude of RF signal and given by [7]

$$q_o = (\hat{R}/\sqrt{5}) \times 10^{(-B_o/20)} \qquad (25)$$

Bo is the degree of back-off from saturation to the given TWT power amplifier in dB.

d' and ε which are the decision threshold level and phase compensating angle respectively, can be arrived at by minmizing the average SER. The up-and down-link noise variance σ_u^2 and σ_d^2 are expressed in terms of the uplink and downlink SNR's ρ_u^2 and ρ_d^2 as follows :-

$$\rho_u^2 = \bar{P}_T/\sigma_u^2 \qquad (26)$$
$$\rho_d^2 = \bar{P}_R/\sigma_d^2$$

where $\bar{P}_T$ and $\bar{P}_R$ denote the average transmitted and received power.
For 16-ary offset-QAM signal with sinusoidal shaping, the amplitude q_o is related to the average symbol energy by ;-

$$E_Q = q_o^2 \left[1^2 + 3^2 \right] \int_{-T}^{T} P^2(t)\, dt/2 = 5\ q_o^2\ T \qquad (27)$$

where p(t) given by equation (3) and $A=q_o$ for widehand channels. The average transmitted power = $E_Q/2T$

$$\bar{P}_T = 2.5\ q_o^2 \qquad (28)$$

For all values of up-link SNR ρ_u^2 used in computation, the up-link AGN power is much smaller than the power in the transmitted QAM signal, hence negligible error is introduced by neglicting the up-link AGN in calculation of average power output from transponder power amplifier $\bar{P}_R$. Considering all states of the transmitted signal and using the approach of ADEL-A.H. SALEH [10] to calculate the output signals for two input signals to the BPNL, the average power $\bar{P}_R$ is given by [7] :-

$$\bar{P}_R = \frac{1}{8} \left[\{f(q_o)\}^2 + \{f(\sqrt{10}\, q_o)\}^2 \times 2 + \{f(3q_o)\}^2 \right] \qquad (29)$$

The appropriate values of d' and ε can only be arrived at by minimizing the average symbol error probability (SER)
For accepted precision it is found that an approximation of the double integration by 44-points, degree 15 [12] is adaquate.
The minimum average symbol error probability as a function of up-and down-link SNR's (ρ_u^2 and ρ_d^2) for various amounts of back-off are computed.

It is , found that the final values of d' and ε depend on the back-off parameter Bo, up-link and down link SNR'S. The optimum values of d' and ε for various degree of back-off are given below for ρ_u^2 = 3o dB and ρ_d^2 = 24 dB:-

Bo = 3 dB	$d' = 0.9\ q_o$	$\varepsilon = 63^o$
Bo = 6 dB	$d' = 1.3\ q_o$	$\varepsilon = 45^o$
Bo = 9 dB	$d' = 1.7\ q_o$	$\varepsilon = 26^o$

It is noted from the above values and other values not included her that the values of d' and ε depend mainly on the back-off (Bo) values and d' increases with increasing back-off degree tell it takes the value of $2q_o$ when BO $\geq$15 dB. The compensation angle decreases with increasing the back-off values, it approachs a zero value when the back-off 15 dB (linear operation of TWT). Figure (2) and (3) show the results of computation of symbol error probability versus down-link SNR ρ_d^2 , with uplink SNR ρ_u^2 as a parameter for different values of back-off degree (3,9,12, and 15 dB). It is noted from figure (3) that the system performance is improved as the amount of back - off from saturation is increased. Figure (3) shows that the effect of back-off values on performance curves is decreased as the degree of back off takes higher values ($\geq$ 15 dB). In this case the transponder power amplifier operates near linear region. Other computations, not included here, reveal that the system performance ultimately reaches its optimum when the back-off is greater than 15 dB.

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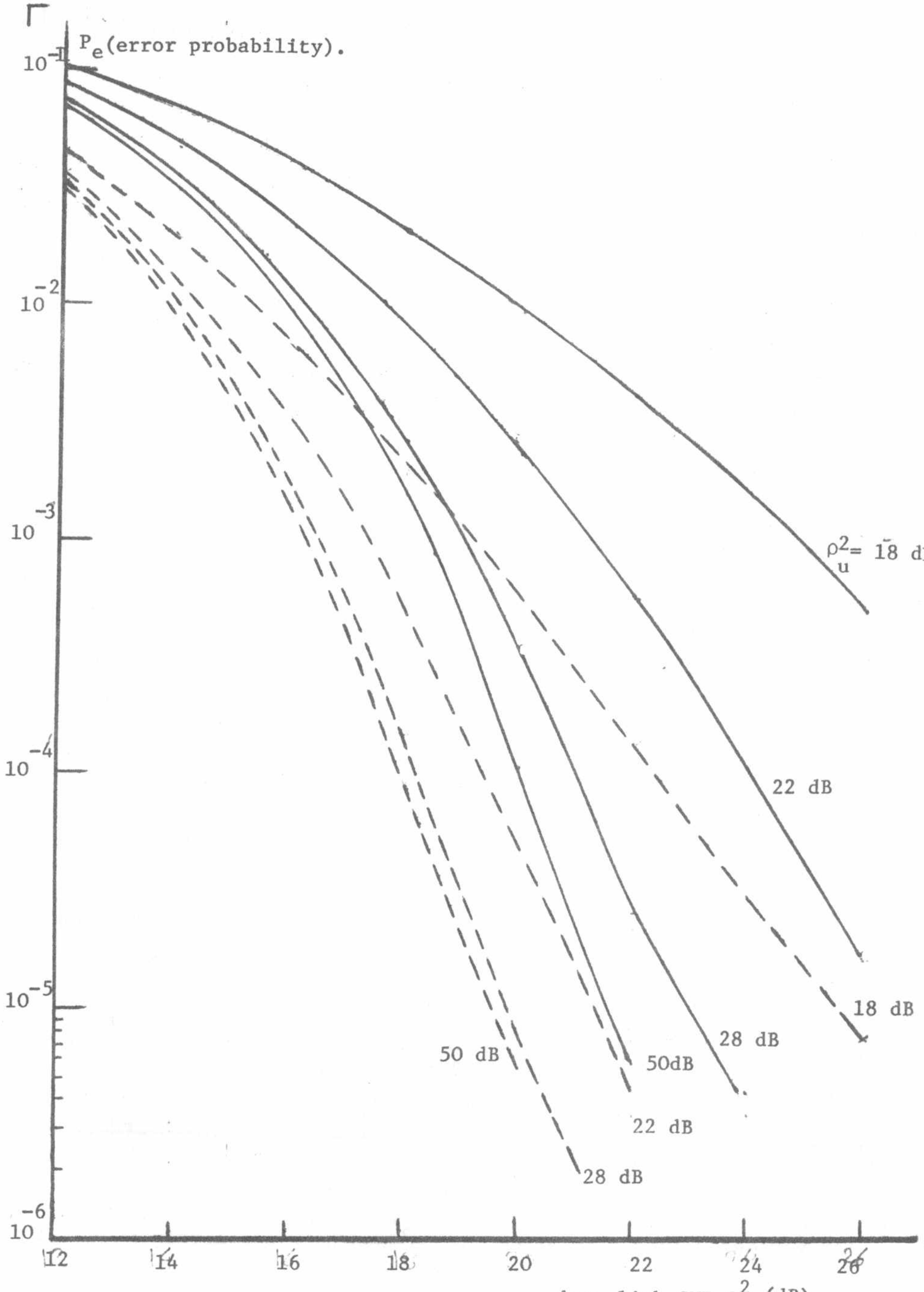

Fig.2. Symbol error probability of 16-ary offset QAM signal transmission for various values of up-link SNR ρ_u^2 and back-off levels

——— 3 dB

- - - - 9 dB

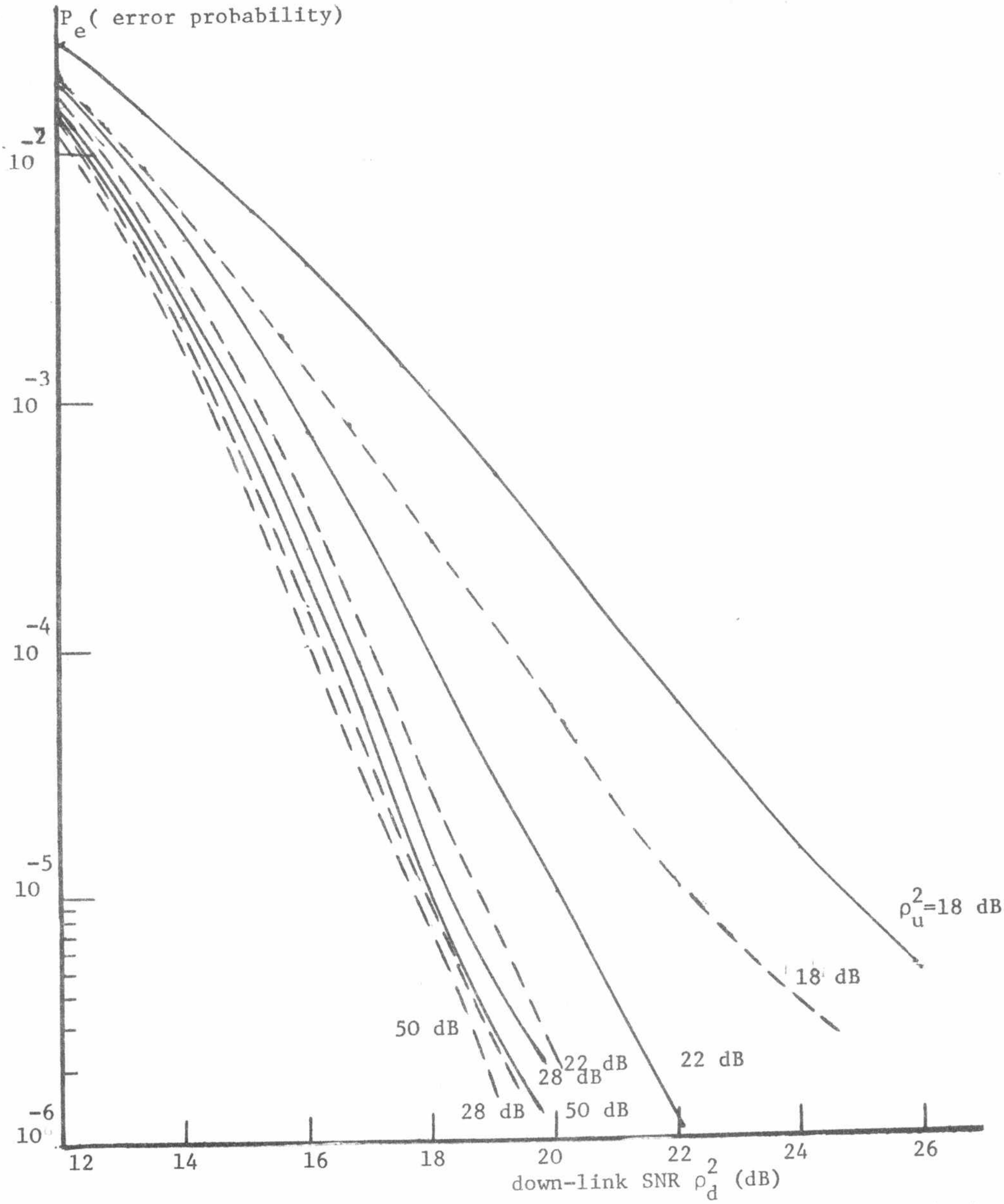

Fig.3. Symbol error probability of 16-ary offset QAM single transmission for various values of up-link SNR ρ_u^2 and back-off levels :-

———— 12 dB
-------- 15 dB

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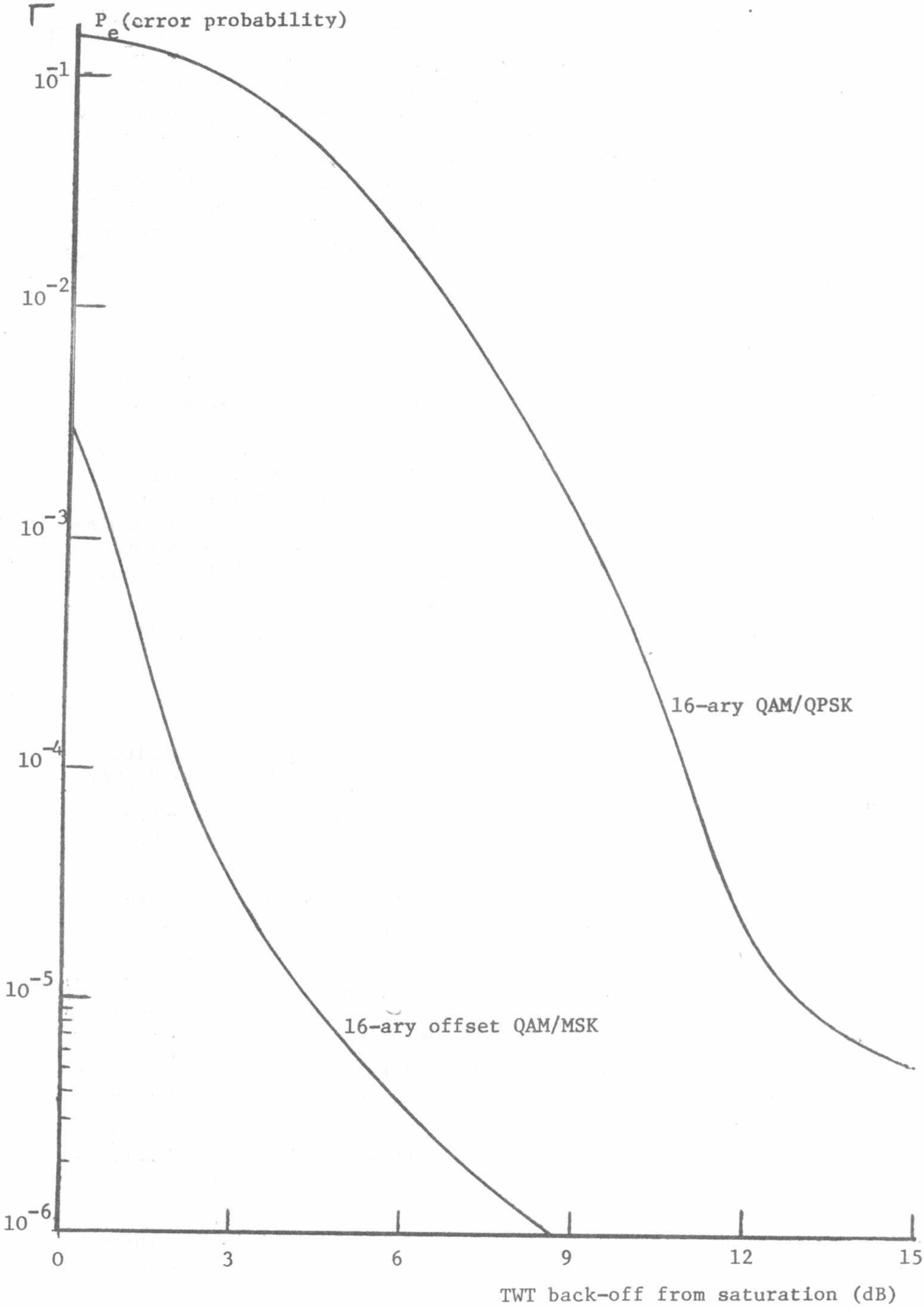

Fig.4. Error probability as a function of TWT back-off from saturation for 16-ary offset QAM/MSK and 16-ary QAM/QPSK transmission with ρ_u^2 =28 dB and ρ_d^2=22 dB

For the purpose of comparison, figure (4) shows the effect of back-off on performance of NLA-16-ary offset QAM/MSK and NLA-16-ary-QAM/QPSK(or conventional 16-QAM) transmission. The up-and down link SNR's are fixed at 28 dB and 22 dB, respectively , and the average symbol probabilities for both signals as a function of back-off (Bo) are computed. It is clearly evident from figure (4) that : at 10^{-4} symbol error rate, 9 dB improvement in back-off degree is obtained by 16-ary offset QAM generated by two MSK submodulator compared with 16-ary QAM generated by two QPSK submodulator when transmitted over wideband nonlinear satellite channel.

III CONCLUSION

A method has been presented for evaluating the performance of NLA-16-ary offset QAM/MSK(that generated by two MSK submodulators) transmission through a two link nonlinear satellite channel. The up-and down-link Gaussian noise and AM-to-AM and AM-to-PM distortions of the transmponder power amplifier have been taken into consideration in the analysis. A study of the effect of back-off of the TWT from saturation is included and the system performance is presented. Compared to conventional 16-QAM transmission, an improvement in system performance in order of 9 dB in back-off degree at symbol error rate of 10^{-4} when up-link SNR ρ_u^2 = 28 dB and down-link SNR ρ_u^2 = 22 dB.

Also optimum performance is obtained at 15 dB back-off compared to 21 dB back-off in the case of conventional 16-QAM signal [4] .

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