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PARAMETRIC ESTIMATION OF A DC MOTOR

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ABSTRACT

There is a continuing need to develop new experimental techniques for identifying linear models to represent the dynamic behaviour of electric motors. The traditional methods, based upon the application of sinusoidal and step input functions, have yielded limited information due to the presence of noise and other factors. The author has considered the particular case of a dc motor and shown that the experimental determination of the linearised dc motor parameters may be reformulated as a problem of estimating the parameters in a state-space model of known structure.

The present paper extends this work to a dc motor when a pseudo-random binary sequence signal used to perturb the rotor. In previous experimental investigation of these motors, it does place restrictions on the form of input signal, whereas here restrictions are focused on the sequence length and clocking interval of these signals. A discrete-time state-space model of the same structure is fitted here to sampled observations from laboratory experiments. The estimated discrete-time parameter matrix is then transformed to obtain estimates of the linearised dc motor parameters. It is shown that this new formulation eradicates possible errors in the interpretation of results which may arise from the steady-states appearing in the measurement. A formula which describes the relation between a system bandwidth and clocking interval, is outlined. The convergence of the obtained estimates is proved. Lastly, experimental results for the estimates of the motor parameters are presented.

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INTRODUCTION

A satisfactory mathematical model for a physical system must often reflect a compromise. It must not attempt to mirror the real system in such great detail that the model becomes cumbersome; on the other hand, it should not be so simplified that predictions and explanations based on it are either trivial or far from reality. The choice of model is critical because many factors can affect the performance of such devices.

It is interesting to note that although, direct current motors are used in a large variety of industrial control systems, mainly due to their high starting torque, simple regulation properties and reliability.

In practice, dc motor parameters are nonlinear [1], however, it has often been found that such a model does not adequately described the transient conditions and for many applications a more accurate model is desirable. Furthermore, a realistic method for identification must take into account the fact that in practice all measurements are contaminated with noise.

A technique applied to process type systems is that of the Kalman technique suggested by Mayne and developed by the author [2]. The developed technique accepts noisy measurements of the motor state vector due to pseudo-random binary sequence (PRBS) input signal and produce up-dated estimates of the motor parameters.

Whilst the proposed technique offers an efficient solution to the estimation of dc motor parameters, it does place restrictions on the clocking interval, sequence length, and signal amplitude of the PRBS signal used to perturb the armature of dc motor.

The author has attempted to develop a more coherent experimental strategy. In particular, he has been concerned with relaxing those prior assumptions which can lead to the misinterpretations of experimental results. The present paper describes the practical application of this new approach to the estimation of a dc motor when PRBS measurements are taken and it shows to possess several adjustments to its frequency spectrum. Using the digital computation, the effects of varying the PRBS parameters, are illustrated.

NOTATION

A	Dynamical parameter matrix
B	Input transducer matrix
C	Output transducer matrix
F	Discrete parameter matrix
G	Discrete input transducer matrix
g(s)	Transfer function
H	Discrete output transducer matrix
I	Identity matrix
k	Index integer.
PRBS	Pseudo-random binary sequence
S	Laplace variable
T	Developed torque
T_s	Sampling time

Δt	Clocking interval of PRBS
u	Input vector
V	Armature voltage
x	State vector
x(k)	Discrete state vector
y	Output vector
y(k)	Discrete output vector

THEORETICAL ASPECTS

Consider a separately excited dc motor and it contains all linear parameters of interest, as a minimum, the effective values of armature resistance (R), inductance (L), inertia (J), viscous damping (B), torque constant (K_T), and voltage constant (K_b). For small perturbation about a steady-state operating speed, the dynamic state of the motor can be represented by the following equations [2];

$$\frac{di}{dt} + \frac{R}{L}\, i = \frac{V}{L} - \frac{K_b}{L}\,\omega$$
$$\frac{d\omega}{dt} + \frac{B}{J}\,\omega = \frac{T}{J} = \frac{K_T}{J}\, i \qquad (1)$$

By defining a suitable set of state variables, for example:

$x_1 = \omega$ the angular speed in radians per second
$x_2 = i$ the armature current in amperes.

then equation (1) may be reformulated as

$$\dot{\underline{x}} = \underline{A} \cdot \underline{x} + \underline{B} \cdot \underline{u} \qquad (2)$$

where

$$\underline{A} = \begin{bmatrix} -B/J & K_T/J \\ -K_b/L & -R/L \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} o \\ 1/L \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where x_1 and x_2 are the perturbations in the motor speed and current due to the random component of the input voltage V.

The corresponding output equation may be written as;

$$y = \underline{C} \cdot \underline{x} \qquad (3)$$

where

$$y = [y_1 \quad y_2]^T$$

and $\underline{C}$ may be taken as the identity matrix since all elements of the state vector $\underline{x}$ are directly available for measurement. The state variable diagram is shown in Fig. 1.

The element of direct interest for system identification is g(s) relating speed to the PRBS forcing, that is

$$g(s) = \frac{y_1(s)}{u(s)} = \frac{K_T}{(LS+R)\,(JS+B) + K_T K_b} \qquad (4)$$

If all six parameters are unknown then applying the concept of structural identifiability [3] it can be shown that the identification of g(s) will not yield sufficient information to assign numerical values to all the parameters. Alternatively, if the parameters are to be determined by estimating the parameter matrices A and B, then if all the states are directly observable, as in the current application, identification is only possible if the system is controllable [3].

An estimate of the motor parameter matrices can be obtained by fitting a discrete-time state-space model to sampled observations of the input signal u and output vector y. Equations (2) and (3) can be rewritten as:

$$x(k+1) = F.\,x(k) + G.u(k) \tag{5}$$
$$y(k) = H.x(k) \tag{6}$$

where u(k), x(k), y(k) are the sampled versions of u, x, and y respectively at the k th sampling instant and H = I without loss of generality.

The discrete-time parameter matrices, F and G are estimated from the sampled records y(k) and u(k) inside the developed Kalman technique. The estimated F matrix is transformed to the continuous-time by means of a similarity transformation [4]. This transformation requires that F is nonsingular and the particular algorithm used assumes that the eighenvalues of F are distinct so that diagonalization is possible. This technique avoids most of identification problems and there are many other reasons, some of which are: **(i)** it can be used with the process under operation; **(ii)** it converges in fewer iterations than other techniques even in the presence of large noise; **(iii)** it is based on input/outputs measurements that may be obtained during normal process operation, thus facilitating on-line identification as long as a transient appears in the measurement; **(iv)** real-time identification became accepted with the advancement and availability of fast digital computer.

The use of the above technique requires a persistently exciting input perturbation if the estimates are to be consistent [5]. Signals derived from PRBS are capable of fulfilling this condition providing a suitable voltage transducer is available.

THE FREQUENCIES OF PRBS

It is well known that **white** noise possesses constant power per unit bandwidth at all frequencies. Therefore this signal is capable of persistently exciting all of a system's modes. It is impossible in practice to generate **white** noise, but a random signal may be considered **white** if it possesses a uniform power spectrum over the complete frequency range to which a system will respond. Hence **colored** or bandlimited **white** noise is a suitable test signal for determining the frequency response characteristic of a system. Alternatively a pseudo-random signal can be used. In their simplest form these signals assume the values $\pm$ **a**, and this amplitude can be made sufficiently small to avoid saturation. They are termed pseudo-random binary sequences because their auto-correlation functon approximates to the ideal impulse obtained by auto-correlating **white** noise.

As has been mentioned, the period of PRBS obtained from an n-stage shift registers has value N. Δt where N is the maximum length sequence given by (2^n-1) and Δt the period of the clock generating the sequence. The outputs of the PRBS have continuous line spectra extending down to dc and the harmonic spacing being established by the combination of Δt and N settings.

The power spectrum $\phi(\omega)$ for a pseudo-random binary sequence is given by [6].

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$$\phi(\omega) = \frac{a^2}{N} \left(\frac{\sin \omega\Delta t/2}{\omega\Delta t/2}\right)^2 \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N\Delta t}\right) \qquad (7)$$

where δ denotes Dirac's delta function. Hence the PRBS is a suitable multi-frequency test signal having a bandwidth $f_b = 0.45/\Delta t$, which is controlled by the clocking interval Δt. We notice that the three parameters Δt, a, and N completely define this signal. Null occur at multiples of the clock frequency $f_c = 1/\Delta t$ and the fundamental occurs at $f_r = 1/N.\Delta t$ as shown in Fig. 2.

The line spectrum given by equation (7) contains significant components above the bandwidth f_b, and low-pass fittering is often required to avoid contamination of the system response.

RANGE OF THE CLOCKING INTERVAL

Given that a PRBS signal was to be used, it was necessary to arrange for the sampling process to be synchronized with the input sequence. This is calculated as folows;

Assuming that, in general, n_s samples were required during each PRBS clocking interval, the desired sampling frequency, f_s is given by

$$f_s = \frac{n_s}{\Delta t} \qquad (8)$$

The sampling rate must be at least twice the desired closed-loop bandwidth and possibly as much as 20 times the system bandwidth, depending on the particular performance requirements of the system [7]. Thus, the number of samples per clocking interval is an integer with a minimum value of 2.

Previous computational investigations [6] show that the value of Δt should be smaller than the system time constant and $N.\Delta t$ should be bigger than the settling time. The settings of the sampling time and the clocking interval require the system bandwidth which gives the dynamic of interest. Thus, to excite all the system's modes, the bandwidth of PRBS input signal is adopted from 2 up to 5 times the system bandwidth, or

$$2 f_o < \frac{0.45}{\Delta t} < f_o$$

or

$$\frac{90}{f_o} \leq \Delta t < \frac{225}{f_o} \qquad (9)$$

where Δt in m.sec and the system's bandwidth, f_o in Hz.

The sampling time is obtained from the reciprocal of equation (8) and is given by

$$T_s = \frac{\Delta t}{n_s} \qquad (10)$$

where $n_s = 2, 3, 4,$

COMPUTATIONAL RESULTS

Before performing the experimental tests, it was necessary to estimate the best settings of Δt, N, and T_s. The problem has been examined with the aid of a computer simulation of a separately-excited dc motor model. The model equations (5) and (6) were solved to yield the states and the Kalman technique procedure used to reconstruct the original equations. The computational scheme developed in the previous paper [2] has been programmed for digital computation. Five models were chosen for investigation. The log-magnitude curves are used to obtain the bandwidth for each model. The computational tests are carried out for various values of clocking interval. Best estimated results are obtained when these values are satisfied equation (9). In fact, there is no noticeable difference between the cases for $N = 127$ and $N > 127$. Fig. 3 shows the effect of n_s on the estimated parameter. Referring to Fig. 3, it is apperent that in the case when $n_s = 2$, the convergence of the estimated parameter is slower due to the low density of the power spectrum. This density is held constant for all values of $n_s > 5$. For $n_s > 10$, thousands of iterations are required for good estimates.

Now consider a dc motor model described by the following parameters: $R = 1.0$, $L = J = 0.08$, $B = 0.04$, $K_T = K_b = 1.2$ in SI system. The log-magnitude curve of the motor with these parameters is yielding a motor's bandwidth of 2.95 Hz. The analysis obtained in the previous section (equations (9) and (10)) was used to adopt the PRBS parameters. A sequence length of 127 bits/period, clocking frequency of 20 Hz, and sampling time of 10 m.sec. were found suitable for this model. With these settings, a minimum number of iterations is required. The estimated motor parameters are shown plotted against number of iterations used in Fig. 4. These estimates were used in conjunction with the exact values of the motor parameters.

EXPERIMENTAL PROCEDURE AND RESULTS

The main parameters of the separately-excited dc motor used in this work are: 4KW, 220V, 20A, 1450 rpm. The PRBS signal source was obtained from the random noise generator design and built by the author as shown in the hardware circuit of Fig. 5. A multi-frequency range function generator is used for controlling the clock interval of the PRBS.

The perturbation of the rotor by a voltage derived from the PRBS was not straight forward due to the unfavourable transmission characteristics of the available random generator. The problem was solved by using the circuit shown in Fig. 6. This circuit contains variable power resistance, power transistor, and opto-coupler. The voltage drop, V_B, represents the peak-to-peak value of PRBS signal. The power transistor (N-P-N type 2N 3055) is operated as a parallel switch across the power resistance R_B. The opto-coupler (type 4N 26) is used to isolate the PRBS signal generator from the dc power source. The output of the random generator is supplied to the base-emitter terminals of the power transistor through the opto-coupler with sufficient magnitude. The current is monitored by measuring the voltage drop across R_D resistor. A microprocessor is interfaced with the motor to sample the random variations of the motor state signals (speed and current).

A series of tests was performed at the natural frequency of the motor to determine the interval time / sampling time characteristics. Also, it was found that an input voltage magnitude of $\pm$ 10V gave a good compromise between the need to obtain measurable deflection (approximately 75 rpm) whilst avoiding nonlinear behaviour.

Using the real-time spectrum analyser, the effects of varying sequence length and clocking frequency were studied. Given that the natural frequency (about 3 Hz), a sequence length of 127 bits/period, interval time of 50 m.sec., and smapling frequency of 100 Hz were found appropriate. The bandwidth of such a sequence is approximately 9 Hz. These selected PRBS parameters are confirming the relations of equations (9) and (10). The motor state signals at these parameters are shown in the photograph of Fig. 7. The estimates of the motor parameters obtained from the discrete-time modelling are shown plotted in Fig. 8.

DISCUSSION

It has been shown that the spectra analysis applied to a dc motor excited with a PRBS signal provides excellent estimates of the true parameters of the dc motor as obtained theoretically from **white** noise analysis.

Referring to Fig. 8. there are considerable agreements between numerical values of the estimated parameters and those representing the real system. Also there appears to be computational scatter in the estimates, thus raising questions about the accuracy of the transformation from the discrete-time matrices to the continuous matrices.

CONCLUSIONS

It has been shown that the parameters of a separately-excited dc motor can be estimated using discrete-time modelling. This has been achieved using speed perturbations of less than 6 per cent of the nominal motor speed. Given the dynamics of interest then the suitable sequence length, clocking frequency and sampling rate of the PRBS input signal can be determined from a formula derived here. The experimental ability of the PRBS testing to provide persistent excitation of the dc motor modes was checked. Using the technique mentioned here, dc motor parameters can be estimated from measurements of the applied voltage and corresponding rotor speed and current. The estimates converge to their correct values very fast and the amount of computation is smaller than that required in other algorithms with the exception of stochastic approximation. Using the chosen PRBS excitation, the time taken to acquire the experimental data was approximately 45 seconds.

It is the author's opinion that this scheme offers considerable promise. It could be applied to practical systems with further investigations.

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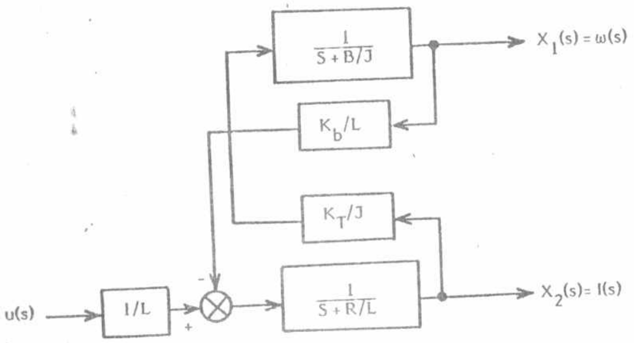

Fig. 1. State variable of dc motor model.

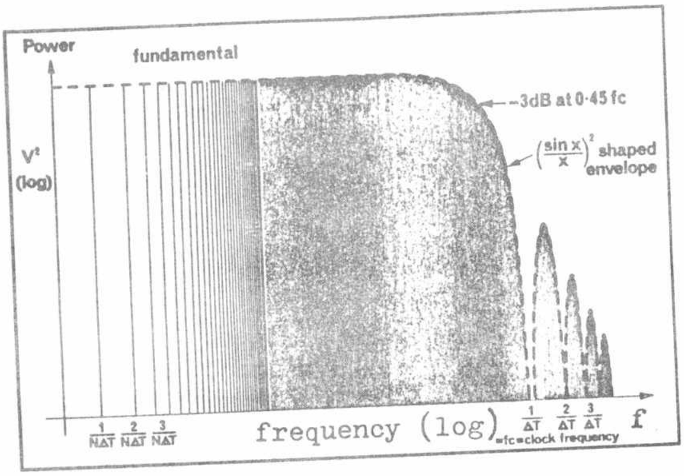

Fig. 2. Power spectral of PRBS.

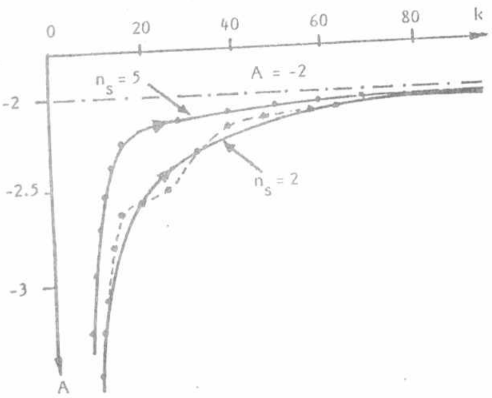

Fig. 3. Effect of n_s on the estimated continuous time parameter.

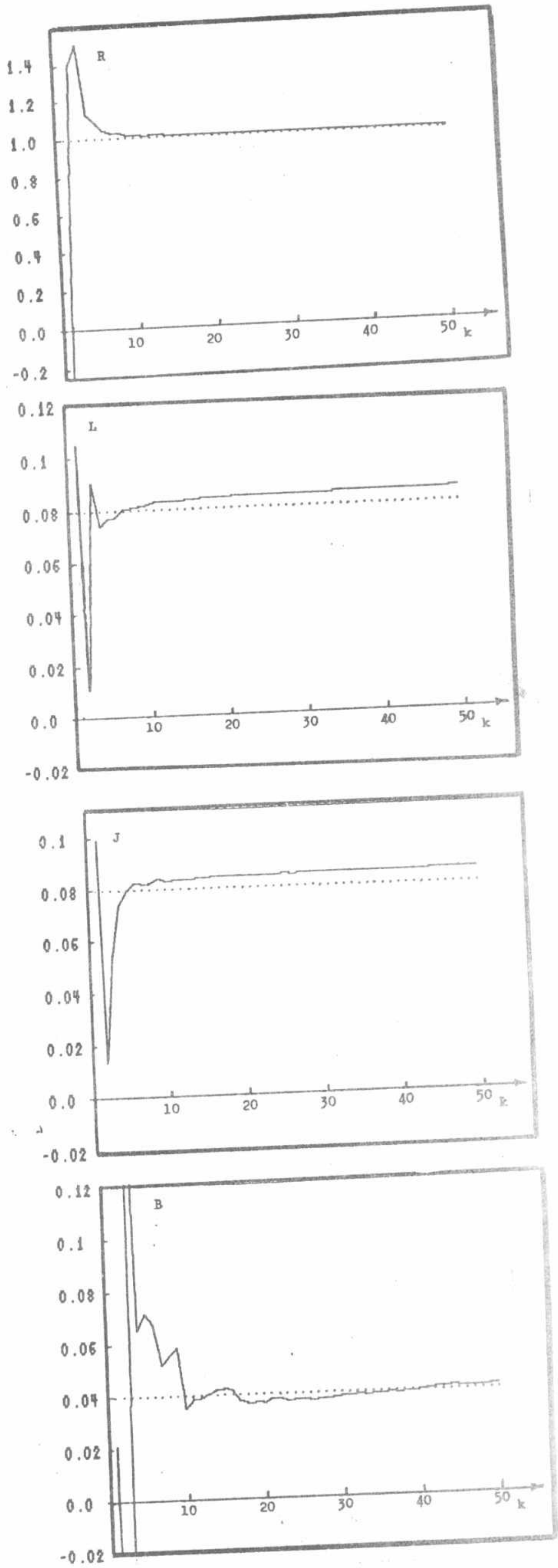

Fig. 4. The estimates of dc motor parameters.

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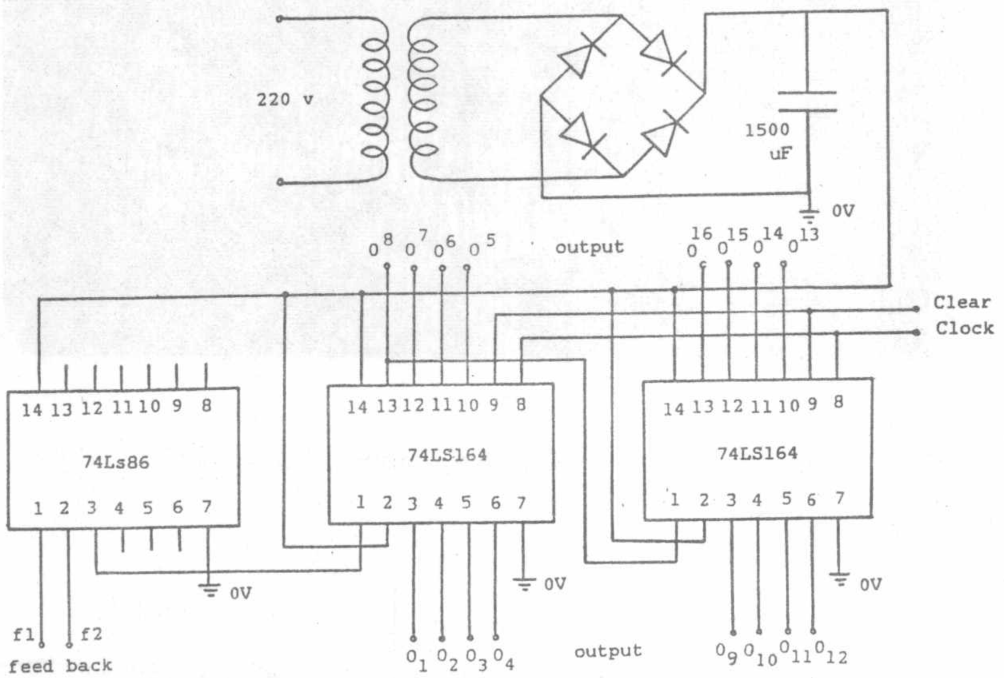

Fig. 5. The hardware circuit of PRBS signal generator.

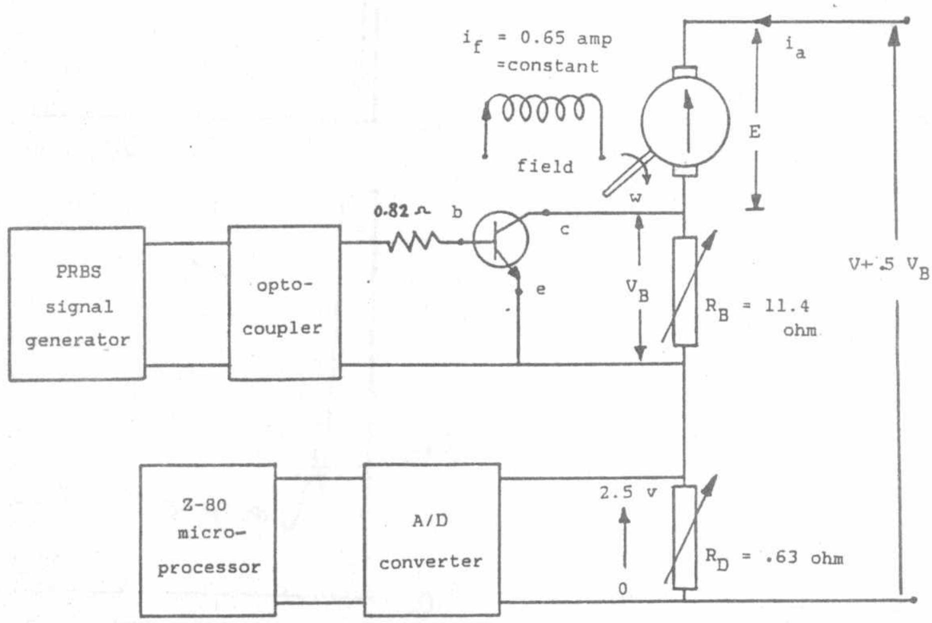

Fig. 6. Circuit diagram of the experimental procedure.

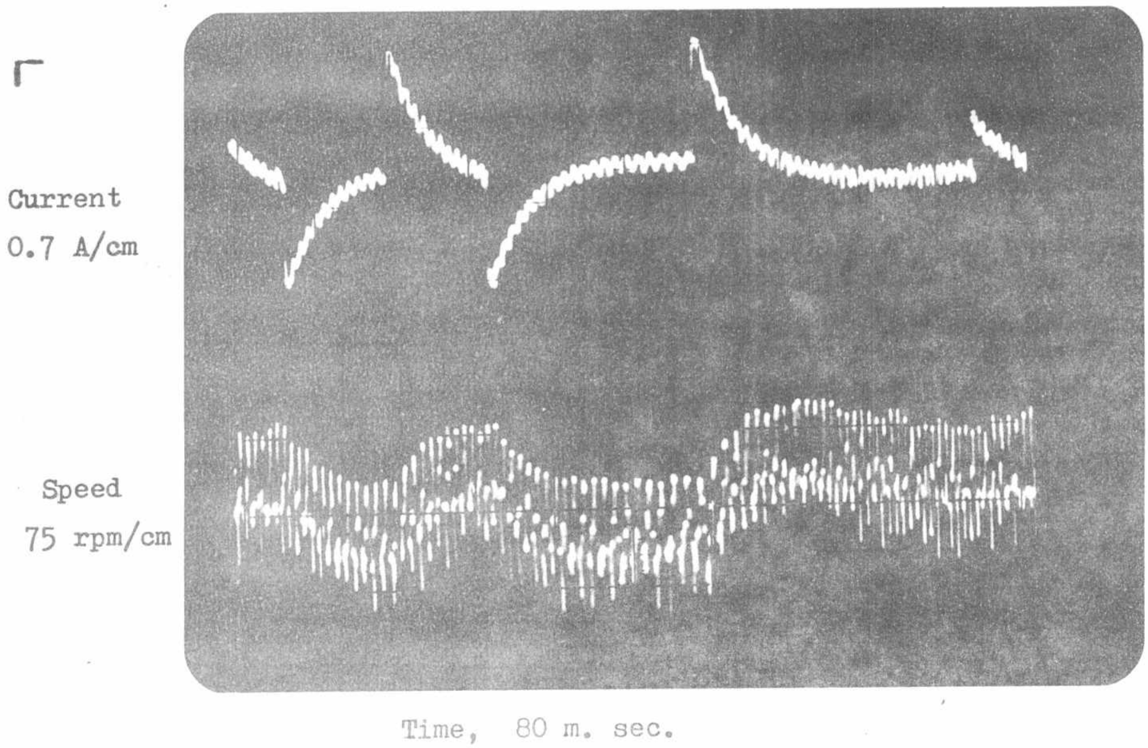

Fig.7. The motor state signals, current & speed.

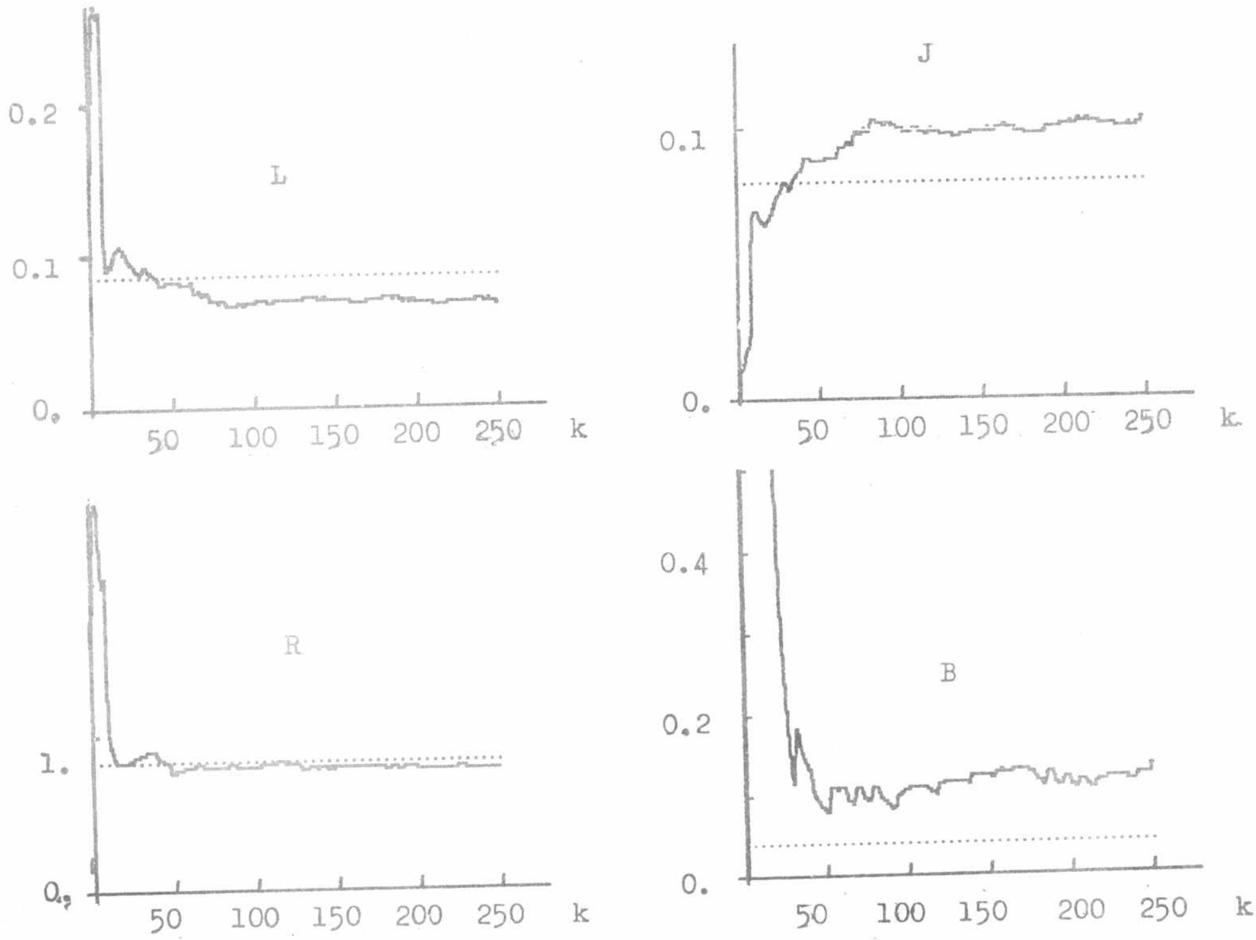

Fig.8. The experimental estimates of motor parameters.