SIMULATION OF BALLISTIC MISSILE MOTION IN FREE-FLIGHT (BALLISTIC) PHASE USING NAVSTAR (GPS) NAVIGATION MODEL

S.A.Gadalla* M.A.El-lithy* A.H.Makarious*

ABSTRACT

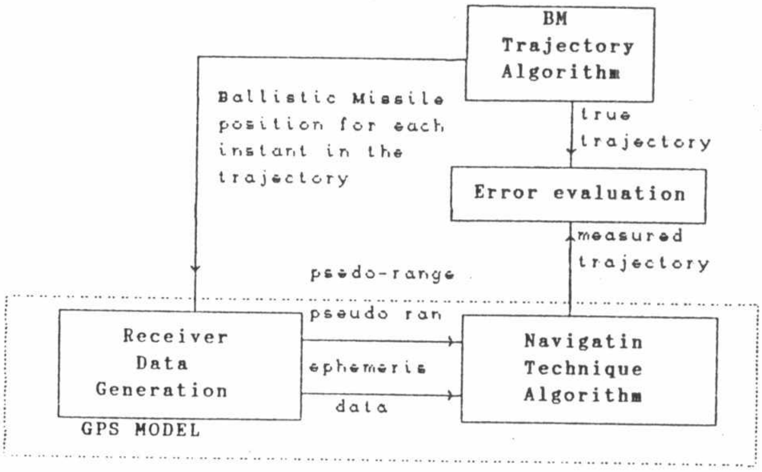
The accuracy of Ballistic Missile (BM) guidance is mainly dependent on the accuracy of the navigation system used. Majority of BM's apply the inertial navigation system (INS) where the inaccuracies of its sensors (accelerometers and gyros) induce errors which may accumulate to intolerable values due to increased flight time. The guidance is restricted only to the powered flight phase of the whole BM trajectory. In the present paper we navigate the BM motion in the free-flight phase (ballistic phase) using the NAVSTAR/GPS model and an implementation of the BM motion in the ballistic phase . The simulation includes several cases of ballistic trajectories applied on the GPS model that includes 24 satellites constellation. pseudo-range and ephemeries data are simulated, and the application of the principle of minimum geometric dilution of precision (GDOP) is performed. The implementation of the BM motion in the free-flight phase is applied on the optimum trajectories using the Keplerian orbit (elliptical section) in which Kepler's equation is solved at each instant on the trajectory by Newton Raphson method.The results of the different algorithms of implementation are evaluated. The solution of GPS equations is executed for BM motion in the ballistic trajectory using the iterative method. Different algorithms for selection of GPS model are given.

Introduction

Three main simulation model are formulated for the implementation of the GPS navigation system on the ballistic missile (BM) free-flight phase of trajectory. The first is the BM trajectory generation program based on the solution of Kepler's equation for different shut-off conditions. The second, is a model program that simulates the performance of GPS navigation receiver supposedly placed on a BM flying on the optimum trajectory generated. The third, is the calculation of the root sum square (RSS) error between the measured trajectory by the GPS model and the calculated true trajectory. Fig(1) presents the overall scheme of the simulation model.

---

* Department of Guidance, M.T.C.,Cairo,Egypt.



Fig(1) *Simulation Block Diagram*

2. Ballistic Missile Trajectory

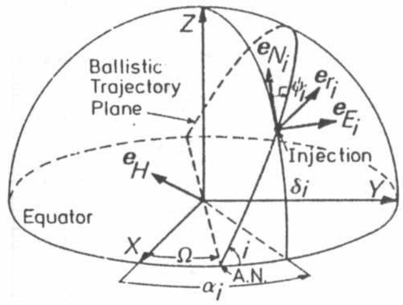
The ballistic missile trajectory is composed of three parts (phases). The powered flight phase lasts from launch to the burn-out point. The free flight (ballistic) portion constitutes most of the trajectory (80%) and will be targeted in this paper. The re-entry part begins at some ill-defined point, where atmospheric drag becomes a significant force in determining the missile path, and lasts until impact.

2.1. Free-Flight (ballistic) Phase Analysis

It starts from shut-off/burn-out point and terminates a hypothetical re-entry point. The missile follows an elliptical trajectory whose geometrical configuration depends entirely upon the burn-out parameters (vectors position and velocity). The only force acting on the missile is the gravitational force of the earth. So the trajectory will be a Keplerian orbit trajectory [1-3] .

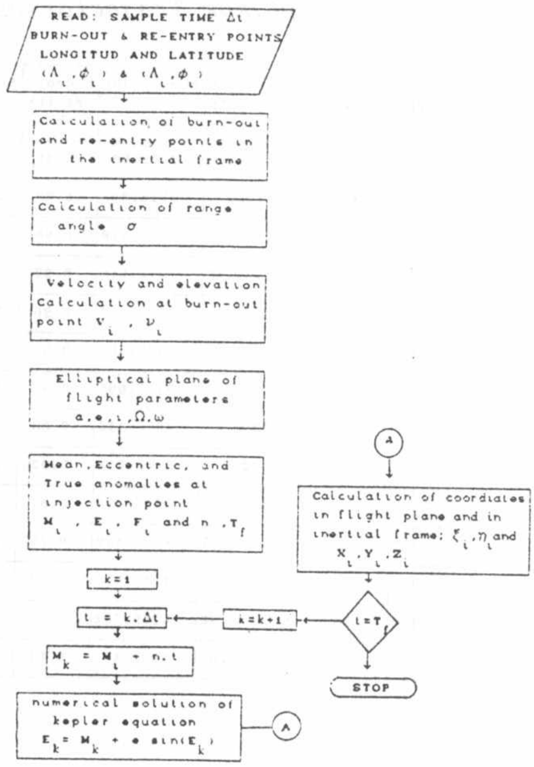
2.2. Ballistic Trajectory Algorithm

The ballistic trajectory plane is specified by three points, burn-out (injection) point, the re-entry point and the mass center of the earth. This plane is inclined to the equator by an inclination angle, i, and a right ascension angle, Ω, form the reference meridian (may be Greenwich meridian). there are several ballistic trajectories configured between the injection and re-entry points, depending on the injection state fig(2) at time t_i (related to sidereal time [3-4]), which are the geocentric latitude, δ_i, longitude, α_i, injection azimuth ψ_i and injection velocity V_i.



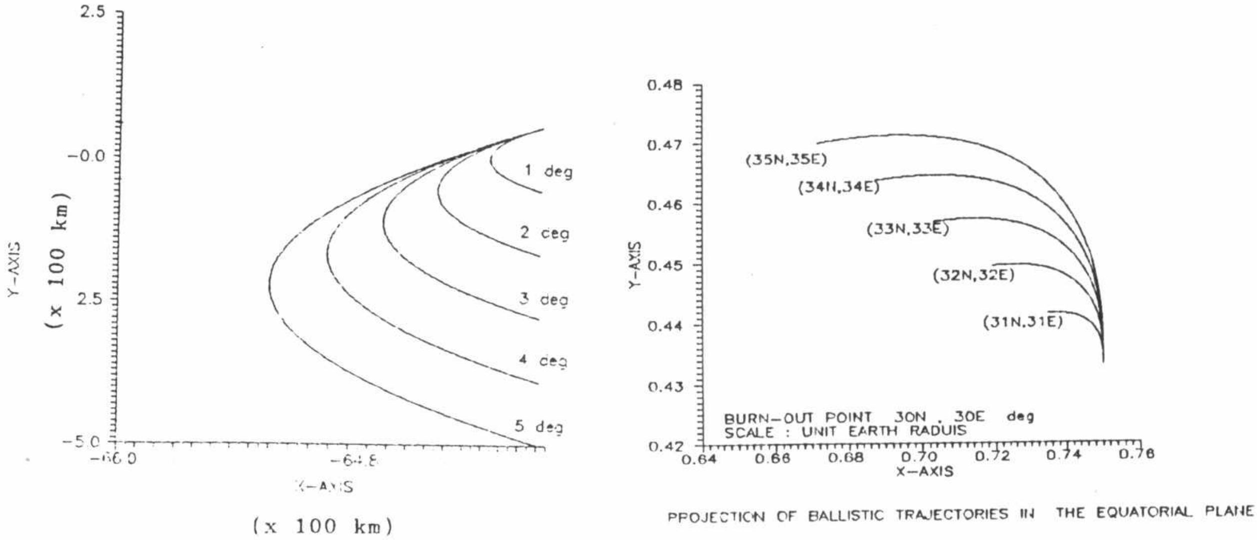
Fig(2) *Injection State of The BM Trajectory*

The ballistic trajectory used in this simulation is choosed for the optimum trajectory, which is the trajectory setup from the flight path angle, ν_i, and the injection velocity, V_i, that verifies the maximum range angle, σ. The algorithm for generation of such optimum BM trajectory in terms of the optimum injection state derived from positions of burn-out (injection) and re-entry points is formulated in the flow chart of fig(3).



Fig(3) *Flow Chart For The Generation of optimal BM trajectory*

Fig(4) shows the different trajectories in the plane of flight and the projections in the equatorial plane.



Fig(4) *Ballistic Trajectories*

Table(1) illustrates the optimum parameters resulting from the application of previous algorithm for different trajectories.

Table(1) *Optimum Trajectories Parameters*

Down Range KM	Fligh path Angle ν_i deg	burn-out velocity V_i m/s	semi-major a Km	Eccent-ricity e	Mean motion rad/sec	t_f sec	Q_v
60	44.88	737	3202.98	0.99	3.48×10^{-3}	106	0.009
100	44.7	1039	3216.892	0.99	3.46×10^{-3}	151	0.020
180	44.49	1464.2	3244.718	0.98	3.42×10^{-3}	215	0.030
260	44.25	1785.62	3272.540	0.97	3.37×10^{-3}	265	0.050
370	43.99	2053	3300.357	0.96	3.43×10^{-3}	308	0.067
500	42.7	2285.63	3328.165	0.95	3.29×10^{-3}	346	0.083
1200	42.5	3165	3467.005	0.91	3.09×10^{-3}	503	0.160
1800	41.25	3798.7	3605.316	0.88	2.9×10^{-3}	632	0.230
2300	40	4300	3742.834	0.839	2.7×10^{-3}	748	0.295
2825	38.75	4715.9	3879.299	0.8	2.6×10^{-3}	855	0.356
3350	37.5	5069.37	4014.449	0.767	2.48×10^{-3}	957	0.411
3900	36.25	5375.5	4148.028	0.73	2.3×10^{-3}	1054	0.462
4400	35	5643.96	4279.782	0.7	2.25×10^{-3}	1148	0.510
5050	33.75	5881.6	4409.46	0.67	2.15×10^{-3}	1239	0.554

3.GPS Model Simulation

The GPS navigation technique is mainly an interaction between the space segment and user segment of the system, so the key features for the GPS simulation model are :-

1- The 24 satellites of GPS constellation, uniformly distributed in 6 orbital planes inclined at an angle 55° to the equatorial plane.

2- Satellites selection based on the application of the minimum geometric dilution of precision coefficient (GDOP).

3- The generation of receiver data is prepared by the geometric calculation of the slant ranges of missile to the selected satellites at each instant of flight time.

4- Ephemeries evaluation, is executed for the dynamic constellation and the launch instant is considered in the time of the day.

5- Navigation algorithms are formulated to solve the navigation equations by the iterative method.

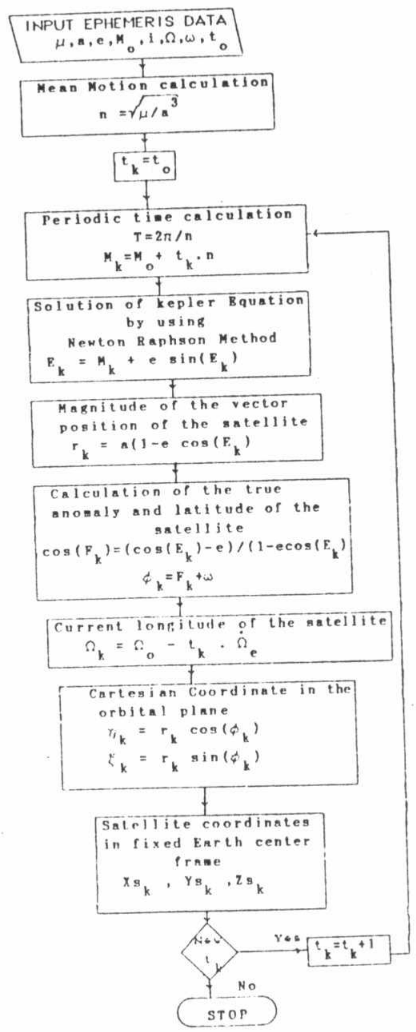
For The simulation of GPS, the following assumptions are taken into consideration without affecting the generality of the model.

1. No built-in ionospheric delay
2. Noise-free environment
3. Zero clock bias error
4. There is no satellite shielding
5. Frame of the coordinate system used is the earth centered earth fixed cartesian coordinates
6. The earth's universal gravitational parameter and the earth's rotation rate taken from WGS72(world geodetic standard) are given by $\mu=3.986008\times10^{14}\ m^3/sec^2$ and $\Omega_e=7.292115147\times10^{-5}$ rad/sec.

3.1. GPS Constellation Model [4-5]

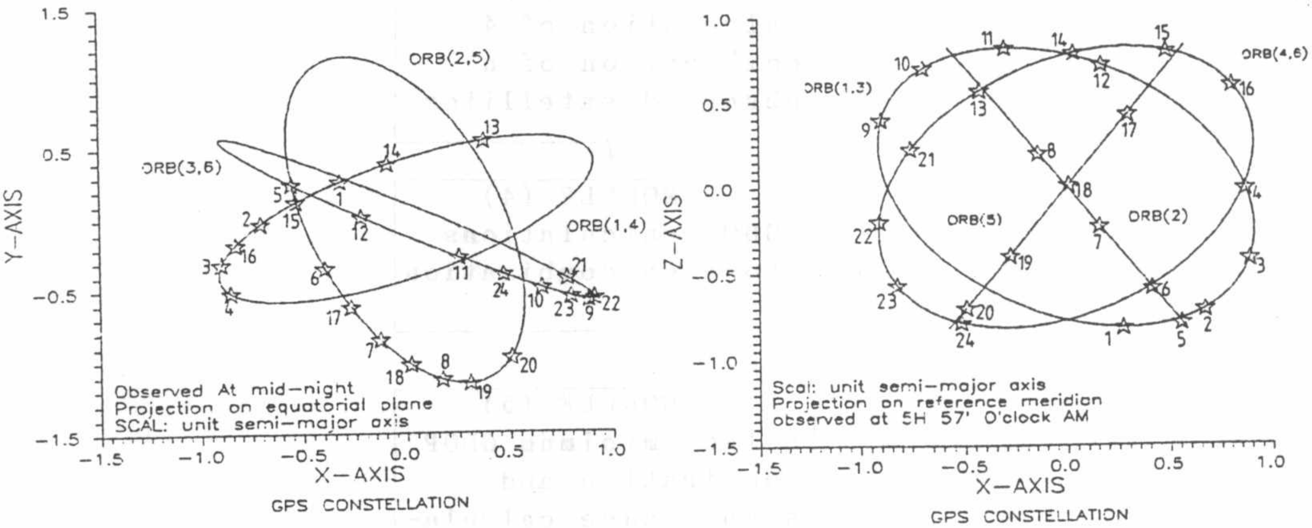
The constellation we used in this model that of 24 satellites, where each four are uniformly distributed in one of 6 orbital planes, inclined on the equatorial plane by 55°, as mentioned before. The satellites longitude w.r.t. the ascension node are given in [6].

Fig(5) introduces the algorithm for 24 satellites-orbits generation using these ephemeries parameters. The generation is based on the dynamic analysis of free-fall flight using the principle of Keplerian orbit computation [2].



Fig(5) *GPS Module No.1*

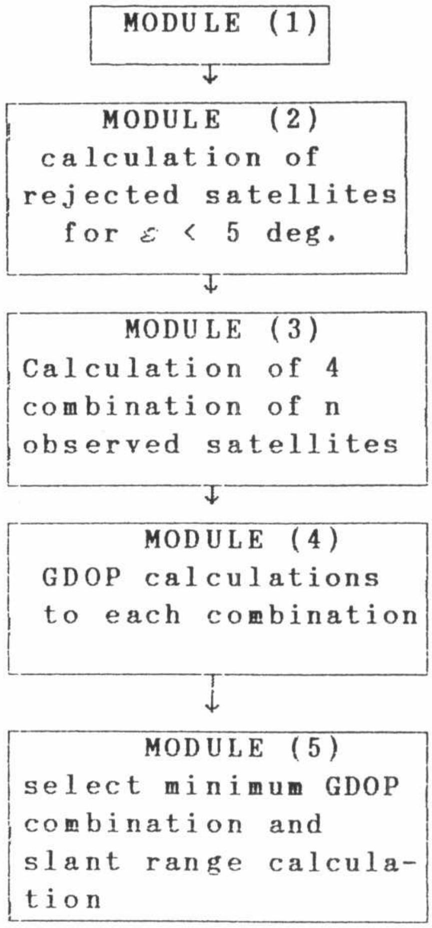
The previous algorithm is applied after the updated ephemeries data has been extracted from the navigation message coming from the tracking loop of the receiver. At the time of the say t_k, this data includes, M_o, a, i, Ω, ω, e which are mean anomaly at t_o, semi-major axis of orbit, inclination angle to the equator of the orbital plane, right ascension angle, angle of orbit perigee, and orbit eccentricity respectively. We denote this algorithm by module number 1 of the GPS model, that is used to calculate the 24 satellites positions at time t_k from the day time (epoch time). Fig(6) shows the projections of all 24 satellites constellation on the equatorial plane and vertical plane respectively at zero epoch time (mid night), and (5h,57').



Fig(6) *Projection of the 24 satellites in both equatorial plane and reference meridian at zero epoch and 5h,57'*

3.3 Satellite Selection Algorithms

It is important to select four appropriate satellites from the set of viewed satellites, over the horizon of user position, to be utilized for the preparation of the receiver data (satellites slant ranges and positions) and the solution of GPS equations. This selection is executed in 5 modules rearranged in fig(7).



Fig(7) *Satellites Selection Modules*

Module 2 : is used for calculation of rejected satellites with elevation angle ε from user position less than 5°.
Module 3 : is used for calculation of the combination $\binom{n}{4}$; where n is
angle $\varepsilon > 5^{\circ}$.
Module 4 is used to calculate the GDOP coefficient for all of these combinations. The GDOP is defined by the geometrical relation which is derived in [7].
Module 5 is used for the calculation of minimum GDOP and candidates its satellites for the calculation of the corresponding slant ranges to be used as receiver data preparation.
All the previous algorithms are detailed in [6].

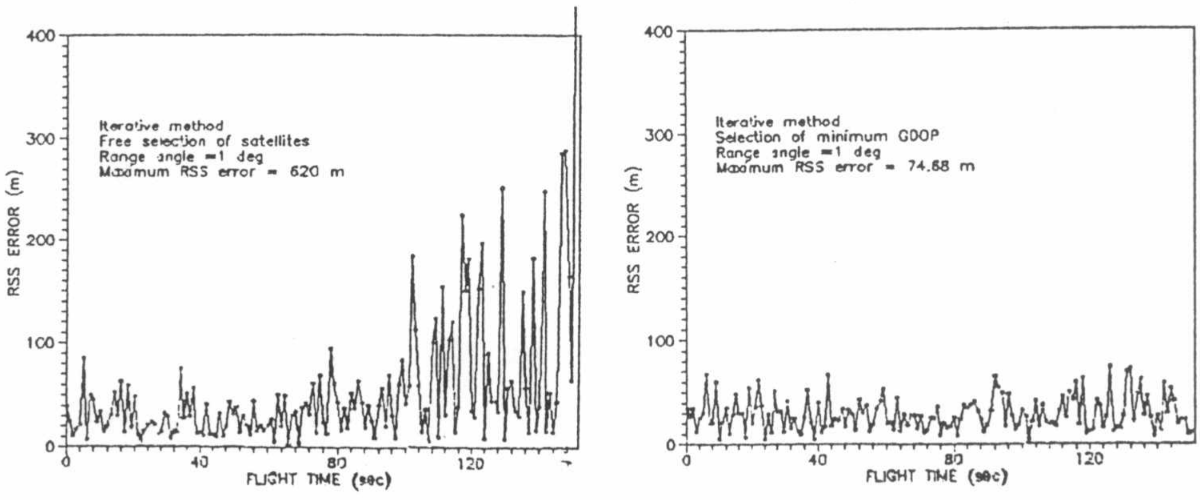
3.4 Navigation Technique Algorithm

The iterative method for solving the GPS equations begins with an estimate of the user's position. The method uses the linearization of the GPS equations about the current estimated user position and solve successively for position corrections based on measuring the residuals resulting in the user processor [8].

4. Results And Conclusions

The BM trajectory was used as the reference kinematic trajectory for the guidance loop and the GPS navigation system applied to this case of flight path.

The resulting RSS error of the GPS positioning process is random in nature and it requires the application of Kalman filter to be smoothed. It is noticed as expected that, the error is smaller for the case of selection the satellites according to the principle of minimum GDOP than the free selection of satellites. Fig(8) (a),(b) compares the RSS error for the two cases. The number of iterations for the process of positioning ranges from 4 to 6 iteration.



Fig(8) *RSS Error For Both Free Selection And Optimum Selection*

References

[1] J.W. Cornelisse,etal,"Rocket Propulsion And Spaceflight Dynamics", Pitman,London (1979).

[2] Archie E. Roy,"Foundation of Astrodynamics", Mac Millan Co., New York (1965).

[3] Roger R. Bate, etal,"Fundamentals of Astrodynamics", Dover Puplication, Inc. New York (1988).

[4] Barry A. Stein, Col. Eric Wheaton,"Graphic Depiction of Dynamic Satellite Constellation Accuracy/Coverage Over Time", NAVIGATION : Journal of The Institute of Navigation, Vol.34, No.3 Fall (1987).

[5] P.S. Jorgensen, "NAVSTAR/Global Positioning System 18-satellite Constellation", Global Positioning System Papers, Vol.II, NAVIGATION, USA (1984).

[6] S.A. Gadalla,"Application of Modern Navigation System For Ballistic Missile Guidance", M.Sc. Thesis, M.T.C., Egypt Cairo (1992).

[7] Paul Massat and Karl Rundnick,"Geometric Formulas For Dilution of Precision Calculations", NAVIGATION : Journal of The Institute of Navigation, Vol.37, No.4, Winter (1990-91).

[8] P.S. Noe, etal,"A Navigation Algorithm For The Low-Cost GPS receiver" Global Positioning System Papers, NAVIGATION : Vol.I USA, (1980).