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# MODERN AND CONVENTIONAL GUIDANCE AND CONTROL PROBLEM IN AIRCRAFT GUIDED MISSILES

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ABSTRACT

The increased accuracy requirements, more dynamic battlefield tactics of modern warfare, arbitrary and unpredictable nature of target maneuverability render classical guidance laws (such as proportional navigation) unsatisfactory in many applications. The solution involves a critical comparison between our current modern approach and the classical approach to missile guidance and control. Single plane analysis is used throughout and simulation result shows that even an accelerating target can be intercepted with shorter miss distance and less time duration for homing in spite of the dynamic lags in the target tracker and the missile dynamics.

## I. INTRODUCTION

The classical technique developed in this paper consists of low pass filter to attenuate the noise inherent in the guidance signal and using proportional navigation (PN) as the guidance law to steer the missile toward the target. Although proportional navigation (PN) guidance results in intercept under a wide variety of engagement conditions, its control-effort-efficiency is not optimum in many situations especially in the case of maneuvering targets. For improving the efficiency, this leads to apply the modern control theory to the tactical guided missiles.

In the modern technique, we replaced the low pass filter with an optimal estimator such as the Kalman filter, which "optimally" separate the signal from the noise by using information about the noise covariances and the detailed dynamics of the threat (target) and the interceptor (missile). In this paper an extended Kalman filter (EKF) has been developed [1] to estimate target maneuver, and a guidance law using these estimates has been implemented. This strategy is compared with a conventional proportional navigation guidance law over a number of simulations for an engagement scenario.

The remainder of this paper is divided into five sections. Section II contains the problem statement, the basic tracking algorithm (EKF) design is presented in Sec. III. In Sec. IV, a system model and missile guidance law design is suggested. Simulation results are presented in Sec. V. A summary and conclusions are given in the last section.

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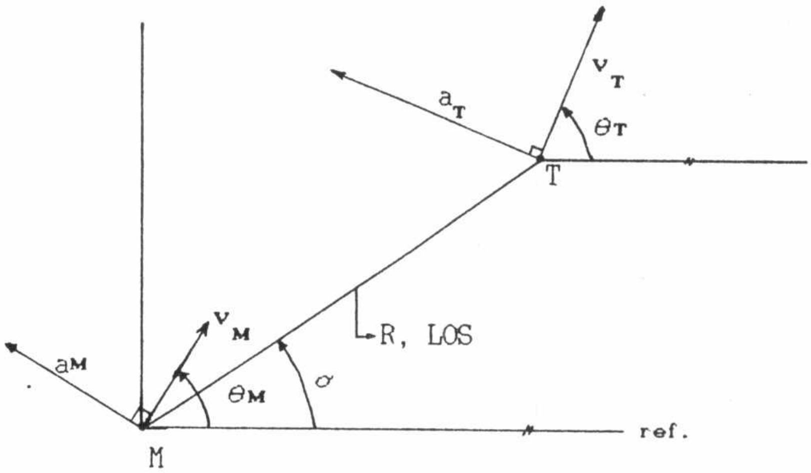
## II. PROBLEM STATEMENT

Consider the planar intercept problem depicted in Fig. 1. The scenario involves a homing missile M and a highly maneuvering target T. Commonly M is equipped with a passive seeker providing bearing angle or bearing rate measurements. In the sequel, it will be assumed that bearing rate measurements are available. These measurements are the only information provided about the missile-target relative motion. The main goal is the synthesis of a tracking filter which generates an estimate $\hat{a}_T$ of target lateral acceleration $a_T$. This is used with the sensed rate of change of bearing Z, the missile lateral acceleration $a_M$, and its first derivative, $\dot{a}_M$ to generate a demanded lateral missile acceleration $a_{MC}$, according to a precomputed suboptimal control law.




M : missile.
$a_M$ : Missile lateral acceleration.
LOS : line of sight.
$\sigma$ : LOS angle.
$\theta_M$ : missile velocity angle.
$V_M$ : missile forward velocity.

T : target.
$a_T$ : target lateral acceleration.
R : range.
$\theta_T$ : target velocity angle.
$V_T$ : target forward velocity.

Fig. 1 Geometry of the planar missile-target system.

### Equations Of The Simplified System Model.

The design of an EKF requires models of the system dynamics. With simplifying assumptions of constant missile and target forward velocities $V_M$, $V_T$, and zero acceleration of the missile relative to the target, using the polar coordinates defined in Fig. 1, the following equations of relative motion of missile and target are obtained:

$$\dot{R} = V_T \cos(\theta_T - \sigma) - V_M \cos(\theta_M - \sigma) \quad (1)$$

$$R\dot{\sigma} = V_T \sin(\theta_T - \sigma) - V_M \sin(\theta_M - \sigma) \quad (2)$$

Differentiating (2) with respect to $t$ and substituting from (1):

$$\ddot{\sigma} = -2\frac{\dot{R}}{R}\dot{\sigma} - \frac{\cos(\theta_M - \sigma)}{R}a_M + \frac{\cos(\theta_T - \sigma)}{R}a_T \qquad (3)$$

Also,

$$\ddot{R} = 0 \qquad (4)$$

For the design of the tracking filter, it will be assumed that the missile acceleration can be measured precisely. In practice, a prefilter may be necessary to account for accelerometer noise. To complete the model [Eqs. (3) and (4)], the dynamics of the target lateral acceleration must be formulated by the relationship:

$$\dot{a}_T = W_T \qquad (5)$$

Where $W_T$ is white Gaussian noise. Notice that Eq.(5) is exact for constant-acceleration maneuvers, but modeling errors will occur when the target acceleration varies with time. In the latter case, more accurate and more elaborate model is required to stabilize the filter [1].

With the further simplifing assumptions that $\theta_M \cong \theta_T \cong \sigma$, i.e., $\cos(\theta_M-\sigma) \cong \cos(\theta_T-\sigma) \cong 1$, Equations (3), (4), and (5) may be cast in the following state-space representation:

$$\dot{x} = f(x, a_M, t) + W(t)$$

$$= \begin{bmatrix} -\frac{2x_1x_2}{x_3} - \frac{a_M}{x_3} + \frac{x_4}{x_3} \\ 0 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ W_T \end{bmatrix} \qquad (6)$$

The measurement equation is

$$z = x_1 + v_{\sigma} \qquad (7)$$

Where

$x \;\underline{\Delta}\;$ state vector of the system, $x = (\dot{\sigma} \;\; \dot{R} \;\; R \;\; a_T)^T$,

$\dot{x} \;\underline{\Delta}\;$ the time derivative of the state vector,

$a_M \;\underline{\Delta}\;$ the "input" forcing function to the missile,

$f(\cdot) \;\underline{\Delta}\;$ a vector function whose components are nonlinear functions of the state and control vector components and of time,

$W \;\underline{\Delta}\; (W_{\sigma} \;\; W_{\dot{R}} \;\; W_R \;\; W_T)^T$ is a vector of independent Gaussian white noise elements,

$z \;\underline{\Delta}\;$ the observed, or measured, rate of change of bearing,

$v_{\sigma} \;\underline{\Delta}\;$ the white Gaussian (measurement) noise, and $(\;)^T$ denotes the transposed.

## III. EXTENDED KALMAN FILTER (EKF) DESIGN

In this section, the extended Kalman filter is derived for the target model defined in the previous section [Eqs. (6) and (7)]. Given the continuous-time dynamics and discrete-time measurements, construction of the filter is completed by specifying the "time" propagation and "measurement" update procedures [1].

### A. Time Propagation

Since the filter is implemented on a digital computer, the tracking equation will be cast in discrete form. For this purpose, equation (6) must be integrated across the sampling interval $(n\tau \leq t \leq (n+1)\tau)$, where $n\tau$ denotes the nth sampling time and the sampling time $\tau$ is chosen to be small compared with the time constants of the system.

Between measurements $(n\tau \leq t \leq (n+1)\tau)$, an estimate of the state at the following sampling instant is calculated:

$$\hat{x}(n+1/n) = \hat{x}(n/n) + \int_{n\tau}^{(n+1)\tau} f(x, a_M, t)\, dt$$

The integration here is approximated using a standard 4th order RUNGE-KUTTA method to give:

$$\left.\begin{aligned}
\hat{x}(n+1/n) &= \hat{x}(n/n) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\
k_1 &= \tau f(\hat{x}(n/n), a_M, t) \\
k_2 &= \tau f(n\tau + \frac{\tau}{2}, \hat{x}(n/n) + \frac{k_1}{2}) \\
k_3 &= \tau f(n\tau + \frac{\tau}{2}, \hat{x}(n/n) + \frac{k_2}{2}) \\
k_4 &= \tau f(n\tau + \tau, \hat{x}(n/n) + k_3)
\end{aligned}\right\} \quad (8)$$

The covariance of the error of that estimate is also calculated:

$$p(n+1/n) = \phi\, p(n/n)\, \phi^T + Q(n) \quad (9)$$

Where $\phi = e^{A\tau} \cong 1 + A\tau$ and

$$A = \left.\frac{\partial f}{\partial \hat{x}}\right|_{\hat{x}(n/n)}$$

$$= \begin{bmatrix} \frac{2\hat{x}_2}{\hat{x}_3} & -\frac{2\hat{x}_1}{\hat{x}_3} & \frac{2\hat{x}_1\hat{x}_2 + \hat{a}_M - \hat{x}_4}{\hat{x}_3} & \frac{1}{\hat{x}_3} \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{\hat{x}(n/n)}$$

$$Q(n) \cong \tau \cdot \mathbb{E}[W \cdot W^T].$$

B. Measurement Update

When a new measurement $[Z(n+1)]$ is available at time $\tau(n+1)$, the estimate $\hat{x}(n+1/n)$ is corrected using the equations:

$$\hat{x}(n+1/n+1) = \hat{x}(n+1/n) + K(n+1)[Z(n+1) - C\hat{x}(n+1/n)] \quad (10)$$

$$P(n+1/n+1) = [I - K(n+1)\cdot C]\cdot P(n+1/n) \quad (11)$$

Where

$$K(n+1) = P(n+1/n)\cdot C^T[C\cdot P(n+1/n)\cdot C^T + \Sigma]^{-1}$$

$$C = (1 \quad 0 \quad 0 \quad 0)$$

$$\Sigma = \mathbb{E}(\upsilon_\delta^{\,2}). \quad (12)$$

The Kalman filter algorithm thus consists of the sequence of (8), (9), (12), (10), (11) with input of new data between (12) and (10).

## IV. SYSTEM MODEL AND MISSILE GUIDANCE LAW DESIGN

The missile guidance law design is based on a model of the missile behavior, the kinematics of the system, and on a model of the target behavior more elaborate (and accurate) than that of (6) and (7). This can be achieved without increasing the on-board computational load by precomputing and storing certain aspects of the strategy.

### A. Missile Behavior

The missile behavior (including its autopilot) is represented approximately by the second-order model and constraint:

$$\frac{a_M}{a_{Mc}} = \frac{\omega_M^{\,2}}{s^2 + 2\,\omega_M\,\zeta\,s + \omega_M^{\,2}}$$

$$\ddot{a}_M = -\omega_M^{\,2}a_M - 2\,\omega_M\,\zeta\,\dot{a}_M + \omega_M^{\,2}a_{Mc}$$

$$|a_M| < a_{M\,max} \quad (13)$$

Where $\omega_M$ is an undamped natural frequency, $\zeta$ is a damping factor, $a_{Mc}$ is the commanded missile lateral acceleration, and $a_{M\,max}$ is the maximum attainable lateral acceleration.

### B. Kinematics Of the System

The kinematics of the system are described by (3), repeated here for completeness:

$$\ddot{\sigma} = -2\,\frac{\dot{R}}{R}\,\dot{\sigma} - \frac{\cos(\theta_M - \sigma)}{R}\,a_M + \frac{\cos(\theta_T - \sigma)}{R}\,a_T \tag{14}$$

In contrast to the simplified model used for Kalman filter design, no assumption is made here that $\theta_M \cong \theta_T \cong \sigma$.

## C. Target Behavior

Target maneuvers are modeled by the first-order Gauss-Markov process:

$$\dot{a}_T = -\lambda a_T + W_T + a_{Tc} \tag{15}$$

where $a_{Tc}$ is the target (pilot) commanded lateral acceleration, and $\lambda$ is the reciprocal of the maneuver (acceleration) time constant. This is more accurate-and more elaborate-than the simplified model used for the Kalman filter design.

Combining (13), (14), and (15), the system model may be cast in state equation form:

$$\dot{\bar{x}} = F\bar{x} + Du + \bar{W} \tag{16}$$

where

$$\bar{x} = (a_M \;\; \dot{a}_M \;\; \dot{\sigma} \;\; a_T)^T$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_M^2 & 2\zeta\omega_M & 0 & 0 \\ \dfrac{-\cos(\theta_M - \sigma)}{R} & 0 & \dfrac{-2\dot{R}}{R} & \dfrac{\cos(\theta_T - \sigma)}{R} \\ 0 & 0 & 0 & -\lambda \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \omega_M^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$u = [\; a_{Mc} \quad a_{Tc} \;]^T$$

$$\bar{W} = [\,0 \quad 0 \quad 0 \quad W_T\,]^T.$$

Proportional navigation is concerned with the deviation from a constant-bearing collision course, and it is therefore convenient to describe the kinematics of the system in terms of rectangular coordinates $X, Y$ relative to an inertial reference frame with the origin at the missile and $X$ along the line of sight.

Now the missile-target distance R is governed by the equations:

$$\dot{\theta}_M = \frac{1}{V_M}\,a_M \qquad \dot{\theta}_T = \frac{1}{V_T}\,a_T$$

$$\dot{X} = V_T \cos\theta_T - V_M \cos\theta_M \qquad \dot{Y} = V_T \sin\theta_T - V_M \sin\theta_M$$

$$R = \sqrt{X^2 + Y^2} \tag{17}$$

The system model consists of (16) and (17), are solved by a 4th order RUNGE-KUTTA routine [2].

## D. linear Quadratic Guidance Law

Based on the the system model (16) and (17), a guidance law can be mechanized. In the following, a stochastic guidance law is determined that minimizes a quadratic performance index subject to the stochastic engagement dynamics, including the stochastic target model under the assumption that the target commanded lateral acceleration is zero ($a_{Tc} = 0$). This assumption simplifies the derivation of the guidance law in which the input $a_{Mc}$ is found which derive the state x to zero while minimizing a cost function of the form:

$$J = \mathbb{E}\,[\ \bar{x}^T(t_f)\cdot S_f \cdot \bar{x}(t_f)\ ] \ + \int_{to}^{t_f} (\ \bar{x}\, L\, \bar{x}^T \ + \ a_{Mc}\, M\, a_{Mc})\ dt$$

where to , $t_f$ are the initial and interception times, $S_f$ , L are positive semidefinite constant matrices, M is a positive definite constant matrix, and $\mathbb{E}$ is the expectation operator. the solution of this is a linear control law:

$$a_{Mc} = -K_r \bar{x}$$

where

$$K_r = M^{-1} D^T S \tag{18}$$

and S satisfies the matrix Riccati equation:

$$\dot{S} = -SF - F^T S + SDM^{-1}D^T S - L$$

$$S(t_f) = S_f. \tag{19}$$

To solve (19), the matrices $F$, D, M, L, $S_f$ must be determined, which is impractical since some of them depend on the missile flight path, which in turn depends on target maneuver. $K_r$ is therefore calculated for a "nominal" path determined from (16) under the "nominal" path assumptions that $V_T$, $V_M$, $\dot{\sigma}$, $\theta_T$, $\theta_M$, $\dot{R}$ are all constant.

Select

$$M = 0.025 \qquad L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \qquad S_f = 0$$

which are reasonable values for a typical situation [3], and which have the effect of controlling the rate of change of bearing $\dot{\sigma}$ to zero (only L(3,3) = 0). Under these conditions, (19) is solved for S and the control matrix $K_r(t)$ found as a function of time. This is done before launch and $K_r(t)$ stored on board the missile in the form of a table:

$$\{K_r(n\tau)\} = \{|\ K_{a_M}(n\tau) \quad K_{\dot{a}_M}(n\tau) \quad K_{\dot{\sigma}}(n\tau) \quad K_{a_T}(n\tau)\ |\ \}.$$

$$n = 0,\ 1,\ 2 \ldots\ldots$$

## V. SIMULATION RESULTS

A computer simulation of the system was set up as shown in Fig.2 to compare the strategy described in this paper with conventional proportional navigation.

Some details of the simulation runs are in the following notes [3]:

\# Radar glint is modeled by the equations:

$$\dot{Y}_G = \frac{-1}{T_G}(Y_G - W_G)$$

$$\dot{\varepsilon}_G = \frac{\dot{Y}_G}{R}$$

where $Y_G$ is the glint linear displacement, $W_G$ is Gaussian white noise, $T_G$ is a time constant, $\varepsilon_G$ is the glint angular displacement.

\# The output of the line of sight angular velocity or rate of change of bearing sensor is modeled by the equation:

$$\dot{z} = \frac{-1}{T_s}(z - \dot{\sigma}_G)$$

where $\dot{\sigma}_G$ is the observed rate of change of bearing, z is the sensor output and

$$\sigma_G = \sigma + \varepsilon_G$$

\# The filter and control for conventional proportional navigation are of the form:

$$\ddot{\sigma}_c = -\frac{-1}{T_F}(\dot{\sigma}_c - z)$$

$$a_{Mc} = -K\cdot\dot{\sigma}_c \qquad (20)$$

where $T_F$ is a time constant.

\# In the mission simulations the target is maneuvered with a step function of lateral acceleration $a_{Tc} = 50\text{ms}^{-2}$ at $t = 3.0$s. Interception occurs at $t \cong 6$s.

\# The intercept geometry and terminology outlined in Fig.1. The main parameters of the engagement scenario and constants used in the simulation runs are as follows:

$T_G = 0.1$s $\quad T_s = 0.1$s $\quad T_F = 0.1$s $\quad K = 4.6\text{s}^{-1}$

$\lambda = 0.3\text{s}^{-1}$ $\quad \omega_M = 3\text{s}^{-1}$ $\quad \mathbb{E}(Y_G)^2 = 4\text{m}^2$ $\quad \tau = 0.05$s

$$\mathbb{E}(W\cdot W^T) = \begin{bmatrix} 360/R^2 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 4800 \end{bmatrix}, \qquad \Sigma = 360/R^2.$$

\# Many simulations runs for various missile damping factor $\zeta$, and different initial conditions for each of the two guidance laws, have been completed

\# The results are summarized in Table 1.

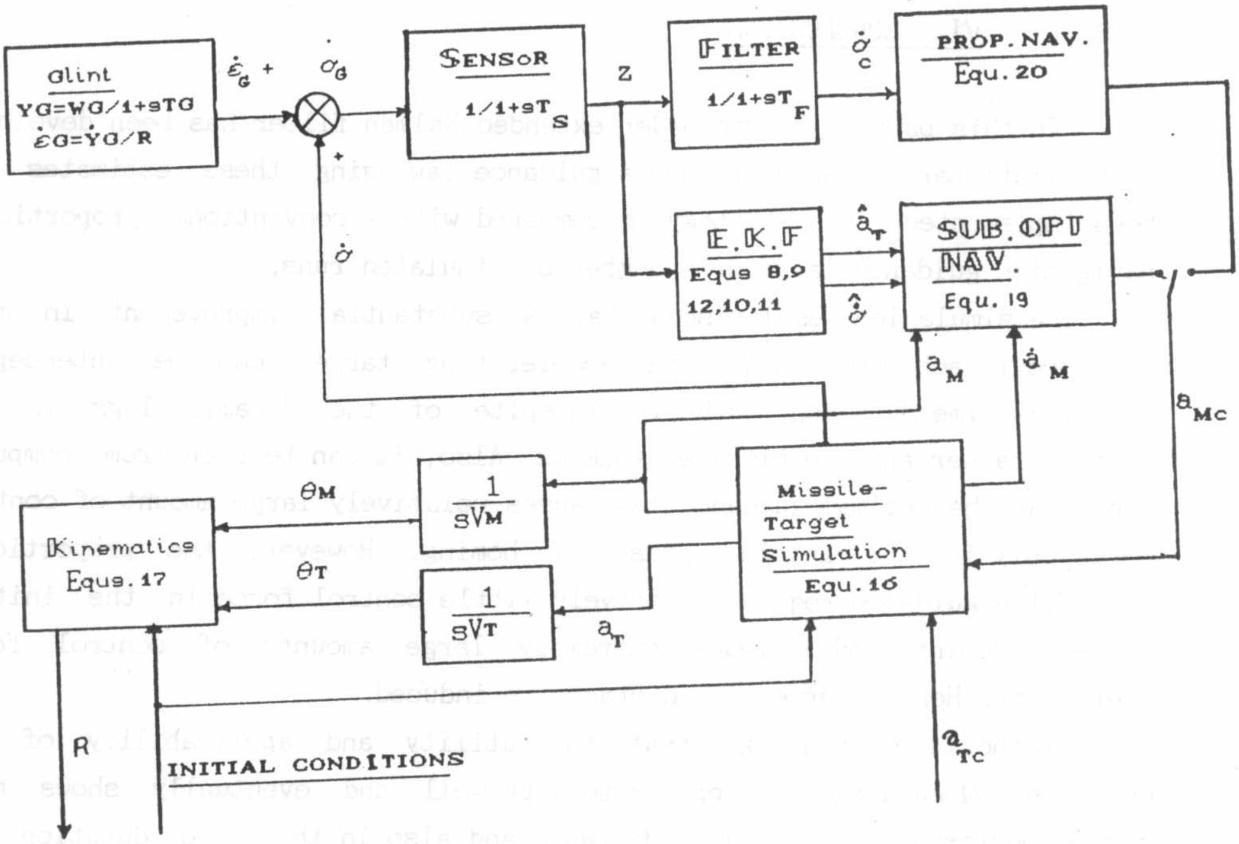



Fig. 2 FUNCTIONAL COMPARISON BETWEEN CLASSICAL AND MODERN GUIDANCE CONTROL.

Table 1 Simulation Results

| | Final miss distance(Rf) & Homing time duration (th*) | | | | |
|---|---|---|---|---|---|
| | PN | | EKF | | |
| NO. | th(s) | Rf(m) | th(s) | Rf(m) | ζ |
| 1 | 5.85 | -91.66 | 5.75 | 4.84 | 1 |
| 2 | 5.85 | -91.73 | 5.75 | 5.13 | .7 |
| 3 | 5.85 | -91.89 | 5.75 | 5.69 | .2 |
| 4 | 5.80 | -91.83 | 5.80 | -3.32 | .7 |
| 5 | 5.80 | -42.59 | 5.75 | 1.21 | .7 |
| 6 | 4.50 | 11.19 | 4.50 | -1.99 | .7 |

*The homing time duration indicates the time interval between initial time and the time of minimum miss distance.

## VI. CONCLUSIONS

In this paper a fourth-order extended Kalman filter has been developed to estimate target maneuver, and a guidance law using these estimates has been implemented. This strategy is compared with a conventional proportional navigation guidance law over a number of simulated runs.

The simulation results show that a substantial improvement in miss distance is achieved and even an accelerating target can be intercepted with less time duration for homing in spite of the dynamic lags in the target tracker and the missile dynamics. Also, it can be seen from computer runs that the optimal guidance law causes relatively large amount of control force only in the initial phase of homing. However, the proportional navigation guidance reqires relatively little control force in the initial phase of homing, and it leads extremely large amounts of control force thereafter. Hence, large miss distance is induced.

It should be emphasized that the utility and applicability of the developed LQ guidance law appears to work well and eventually shows much better performance in the miss distance and also in the time duration for homing, as can be seen from Table 1.

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