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EFFECT OF AERODYNAMIC CROSS-COUPLING ON THE STABILITY OF A ROLLING AIRCRAFT

A.A. EL AZIZ*, M.H. KATARY**, I. MANSOUR***

ABSTRACT

A method for investigation of stability of rolling aircrafts of inertially slender configuration is introduced. It includes the monotonic dependence of lateral-directional derivatives upon angle of attack, which is called "Aerodynamic cross-coupling", and expressed by the derivatives ($Cl_{\beta\alpha}$, $Cl_{r\alpha}$, $Cl_{\delta a\alpha}$, $Cn_{p\alpha}$ and $Cn_{\delta a\alpha}$). An application to a supersonic fighter model is made, in order to emphasize the effect of aerodynamic cross-coupling upon values and stability of pseudosteady roll rates. When results were compared with that obtained by neglecting aerodynamic cross-coupling, it was found that, the stable linear part of response is enlarged and the critical unstable area is completely disappeared. The results of the method were supported by a time domain simulation of the complete nonlinear equations of motion, beside a detailed analysis. Generally, it could be concluded that, the consideration of aerodynamic cross-coupling adds a source of nonlinearity to the response of a rolling aircraft, which may varry the stability margins provided, specially at large aileron deflections.

NOMENCLATURE

g Gravity acceleration.

I_{xz} product moment of inertia about the x and z-axes.

$J_x=(I_z-I_y)/I_x$, $J_y=(I_z-I_x)/I_y$, $J_z=(I_y-I_x)/I_z$ } Nondimensional inertia coefficients.

$\mathcal{L}$ Rolling moment per (I_z-I_y).

$\mathcal{M}$ Pitching moment per (I_z-I_x).

$\mathcal{N}$ Yawing moment per (I_y-I_x).

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*Assistant lecturer, E.A.A, Belbis, **Assistant professor, Dpt. of Aircraft Mechanics, M.T.C., Cairo, ***Professor, Chief of Mechanical Engineering Branch, M.T.C.,Cairo.

p, q, r	Scalar components of aircraft angular velocity w.r.t. the principal axes system.
V	Velocity of the aircraft center of mass.
u, v, w	Components of (V) w.r.t. the principal axes.
y	Side force over aircraft mass and velocity.
z	Aerodynamic force along Z-principal axis over aircraft mass and velocity.
α	Angle of attack of the x-principal axis assumed to be coincide with the zero lift line.
β	Angle of sideslip.
δa	Aileron deflection angle.
δr	Rudder ,, ,, .
δe	Elevator ,, ,, .
ϕ	Angle of bank.
ψ	Azimuth angle.
θ	Attitude angle.

Subscripts:

o Denotes initial steady states.

α, β, $\overset{\circ}{\alpha}$, p, q, r, δa, δr, δe Denotes partial derivatives w.r.t. the respective quantity. (e.g. $y_\beta = \frac{\partial y}{\partial \beta}$, ...etc.)

Superscripts:

– Denotes the pseudosteady states.

o Denotes the time derivatives (e.g. $\overset{\circ}{\alpha} = \frac{\partial \alpha}{\partial t}$, ...etc.)

(I) INTRODUCTION

Since the end of world war II, designers were trying to reach supersonic speeds. This led to the evolution of a new design concept, characterized by long slender fuselage with short and thin wings. This shift of mass causing the relation between moments of inertia to change, and consequently leads to some instability problems at high roll rates. The equations of motion that describe a rolling aircraft are nonlinear, some sources of nonlinearity comes from the condition of inertially slender configuration, the large value of states encountered during the maneuver and the aerodynamic cross-coupling. Due to complication of the problem, several assumptions were imposed in order to facilitate the solution. The first analytical study was given by Phillip in 1948 [1]. He investigated the stability of a particular solution, roll rate is constant and other states are zeros. He derived a simple formula for the critical roll rates based on a simplified 4^{th}-order system. In the last decades, a considerable research effort was conducted to analyze the longitudinal-lateral cross coupling in aircraft dynamics. Attention was focused on phenomena associated with rapid roll rates and high angles of attack, such as roll resonance and autorotation [2-6]. Also, the dynamic and aerodynamic cross-coupling of longitudianl-lateral aircraft dynamics in a spiral-dive phenomena was investigated using time domain simulation, and a criterion for recovery by elevator only was introduced [7]. A.A.Schy. and M.E.Hannah [4] were the first to introduce a method, that determine critical control areas for any combination of control inputs, which is called Pseudosteady state method (PSS). They used a $5^{th.}$-order nonlinear system (only neglect the value of the qr-term in the rolling moment equation), and assuming linear air reaction. The assumption of linear air reaction was ignoring one of the important sources of nonlinearity, which is the aerodynamic cross-coupling. So, it was logic to introduce the monotonic dependence of lateral-directional derivatives upon angle of attack, using the cross-coupling derivatives ($Cl_{\beta\alpha}$, $Cl_{r\alpha}$, $Cl_{\delta a\alpha}$, $Cn_{p\alpha}$ and $Cn_{\delta a\alpha}$).

In this paper, the PSS method is modified, in order to include aerodynamic cross-coupling, using a system of five $1^{st.}$-order nonlinear differential equations, that describe the rolling motion of aircraft (including the qr-term). The method is applied to a fighter model, in order to analyze the effect of aerodynamic cross-coupling upon stability margins provided.

(II)MATHEMATICAL MODEL

The equations of motion for a rigid aircraft will be derived w.r.t. a principal coordinate system. The aircraft velocity is assumed to be held constant during the maneuver (i.e. the $\overset{\circ}{u}$-equation is identically satisfied during the maneuver). The gravity term (g/v) is ignored in order to have constant steady states. This assumption makes the attitude angles θ and ϕ to be disappeared from the dynamic equations i.e. the two kinematical equations involving $\overset{\circ}{\theta}$ and $\overset{\circ}{\phi}$ can be disregarded, and the system order is reduced to a $5^{th.}$-order. The equations of motion are written on the form of perturbations from initial steady straight level flight. The aerodynamic angles α and β are assumed to be small i.e.

$$\alpha = \alpha_o + \Delta\alpha \simeq w/V$$

$$\beta \simeq v/V$$

The aerodynamic air reaction is on the form :

$$y = y_\beta \beta + y_p p + y_r r + y_{\delta r} \delta r$$

$$z = z_\alpha \Delta\alpha + z_q q + z_{\alpha^o} \overset{\circ}{\alpha} + z_{\delta e} \delta e$$

$$\mathcal{L} = \mathcal{L}_\beta \beta + \mathcal{L}_{\beta\alpha} \beta\alpha + \mathcal{L}_p p + \mathcal{L}_r r + \mathcal{L}_{r\alpha} r\alpha + \mathcal{L}_{\delta a} \delta a + \mathcal{L}_{\delta a\alpha} \delta a\alpha$$

$$\mathcal{M} = \mathcal{M}_\alpha \Delta\alpha + \mathcal{M}_{\alpha^o} \overset{\circ}{\alpha} + \mathcal{M}_q q + \mathcal{M}_{\delta e} \delta e$$

$$\mathcal{N} = \mathcal{N}_{\beta}\beta + \mathcal{N}_{p}p + \mathcal{N}_{p\alpha}p\alpha + \mathcal{N}_{r}r + \mathcal{N}_{\delta a}\delta a + \mathcal{N}_{\delta a\alpha}\delta a\alpha + \mathcal{N}_{\delta r}\delta r$$

The aerodynamic cross-coupling derivatives are defined as:

$$\mathcal{L}_{\beta\alpha} = \frac{\partial^2 \mathcal{L}}{\partial\beta\ \partial\alpha}$$

$$\mathcal{L}_{r\alpha} = \frac{\partial^2 \mathcal{L}}{\partial r\ \partial\alpha}$$

$$\mathcal{L}_{\delta a\alpha} = \frac{\partial^2 \mathcal{L}}{\partial\delta a\partial\alpha}$$

$$\mathcal{N}_{p\alpha} = \frac{\partial^2 \mathcal{N}}{\partial p\ \partial\alpha}$$

$$\mathcal{N}_{\delta a\alpha} = \frac{\partial^2 \mathcal{N}}{\partial\delta a\partial\alpha}$$

Finally, the $5^{th.}$-order system is given as :

$$\alpha^{\circ} = \frac{1}{(1-z_{\alpha}\circ)}\left[(\cos\alpha_{o} + z_{q})q - p\beta + z_{\alpha}\Delta\alpha + z_{\delta e}\delta e\right]$$

$$\beta^{\circ} = (y_{r} - \cos\alpha_{o})\ r + (y_{p} + \sin\alpha_{o})p + p\Delta\alpha + y_{\beta}\beta + y_{\delta r}\delta r$$

$$p^{\circ} = J_{x}\Big[-qr + (\mathcal{L}_{\beta} + \mathcal{L}_{\beta\alpha}\alpha_{o})\beta + \mathcal{L}_{p}p + (\mathcal{L}_{r} + \mathcal{L}_{r\alpha}\alpha_{o})r + (\mathcal{L}_{\delta a} + \mathcal{L}_{\delta a\alpha}\alpha_{o})\delta a + (\mathcal{L}_{\beta\alpha}\beta + \mathcal{L}_{\delta a\alpha}\delta a + \mathcal{L}_{r\alpha}r)\ \Delta\alpha\Big]$$

$$q^{\circ} = J_{y}\left[pr + \bar{\mathcal{M}}_{\alpha}\Delta\alpha + \bar{\mathcal{M}}_{q}q - \bar{\mathcal{M}}_{\alpha}\circ p\beta + \bar{\mathcal{M}}_{\delta e}\delta e\right]$$

$$r^{\circ} = J_{z}\Big[-pq + \mathcal{N}_{\beta}\beta + (\mathcal{N}_{p} + \mathcal{N}_{p\alpha}\alpha_{o})p + \mathcal{N}_{r}r + (\mathcal{N}_{\delta a} + \mathcal{N}_{\delta a\alpha}\alpha_{o})\delta a + (\mathcal{N}_{p\alpha}p + \mathcal{N}_{\delta a\alpha}\delta a)\Delta\alpha\Big]$$

To solve for pseudosteady states, all time derivatives are assigned to zero, leaving aside the α° and p°-equations, the other three equations are linear in $\bar{\beta}$, $\bar{q}$ and $\bar{r}$. Solving the 3-equations simultaneously for $\bar{\beta}(\Delta\bar{\alpha},\bar{p})$, $\bar{q}(\Delta\bar{\alpha},\bar{p})$ and $\bar{r}(\Delta\bar{\alpha},\bar{p})$. Inserting the solution into the α° and p° equations, two nonlinear algebric equations in $\Delta\bar{\alpha}$ and $\bar{p}$ are obtained. Solving the two equations for $\Delta\bar{\alpha}$ and $\bar{p}$, then inserting these values into $\bar{\beta}(\Delta\bar{\alpha},\bar{p})$, $\bar{q}(\Delta\bar{\alpha},\bar{p})$ and $\bar{r}(\Delta\bar{\alpha},\bar{p})$, to obtain the other 3 PSS.

The stability of the steady states is examined by assuming small perturbations from each steady state. In a matrix form, the linearized perturbed system is given by:

$$x^{\circ} = Ax + C$$

where A is a constant matrix, that is function of aircraft aerodynamic and stability derivatives and the initial PSS, C is the control vector and x is the state vector. To ensure asymptotic stability of steady states, the eigenvalues of the matrix A must be all negative real or complex with negative real part. It was proved that, the condition of asymptotic stability of solutions of the 5^{th}-order system is necessary for the existance of stable periodic solutions of the 7^{th}-order system with gravity terms included [2].

(III)ANALYSIS

The modified PSS method [8], which includes aerodynamic cross-coupling is applied to a supersonic fighter model [2], at the following regim : M = 0.6, H = 6 [Km].

When comparing figures No.(1) and (2), it is found that, aerodynamic cross-coupling is responsible for many changes in values and stability of PSS, which can be summarized in two main effects:

(1) The basic branch (the branch which starts from the origin) is expanded up to $\delta a = -19^\circ$ instead of $\delta a = -13^\circ$ in case of neglecting aerodynamic cross-coupling.

(2) The unstable area of response $\delta a \in [-21^\circ, -24^\circ]$ is stabilized, when considering aerodynamic cross-coupling.

The time domain simulation of the model supports the results of the modified PSS method. Fig.(3) shows that, the basic branch is attracted to branch No.(II) at $\delta a = -13^\circ$, while Fig.(4) shows the attraction of basic branch to branch No.(I) at $\delta a = -19^\circ$, when considering aerodynamic cross-coupling.

To analyze the effect of aerodynamic cross-coupling, Fig.(5) should be well investigated. The value $\delta a = -19^\circ$ constitutes a boundary to the qualitative behavior of PSS angle of attack variation, from positive values for $|\delta a| < 19^\circ$ to negative values for $|\delta a| \geq 19^\circ$.

For $|\delta a| < 19^\circ$, the PSS angle of attack is positive, so the PSS values of $\mathcal{N}_p$ and $\mathcal{L}_\beta$ are increased to a higher negative values and consequently reducing the dutch roll damping Fig.(6), while the short period root is stabilized Fig.(7). The two derivatives $\mathcal{N}_p$ and $\mathcal{L}_\beta$ have a major effect on the stability of dutch roll and short period roots, while the variation of control derivatives against angle of attack affects the value of PSS roll rate. The stabilization of short period root makes the basic branch to continue up to $\delta a = -19^\circ$.

For $|\delta a| \geq 19^\circ$, the PSS angle of attack is negative, so the PSS values of $\mathcal{N}_p$ and $\mathcal{L}_\beta$ are increased to a higher positive values, which augments both the dutch roll and short period dampings of branch No.(I) Figs.(8,9). As a result, branch No.(I) becomes stable more earlier at $\delta a = -19^\circ$ instead of $\delta a = -24^\circ$ in case of neglecting aerodynamic cross-coupling, and consequantly the unstable area of response $\delta a \in [-21^\circ, -24^\circ]$ is stabilized.

(IV)CONCLUSION & RECOMMENDATIONS

When the modified PSS method (which includes aerodynamic cross-coupling) was applied to a supersonic fighter model, it was found that, the stable linear part of basic branch was enlarged, the critical unstable area of response was completly disappeared, and obviously the values of PSS roll rate were changed. Generally, it could be said that, the aerodynamic cross-coupling has a negligible effect within the small linear part of response, while its effect becomes very significant for more aileron deflections. So, it is recommended to use methods that include aerodynamic cross-coupling, when examining the stability of rolling aircrafts at high roll rates.

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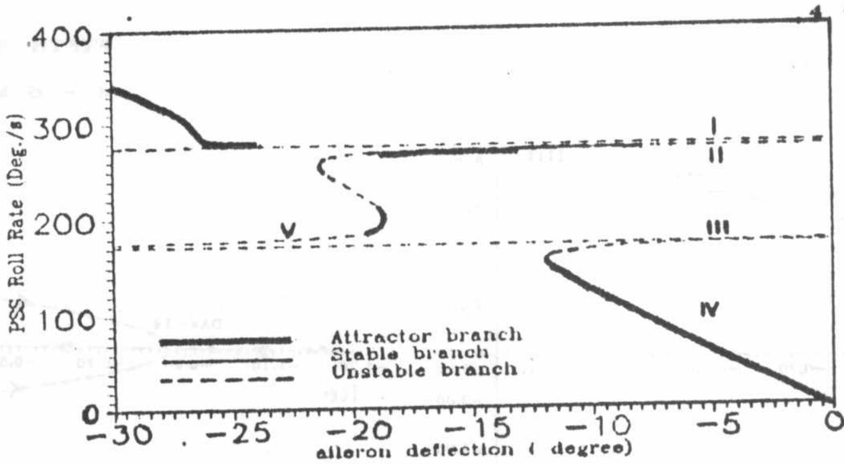

Fig.1 PSS roll rate for initial AOA 1.5 deg. and zero pitch down, and including aerodynamic cross-coupling.

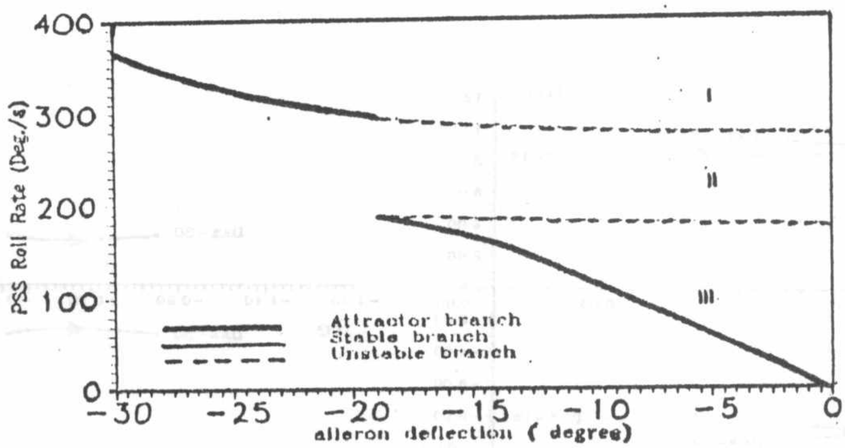

Fig.2 PSS roll rate for initial AOA 1.5 deg., zero pitch down, and including aerodynamic cross-coupling.

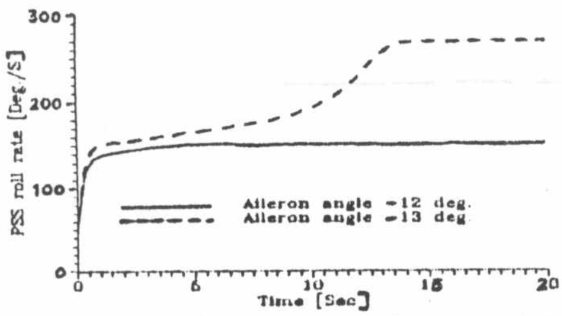

Fig.3 Time domain response for PSS roll rate when neglecting aerodynamic cross-coupling.

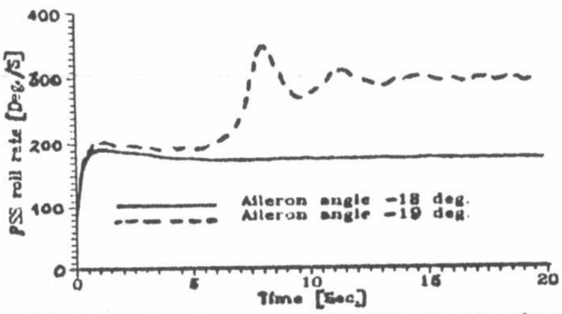

Fig.4 Time domain response for PSS roll rate when including aerodynamic cross-coupling.

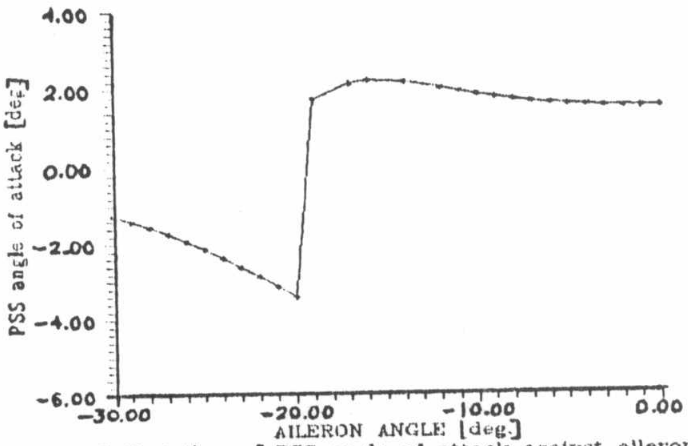

Fig.5 Variation of PSS angle of attack against aileron angle.

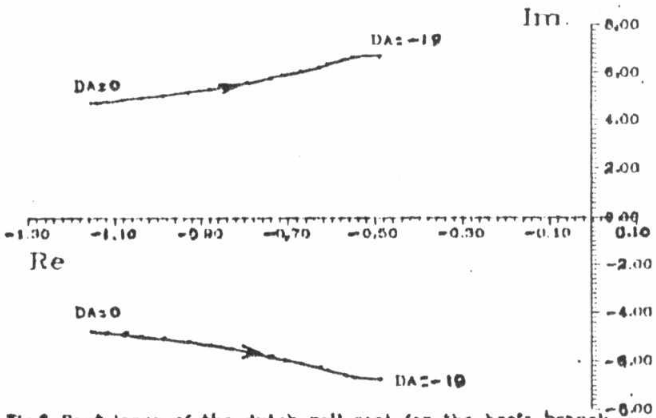

Fig.6 Root locus of the dutch roll root for the basic branch when including aerodynamic cross-coupling.

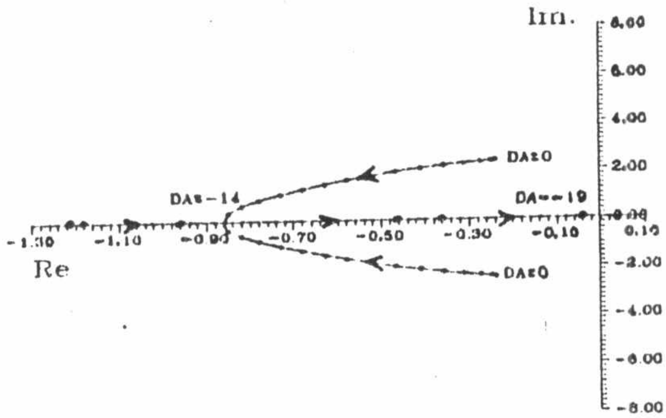

Fig.7 Root locus of the short period root for the basic branch when including aerodynamic cross-coupling.

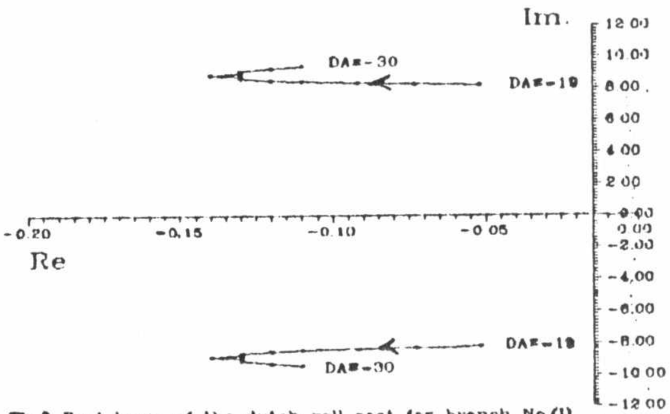

Fig.8 Root locus of the dutch roll root for branch No.(1) when including aerodynamic cross-coupling.

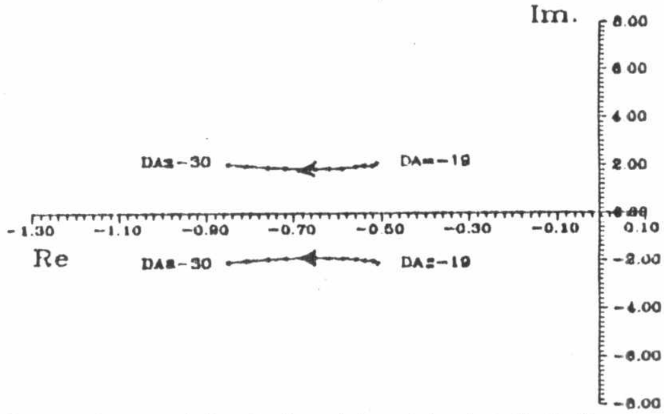

Fig.9 Root locus of the short period root for branch No.(1) when including aerodynamic cross-coupling