MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

INVESTIGATION OF RESIDUAL STRESSES IN A PLATE SUBJECTED TO COMBINED LOAD USING CRACK METHOD

S.A.KHALIL* N.M.ASHOUR* M.M.EL SAYED**

ABSTRACT

The problem of residual stresses in a structural member may play an important role in determining the overall actual stress distribution. A theoretical and experimental investigation of residual stresses in a plate subjected to combined tensile and bending stresses are carried out in this paper. The basic concept adopted in the investigation is to apply known stresses on the loaded plate and then release a certian value of these stresses by making two single edge cracks. Known stresses are considered as residual stresses. The released strain on the relieved portion was measured using strain gages. A comparison between the results obtained using two single edge cracks method and two double edge cracks method is presented for the case of a plate under tension. A good agreement between theoretical and experimental results is obtained for the plate subjected to combined load.

INTRODUCTION

The majority of the various phases of manufacturing processes of structural parts causes residual stresses. Prediction of the magnitude of residual stresses has been focused for considerable attention during the past several decades. There are some known techniques for measuring residual stresses. These techniques involve either destructive, semi- destructive or nondestructive testing. In the destructive technique, partining and sectioning is performed on the specimen while the semi-destructive method is performed using blind hole drilling method and rossette strain gage. Both of these destructive and semi-destructive techniques are based on strain relieving by cutting in the part to relieve the trapped elastic deformation and then measuring it by comparing strain values before and after cutting.

* Assist. prof., ** Assoc. prof., Faculty of Eng. & Tech. Mech. Design Dept., Mataria, Helwan University, Cairo, Egypt.

Nondestructive technique, x-ray or acoustoelastic, is based on the fact that the difference in the speed of propagating shear waves is proportional to the difference in principle stresses. A technique for complete nondestructive evaluation of plane states of residual stresses is proposed in [1].

The technique is based on the acoustoelastic effect in which the presence of residual stress causes a shift in the speed at which a wave propagates through the material.

Little attention was paid to predict the behavior of highly stressed cracked structures. A new calibration method is proposed in [2] using finite element analysis to determine the correlation coefficient. It is observed that the variation of the strains measured on the surface for each incremental change in hole depth is caused by two aspects. One is due to the residual stresses on the surface and the other is due to the change of the hole geometry. In many practical applications rectangular plates found frequently to be subjected to combined loading, tension and bending. Residual stresses have been investigated [3] for a tensile plate using two double edge cracks method. The effect of crack depth and location in measuring residual stresses were considered. Results reported in [3] showed generally a good agreement between numerical and experimental estimates of the strain relaxation.

In this paper, residual stresses for a plate having two single edge cracks are obtained under applied combined tension and bending stresses. Actual strain relaxation of the plate can also be measured using four strain gages. A comparison has been done between the results obtained for the plate under pure tension with two double edge cracks [3] and the one investigated here.

Theoretical Analysis

An application of Tong el at [4] model, hybrid stress model, has been employed to formulate the crack tip super-element stiffness matrix. The regular finite element displacement model was used in all other remaining elements. A list of the original program is given in [5]. After some FORTRAN statement modification in the Frontal solution subroutine the program was successfully run on the IBM PC with microsoft FORTRAN optimizing compiler. The finite element mesh is given in Fig.1 for only a quarter plate due to geometric symmetry as shown in Fig.2.

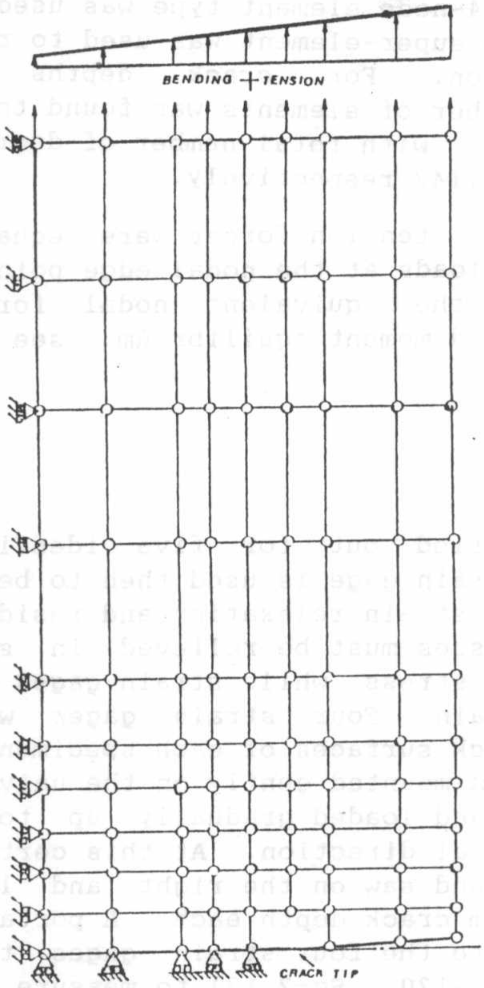

Fig.1 A test specimen with two edge cracks

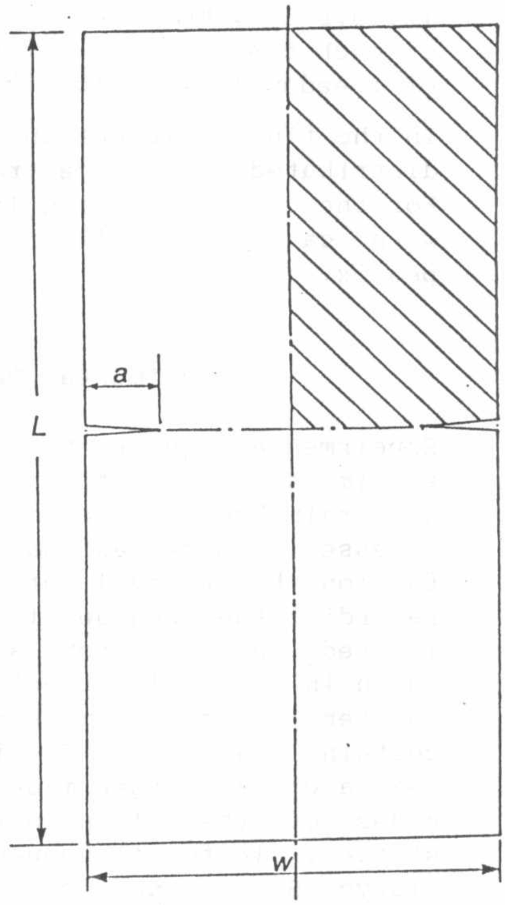

Fig.2 Mesh for one quarter specimen

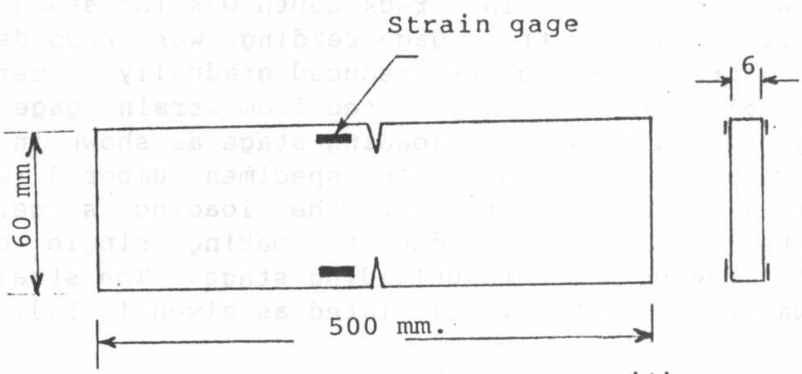

Fig.3 Specimen test with two edge crack

An isoparametric quadilateral 4-node element type was used as a regular element while 5-node super-element was used to represent the crack tip region. For crack depths of 12,10,8,6,4,2mm, the total number of elements was found to be in each case 71,71,62,62,63,54, with total number of degrees of freedom 178,178,158,158,162,142 respectively.

In the finite element analysis, tension forces were equally distributed as concentrated loads at the nodal edge points. For the case of bending load, the equivalent nodal forces were calculated using force and moment equilibrium, see appendix.

Experimental Work

Expeirmental tests were carried out for five identical specimens made of steel 37. Strain gage is used then to be as a strain sensor to measure strain relaxation and residual stresses. These residual stresses must be relieved in some fashion by removal of the stress while strain gages are recording the change in strain. Four strain gages were mounted on the front and back surfaces of each specimen as shown in Fig.3. The specimen was mounted gently on the universal tensile testing machine, and loaded gradually up to a certain load(P=50 KN) in axial direction. At this certain load a crack is then made by hand saw on the right and left sides of the plate with 2mm crack depth each. A portable strain indicator is connected to the four strain gages type (Tokyo Sokki Kenkyujo Fla-3-11-120, Sg=2.12) to measure the relieved strain. The crack width equal to 2mm (saw width +little clearance).

Relaxation of the stretched plate is observed on the strain gages, the average of strain relaxation for each side of the plate was calculated. The crack depth was increased gradually up to 12mm and the strain gage readings were recorded at each crack depth. Loading was reduced gradually to zero, while strain relaxations were estimated from strain gage readings during unloading at each loading stage as shown in Fig.4.One of the loading cycles ($P-\zeta$) for specimen number 3 was drawn in Fig.4. Line oa presents the loading stage, line ab presents relaxation stage due to making single crack and finally line bc presents unloading stage. The strain relaxation was experimentally calculated as given in [3].

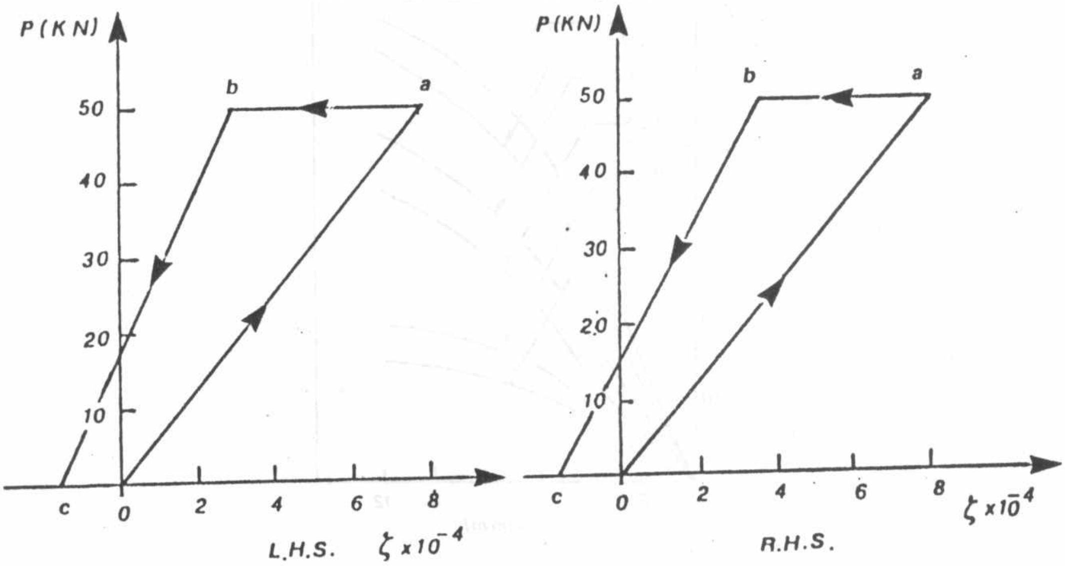

Fig.4 Strain relaxation
with tensile load

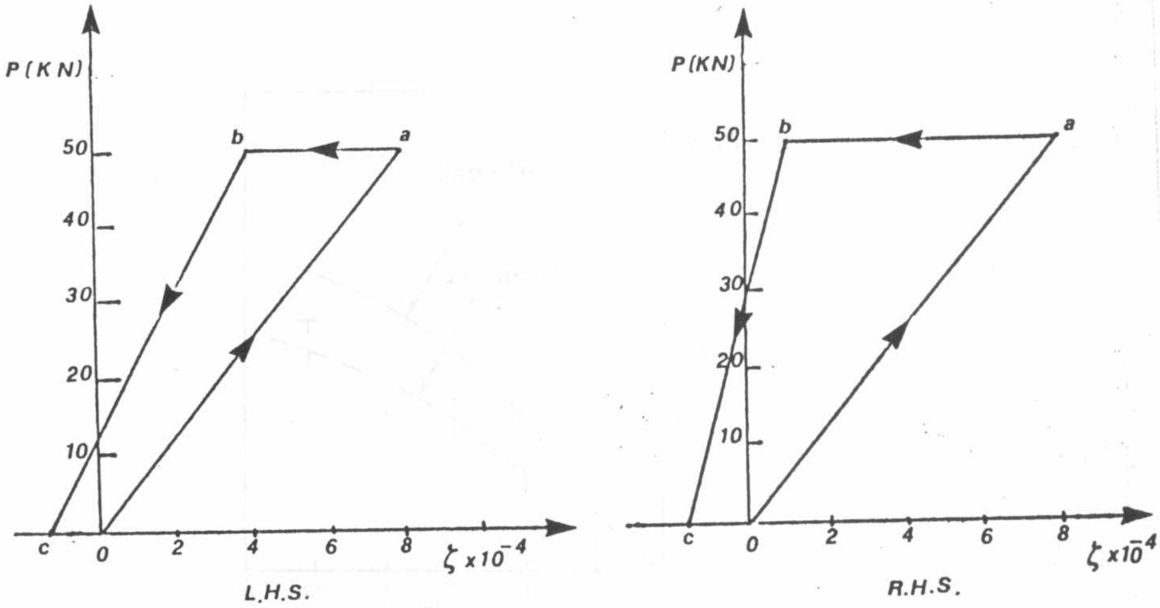

Fig.5 Strain relaxation
with combined load

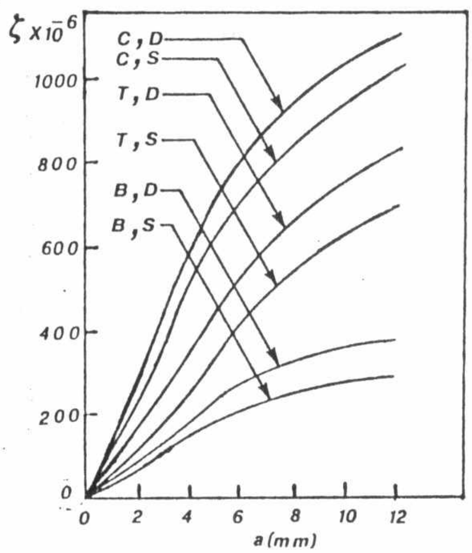

Fig.6 Variation strain relaxation with crack depth under combined load

T Tension S Single crack
B Bending D Double crack
C Combined tension and bending

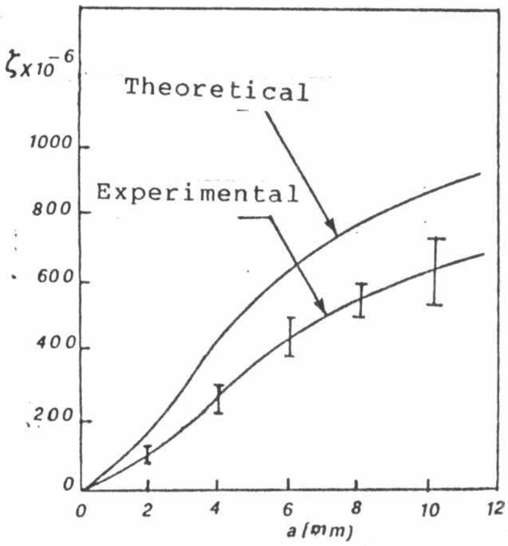

Fig.7 Theoretical and experimental strain relaxation values with crack depth

Results and Discussion

The loading cycle curve (P-ζ) for the right and left side of the tensile test specimen is given experimentally in Fig.4. It is indicated from that figure, that the measured strain relaxation due to making single crack with depth 12mm is equal to 430×10^{-6} for right side and 510×10^{-6} for left side. Also residual strains due to manufacturing (oc) as shown from that figure is equal to 170×10^{-6} and 150×10^{-6} for right and left sides respectively.

The loading cycle (P-ζ) for both sides of the tested plate under combined loading is given in Fig.5 respectively. The strain relaxation value in right and left sides is equal to 700×10^{-6} and 400×10^{-6}. The residual strain due to manufacturing (oc) is equal to 160×10^{-6} and 140×10^{-6} for both sides.

The relationship between the strain relaxation and the crack depth for the combined loading case is shown in Fig.6. It is found that, the maximum deviation between theoretical and experimental results is about 20%. This deviation value is due to reason which were discussed [3]. The theoretical values of strain relaxation calculated by F.E. method and the experimental values with respect to the crack depth are shown in Fig.7.

Conclusion

Residual stresses are determined for a plate subjected to combined load. Method of two single edge cracks is used in the experimental investigation and compared with the results of F.E. method. The following conclusion were observed:

1- The single edge crack is a suitable method for the determination of resuidal stresses and gives less damage than double edge cracks method.
2- Within a variation in crack depth it is observed experimentally that, the small crack depth gives reasonable value of strain relaxation. It is found to be in the range of 2-4mm.
3- In general there is a suitable agreement between strain relaxation predicted by numerical solution and experimentally measured values.

References

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Appendix

Theoretical Analysis

Applied bending stress is converted to concentrated equivalent nodal forces for each loaded element i as in Fig.8 by the following two equilibrium equations:

Force equilibrium

$$R_i^{(i)} + R_{i+1}^{(i)} = \int_{\ell_{i+1}}^{\ell_i} \sigma^{(i)}\, t\, dx$$

Moment equilibrium

$$R_i^{(i)}\, \ell_i + R_{i+1}^{(i)}\, \ell_{i+1} = \int_{\ell_{i+1}}^{\ell_i} \sigma^{(i)}\, t\, x^{(i)}\, dx \qquad \text{for } \ell_{i+1} \leq x^{(i)} \leq \ell_i$$

where t is the thickness, $\sigma^{(i)}$ is bending stress at element and R_i, R_{i+1} represent left and right equivalent nodal force for element number (i).

Now, by taking advantage of the linearty of the bending moment and some trigonometric relations one can easily derive the following expression:

$$y_1 = \sigma_{max} = 6M_{max}/tW^2$$

$$y_2 = 2y_1(W/2 - x_1)/W$$

$$y_i = 2y_1[(W/2 - x_1) - x_{i-1}]/W \qquad \text{for } i=3 \text{ to } N$$

where M_{max} is maximum bending moment and N is the number of loaded elements

$$\left.\begin{aligned} \bar{x}_i &= 1/3\; x_i(y_{i+1} + 2y_i)/(y_{i+1} + y_i) \\ F_i &= t\,(y_i + y_{i+1})\, x_i/2 \\ \beta_i &= \bar{x}_i/x_i \end{aligned}\right\} \quad \text{for } i=1 \text{ to } N$$

$$R_1 = \beta_1 F_1$$

$$R_{i+1} = (1-\beta_i)\, F_i + \beta_{i+1}\, F_{i+1} \qquad \text{for } i=1 \text{ to } N-1$$

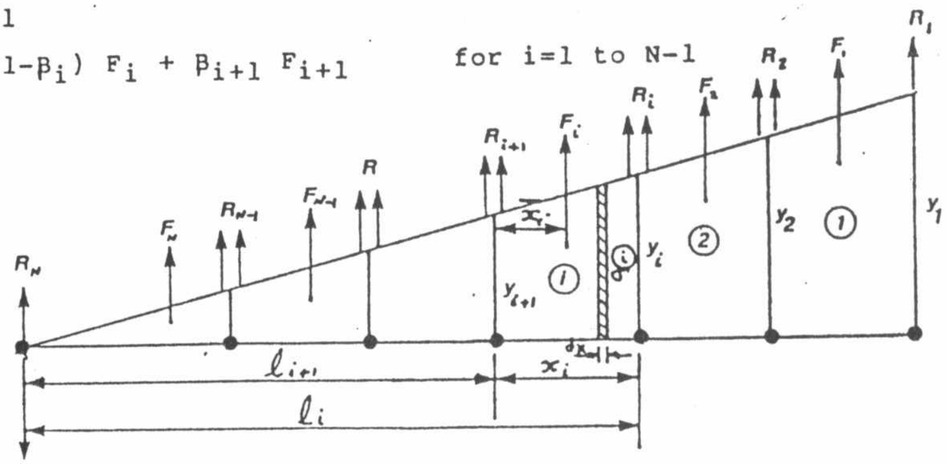

Fig.8 Converting bending stress to concentrated nodal forces