MICROPROCESSOR IMPLEMENTATION OF AN ON-LINE OPTIMAL ESTIMATOR OF GUIDANCE STATES FOR A SHORT-RANGE HOMING SYSTEM

M.A.Fahmy, S.F.Bahgat S.Ghoniemy *

ABSTRACT

An optimal on-line μp-based estimator for the guidance states of a short range homing system is presented, taking into account : the homing head measurement errors, autopilot measurement errors, launch initial heading errors, and wind disturbances. A powerful 16-bit microprocessor system is used. The obtained results illustrate the potential improvement of both the missdistance and normal acceleration as well as the overall system performance. Moreover, the system allows for further modifications through more powerful algorithms.

1- Introduction

The objectives of this work are:

- To design and implement a μp-based on-line optimal estimator for the guidance process state vector from the available set of noisy measurements when the system is driven by an external noise (wind disturbance), and

- To compare the performance of the proposed μp-based system with other estimators for realistic values of the missile dynamics in the presence of measurement errors and wind disturbances.

These objectives are treated in 6 sections:
In section 2, the problem is briefly formulated. In section 3, the error sources are discussed. In section 4 , the optimal estimator is presented. Section 5, deals with the implementation of the proposed system using a suitable microprocessor. Section 6, evaluates the proposed system and compares the obtained results with those obtained previously for the same combat situation. The paper is terminated by a conclusion summarizing the obtained results and proposing problems for future work.

2- Problem Formulation :

Assuming planar motion, the linearized missile model is introduced , the target model is assumed, and the complete guidance process is described through the guidance law as follows, Fig.(1), [4] and [5]:

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_m & -1 \\ 0 & 0 & 0 & -\omega^2 & 0 \\ 0 & 0 & 1 & -2\epsilon\omega & 0 \\ 0 & 0 & 0 & 0 & -\omega_T \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega^2 \\ 0 \\ 0 \end{pmatrix} A_{mc} \qquad (1)$$

* Guidance Dept., M.T.C. Cairo, Egypt.

where

X_1, X_2 are the missdistance (normal to the initial (LOS) and its derivative.

X_3 is the state variable representing the integral of [the missile commanded acceleration - missile turning rate] multiplied by ω^2.

X_4 is the state variable representing missile turning rate.

X_5 is the state variable representing target acceleration.

A_{mc} is the commanded missile acceleration.

V_m is the missile velocity.

ω is the natural undamped frequency of the missile system model.

ϵ is the damping ratio.

ω_T is a variable representing target manoeuver.

Variations of this planar model were used in [3], [4], [5], [6], [7]. [8], and [9].

3 - Sources of Errors:

3.1 Measurement Errors:

The set of measurements, Z(t_i), available at discrete times t_i, and corrupted by zero-mean white additive Gaussian noise V(t_i), where V(t_i) has a (+ve) definite covariance matrix r(t_i), can be expressed as follows :

$$Z(t_i) = Z_i = H(t_i)X(t_i) + V(t_i) \tag{2}$$

The measurement errors are considered to have constant root-mean square (rms) value and by nature they are uncorrelated with each other. Consequently the covariance matrix r(t_i) is a diagonal matrix with elements r_{11} (homing head sensor), and r_{22} (angular rate sensor), where r_{11} = 2.5 10^{-5} (rad/sec $)^2$ and r_{22} = 7.5 x 10^{-5} (rad/sec $)^2$;[4]. The measurement matrix H(t_i) is expressed as:

$$H(t_i) = \begin{pmatrix} \frac{1}{V_c(T_f-t_i)^2} & \frac{1}{V_c(T_f-t_i)^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{3}$$

where V_c is the missile closing velocity and T_f is the nominal time of flight.

3.2 Wind Disturbance Errors

Missile turning rate (pitch rate) is greatly affected by wind disturbances. Thus state X_4 is usually corrupted with an external noise due to this disturbance. The noise n_w affecting the system due to wind disturbance can be characterized by its mean and variance. Typical atmospheric data results in the following statistical parameters:

Mean $$E[n_w(t)] = 0 \tag{4}$$

Variance $$E[n_w(t)n_w^T(t)] = \sigma^2 = 0.00244 \tag{5}$$

i.e the noise strength is $Q_{\text{wind}} = 0.00244$ (rad/sec $)^2$. By introducing the effect of wind disturbance, the system of Eq's (1) can be written in the form

$$\dot{X} = A(t)X(t) + B(t)U_c(t) + G(t)n_w(t) \tag{6}$$

where
$$G^T(t) = [\begin{matrix} 0 & 0 & 0 & 1 & 0 \end{matrix}] \tag{7}$$

4. Optimal Estimation of Guidance States

The estimator is designed as a conventional Kalman filter (KF), and the states of the system of Eq's (6) can be estimated as follows:
At time t_{i-1} the state propagation equation and the covariance propagation equation are given by [2,5] :

$$\dot{\hat{X}}(t/t_{i-1}) = A(t)\hat{X}(t/t_{i-1}) + B(t)U_c(t), \tag{8}$$

$$\dot{p}(t/t_{i-1}) = A(t)p(t/t_{i-1}) + p(t/t_{i-1})A^T(t) + G(t)Q_{\text{wind}}G^T(t) \tag{9}$$

Eq's(8,9) are integrated over the interval from time t_{i-1} to time t_i starting from the initial conditions :

$$\hat{X}(t_{i-1}/t_{i-1}) = \hat{X}(t^+_{i-1}), \tag{10}$$

$$p(t_{i-1}/t_{i-1}) = p(t^+_{i-1}) \tag{11}$$

yielding the estimates $\hat{X}(t^-_i)$ and $p(t^-_i)$.
At measurement time t_i, the set of measurement $Z(t_i)$ becomes available. The estimate is updated by defining the Kalman filter gain $K(t_i)$ and employing it in both the state and covariance relations, where:

$$K(t_i) = p(t^-_i)H^T(t_i)[H(t_i)p(t^-_i)H^T(t_i) + r(t_i)]^{-1}, \tag{12}$$

$$\hat{X}(t^+_i) = \hat{X}(t^-_i) + K(t_i)[Z_i - H(t_i)\hat{X}(t^-_i)], \tag{13}$$

$$p(t^+_i) = p(t^-_i) - K(t_i)H(t_i)p(t^-_i) \tag{14}$$

From which the optimal state estimate $\hat{X}(t_i)$ at time t_i is obtained.
A schematic diagram of the estimator is given in Fig.(2). Starting by the data supplied from different sensors, the microprocessor based device performs the necessary analysis and processing which results in the best estimation of the current values of variables of interest.

5- Implementation of The Microprocessor Based System

The large number of arithmetic operations required to provide an optimal on-line estimation of the guidance process state vector $\hat{X}$ (t_i) within a limited calculation time interval implies the application of a high-speed microprocessor. Therefore, the characteristics of microprocessors, available in the local market, have been comparatively studied and the most suitable μp (at that time) has been picked-up to implement the required estimator.

The principal criteria influencing the choice of the required μp are : Wordlength of data to be treated, Family of logic components, Power supply and power consumption, Clock frequency and number of its phases, Speed of central processing unit (CPU), Execution time, Speed of data interchanging, Possibility of interruption, Arithmetic possibilities, and Set of instructions. For the present application, the values of operands are varying in a wide range of magnitudes, therefore a 16-bit word length microprocessor such as (8086 with its co-processor 8087) seems to be a reasonable choice. The block diagrams of the proposed system and its interfacing are shown in Fig.'s (3), and (4).

There are two types of calculations to be performed at different stages of the optimal estimator design. The 1st is the prefire calculations which are system dependent calculations such as the Kalman filter gains. This type can be carried out before engagement (off-line). The 2nd is the on-line calculations which are target dependent, and are carried out during engagement.

5.1 Off-line Calculations:

These parameters are constant for a certain missile target situation, and therefore they can be precalculated and stored according to the following relations :

$$\dot{p}(t/t_{i-1}) = A(t)p(t/t_{i-1}) + p(t/t_{i-1})A^T(t) + G(t)Q_{\text{wind}}G^T(t)$$

$$K(t_i) = p(t_i^-)H^T(t_i)[H(t_i)p(t_i^-)H^T(t_i) + r(t_i)]^{-1}$$

Each of the gain components, aK_{11}, aK_{12}, aK_{21}, aK_{22}, aK_{31}, aK_{32}, aK_{41}, and aK_{42}, occupies 80 words, resulting in a totality of 640 words size.

5.2 On-line Calculations:

The on-line calculations represent the main task of the proposed microprocessor based system. These calculations are performed as follows:

(1) The optimal estimator state vector is obtained from the following Eq.

$$\begin{pmatrix} \hat{X}_1(t_i^+) \\ \hat{X}_2(t_i^+) \\ \hat{X}_3(t_i^+) \\ \hat{X}_4(t_i^+) \\ \hat{X}_5(t_i^+) \end{pmatrix} = \begin{pmatrix} \hat{X}_1(t_i^-) \\ \hat{X}_2(t_i^-) \\ \hat{X}_3(t_i^-) \\ \hat{X}_4(t_i^-) \\ \hat{X}_5(t_i^-) \end{pmatrix} + \begin{pmatrix} aK_{11}(t_i) & aK_{12}(t_i) \\ aK_{21}(t_i) & aK_{22}(t_i) \\ aK_{31}(t_i) & aK_{32}(t_i) \\ aK_{41}(t_i) & aK_{42}(t_i) \\ aK_{51}(t_i) & aK_{52}(t_i) \end{pmatrix} \begin{pmatrix} e_1(t_i) \\ e_2(t_i) \end{pmatrix} \qquad (15)$$

(2) The measurement residual e(t_i) is generated as the difference between the true measurement value Z_i and the best prediction of it before it is actually taken. Thus the vector $[\ e_1(t_i)\ e_2(t_i)\ ]^T$ is calculated using the following Eq.

$$e(t_i) = Z_i - H(t_i)\hat{X}(t_i) \tag{16}$$

The required storage size for this vector is 160 words size.
The estimated state vector $\hat{X}(t_i^+)$ requires 4 memory words per cycle with the totality of 320 words for the 80 cycles.

Assembly language is essential in programs where the speed of operation is vital, such as in missile guidance systems. The assembly program used for calculating the estimated state vector is implemented for three possible target behaviours; namely; target moving with constant velocity, target manuvering with a constant acceleration, and target manuvering with exponential acceleration [4]. The corresponding flow chart using Runge Kutta method, and the initial state vector $[\ 7 \quad 0 \quad 0 \quad 0 \quad 0\ ]^T$ is shown in Fig.(5). The optimally estimated states $\hat{X}_1$, $\hat{X}_4$ compared with the classically calculated ones, are shown in Fig.(6,7,8).

Referring to the deviced assembly program, it has been found that the program comprises:

59 addition operation.
45 multiplication operation.
92 data movements.

The main used instructions with their execution times are shown in table(1).

Table (1) Set of used instructions and their execution times

Instruction	Bytes	Clock cycles	No. repetition	Total clock cycles	Execution times (ns)
ADD ac,data	2 or 3	4	59	236	47200
MUL	2,3 or4	80-98	45	3600-4410	720000-882000
MOV	3	10	92	920	184000
CMP	2,3 or4	3	1	3	600
JMP	5	15	1	15	3000
JE	2	4/2	1	2	400
Total					955200-1117200

6- Evaluation of The Implemented System:

The most important state variables in the guidance process are the miss distance X_1 and the missile pitch rate X_4. The comparative analysis, table (2), showed that the proposed system produces a much smaller impact error (I.E.) and significantly smaller missile normal acceleration (M.N.A.). In addition, it produces a much smoother trajectory and great reduction in the maximum miss distance during engagement.

Table (2) Performance of Conventional and Proposed Systems For 7m I.H.E.

	Constant velocity		Exponential Acceleration		Constant Acceleration	
	I.E. cm	M.N.A. m/sec^2	I.E. cm	M.N.A. m/sec^2	I.E. cm	M.N.A. m/sec^2
Conventional System	35.10	1.58	11.66	96.81	401.15	26.99
Proposed System	8	2.21	11.23	76.25	20.15	24.96

Conclusion

The implemented 8086-microprocessor based optimal estimator for the guidance states of a short range homing missile performs a very important tool for designing an optimal controller that improves the overall system performance, specially under the quickly developing capabilities of the modern air targets. Calculation time is about 1-msec which is very small compared with the sampling period (75 msec). However, modifications are necessary in both directions : hardware by using new microprocessors (80386, 80486, 80586,..) and software through more powerful algorithms.

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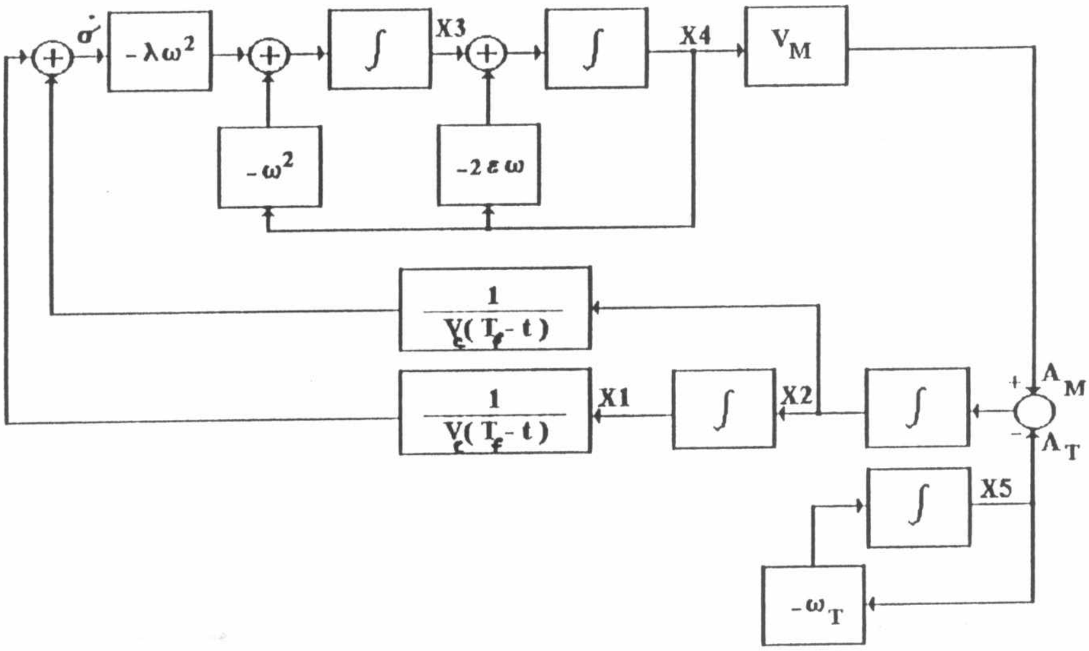

Fig.(1) P. N. Guidance System

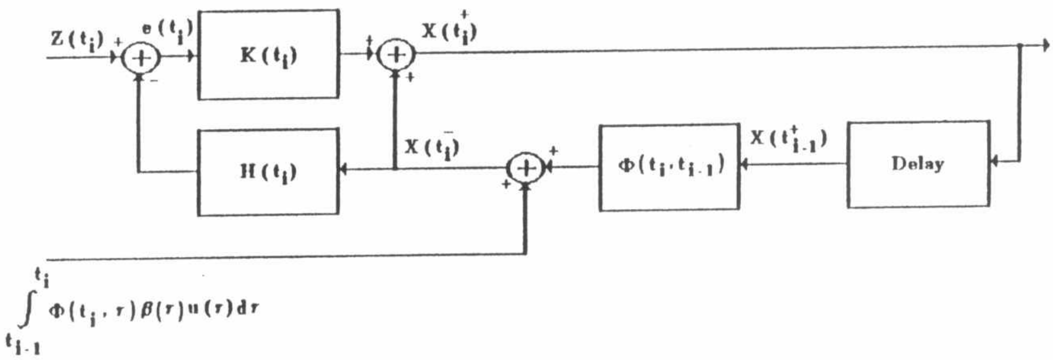

Fig.(2) A Schematic Diagram of the Estimator

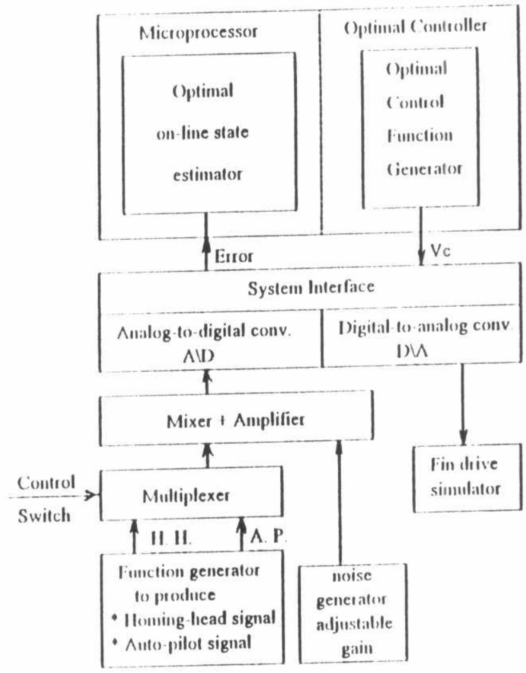

Fig (3) Block Diagram of the proposed System

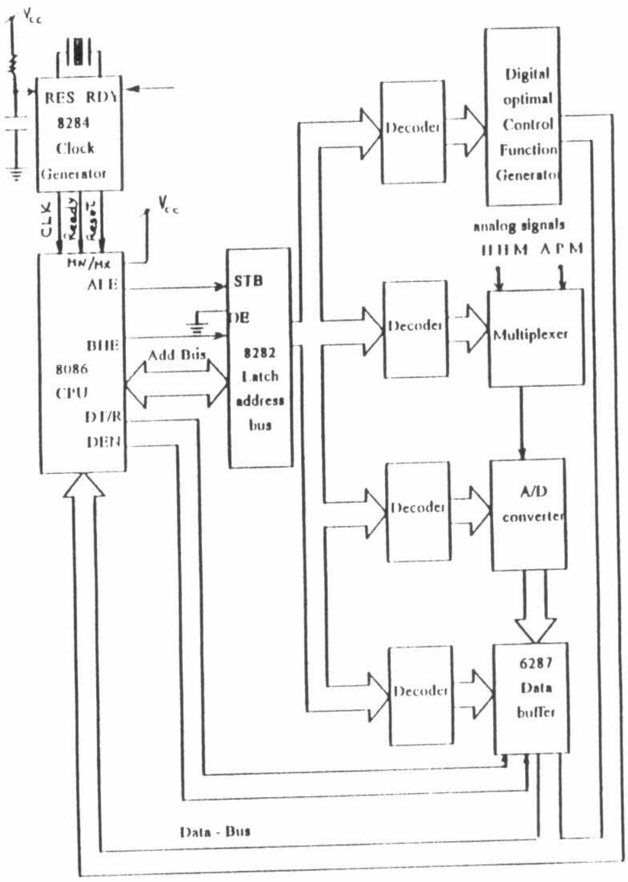

Fig.(4) Microprocessor-based system interface

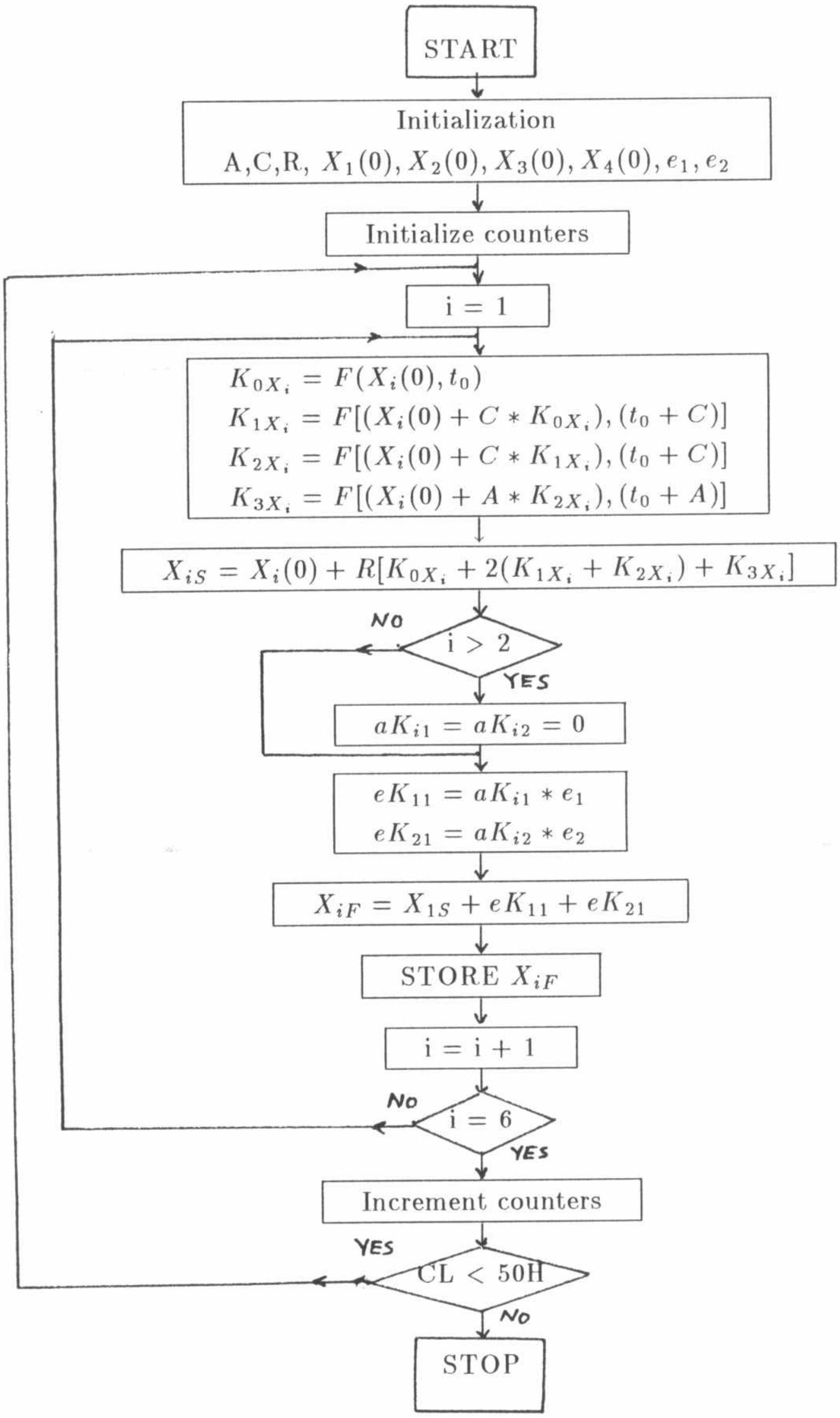

Fig.(5) Flowchart For Calculating State Vector.

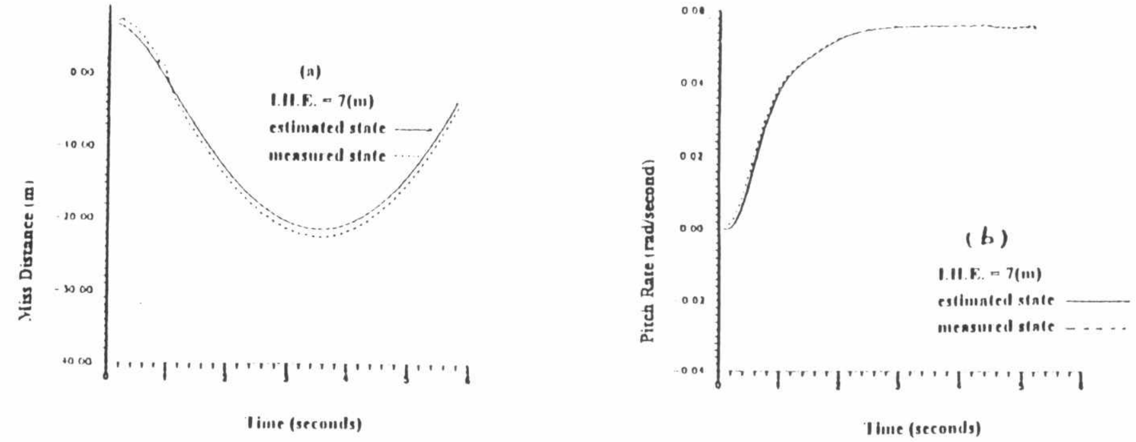

Fig (6) KF Performance for Target moving with constant acceleration 20 m/sec^2
(a) Miss distance
(b) Missile pitch rate

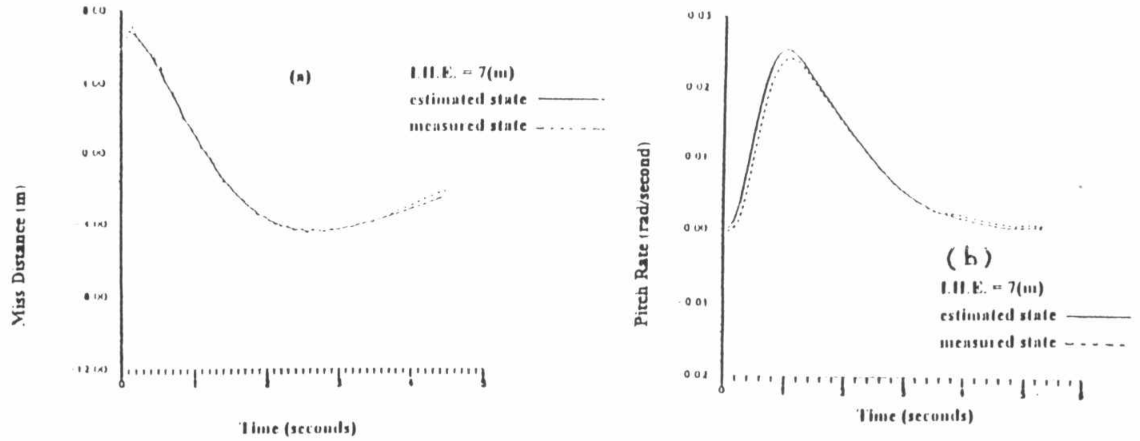

Fig (7) KF Performance for Target moving with exponential acceleration 20 e^{-t} m/sec^2
(a) Miss distance
(b) Missile pitch rate

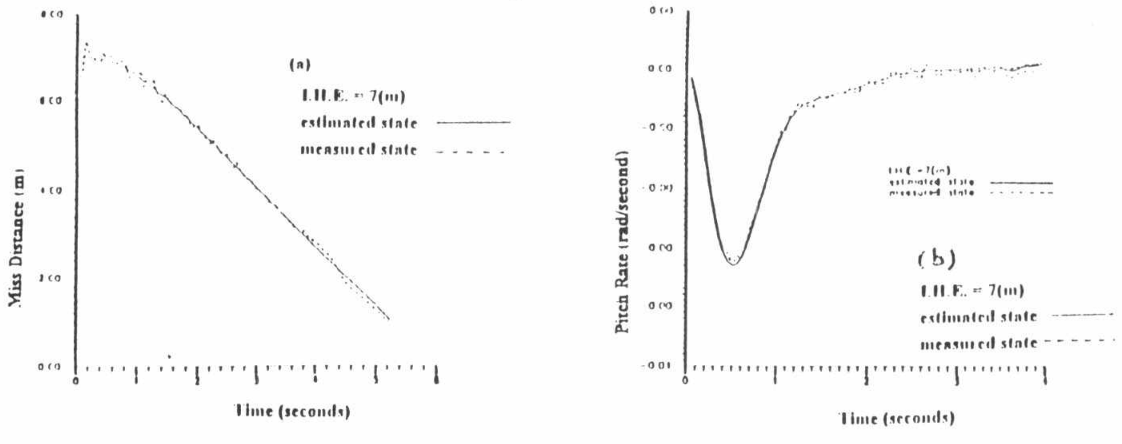

Fig (8) KF Performance for Target moving with constant velocity
(a) Miss distance
(b) Missile pitch rate