ARCHITECTURE FOR FILTER ALGORITHM IN GPS/INS INTEGRATION

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ABSTRACT

منسّق:الخط: (افتراضي) Arial، ١٢ نقطة، دون مائل، خط اللغة العربية وغيرها: Arial، ١٢ نقطة، دون مائل

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محذوف: , and

An inertial navigation system (INS) exhibits relatively low noise but tends to drift ov time. In contrast, Global Positioning System (GPS) errors are relatively noisy, b exhibit no long-term drift. Integrated INS/GPS navigation systems provide the best both worlds: the low short term noise characteristics of INS and the long term stabi of GPS are combined to provide a navigation solution with accuracy, reliability a robustness far beyond the sum of the constituent parts. However, in order to fu evaluate the performance of an integrated INS/GPS system, it is necessary stimulate both the GPS and inertial components of the system simultaneously. In tl paper, the architectures for common filter algorithms in GPS/INS loose integrati are presented. The error dynamics for attitude calculation are derived. Algorithı based on quaternions and direction cosines are used. Simulation results a analyzed to evaluate the different systems. This system evaluation are required help to create specification data; to aid integration algorithm design and tuning; determine if the receiver meets a given specification; to create conditions beyo those which can be created during live trials; and to recreate a known anomaly whi occurred in the real world.

KEYWORDS

Inertial Navigation System , Inertial Measurement Unit, Global Positioning Systems Loose integration ,Direction Cosines Matrix, Skew-symmetric matrix, and Quaternions.

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1- INTRODUCTION

GPS and INS have complementary qualities that make them ideal to use for sens fusion. The limitations of GPS include occasional high noise content, outages wh satellite signals are blocked, interference and low bandwidth. The strengths of GI include its long-term stability and its capacity to function as a stand-alone navigati system. In contrast, inertial navigation systems are not subject to interference outages, have high bandwidth and good short-term noise characteristics, but ha long-term drift errors and require external information for initialization. A combin system of GPS and INS subsystems can exhibit the robustness, higher bandwic and better noise characteristics of the inertial system with the long-term stability GPS. The level and complexity of GPS and INS coupling is dictated by seve factors, including desired navigation accuracy, quality of the inertial measureme unit (IMU) and required robustness of the GPS receiver outputs. The levels integration are usually classified as loose integration, tight integration and ultra-tig or deep Loose integration is the simplest method of coupling [1] where GPS and IN generate navigation solutions independently (position, velocity and attitude). T Kalman filter used in GPS/INS integration module is independent on the Kalman fil of GPS module, which increases the reliability of the system in case of failure GPS INS.

Tight Integration is a more complex level of coupling is tight integration [2],where t raw GPS ephemeris information and the position and velocity from INS algoritl used to predict pseudoranges and Doppler measurement. The tight integrati method contains only a single Kalman filter.

Ultra-Tight Integration is the most complex level of coupling [3]. It occurs at the GI tracking-loop level. It takes the difference between predicted in In-phase a Quatrature-phase of INS and the raw of GPS measurements In-phase a Quatrature-phase to determine the error estimates of the position, velocity a attitude. In terms of performance, ultra-tight integration also offers the most benet in terms of accuracy and robustness improvements to the GPS receiver and over system.

In this paper, the architectures for common filter algorithms in GPS/INS loo integration are presented. The lose integration is used due to its computatioı simplicity comparing with other integration algorithms. More over, it is suitable parallel processing.
The error dynamics for attitude calculation are derived. Algorithms based quaternions and direction cosines are used. Simulation results are analyzed explore the advantages of each algorithm.

1.1- Architecture of Loose Integration

GPS/INS system with loose integration is depicted in Fig. , for which the operation steps are given as bellow:

1- The Kalman filter of GPS extracts the position velocity data processes the rı data received by GPS receiver.
2- The raw IMU measurements, (Specific forces and angular rates) are process through the INS algorithm to determine the position, velocity and attitude.

3- The GPS/INS Integration using Kalman filter takes the difference between t position and velocity from GPS Kalman filter and INS algorithm to determine t error estimates in position, velocity, and attitude.

4- The error estimates from GPS/INS Integration using Kalman filter are feedback correct the position, velocity, and attitude in INS algorithm.

The loose integration method is distinguished by its simplicity in implementation a its robustness. If one of the sensors (INS or GPS) fails, a solution is still given by t other sensor. Other advantage of the loose integration can be seen in the processi time of the algorithm due to generally smaller state vectors. One of the benefits loose integration is that the INS/GPS Integration using Kalman filters has better noi characteristics than the GPS solution alone.

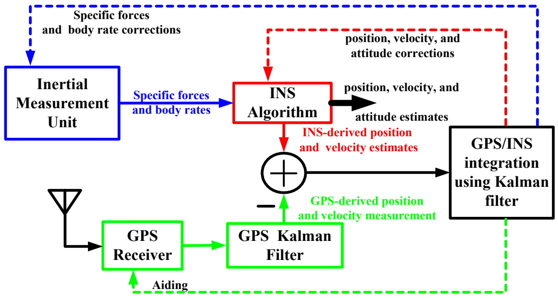

Fig.1. GPS/INS system with loose integration

The disadvantage is mainly that it is impossible to provide measurement update fro the GPS filter during poor GPS cover (less than four satellites). Loose integration best implemented with higher quality inertial sensors (navigation-grade or tactic grade) if the GPS outages are long in duration. Lower quality inertial sensors c also provide some immunity against momentary GPS outages, especially if th various biases were calibrated using GPS prior to the outage. In general, low quality inertial sensors are suited for applications where GPS outages are infreque and short in duration.

2- POSITION ERROR DYNAMICS

The error dynamics equations for positions in the N-frame are functions of positi and velocity error [4,5].

$$\delta \dot{r}^n = F_{RR}\,\delta r^n + F_{RV}\,\delta v^n$$

where: $\delta r^n = (\delta\varphi\ \delta\lambda\ \delta h)^T$ is the error in position in N-frame, $\delta v^n = (\delta v_N\ \delta v_E\ \delta v_D)^T$ the error in velocity in N-frame, F_{RR} and F_{RV} are described by the followi equations.

محذوف: $F_{RV} = \begin{pmatrix} \frac{\partial \dot\varphi}{\partial v_N} \\ \frac{\partial \dot\lambda}{\partial v_N} \\ \frac{\partial \dot h}{\partial v_N} \end{pmatrix}$

محذوف: $F_{RR} = \begin{pmatrix} \frac{\partial \dot\varphi}{\partial \varphi} \\ \frac{\partial \dot\lambda}{\partial \varphi} \\ \frac{\partial \dot h}{\partial \varphi} \end{pmatrix}$

$$F_{RR} = \begin{pmatrix} \frac{\partial \dot\varphi}{\partial \varphi} & \frac{\partial \dot\varphi}{\partial \lambda} & \frac{\partial \dot\varphi}{\partial h} \\ \frac{\partial \dot\lambda}{\partial \varphi} & \frac{\partial \dot\lambda}{\partial \lambda} & \frac{\partial \dot\lambda}{\partial h} \\ \frac{\partial \dot h}{\partial \varphi} & \frac{\partial \dot h}{\partial \lambda} & \frac{\partial \dot h}{\partial h} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-v_N}{(M+h)^2} \\ \frac{v_E \sin(\varphi)}{((N+h)\cos^2(\varphi))} & 0 & \frac{-v_E}{((N+h)^2\cos(\varphi))} \\ 0 & 0 & 0 \end{pmatrix}$$

$$F_{RV} = \begin{pmatrix} \frac{\partial \dot\varphi}{\partial v_N} & \frac{\partial \dot\varphi}{\partial v_E} & \frac{\partial \dot\varphi}{\partial v_D} \\ \frac{\partial \dot\lambda}{\partial v_N} & \frac{\partial \dot\lambda}{\partial v_E} & \frac{\partial \dot\lambda}{\partial v_D} \\ \frac{\partial \dot h}{\partial v_N} & \frac{\partial \dot h}{\partial v_E} & \frac{\partial \dot h}{\partial v_D} \end{pmatrix} = \begin{pmatrix} \frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{1}{((N+h)\cos(\varphi))} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

3- VELOCITY ERROR DYNAMICS

The velocity dynamics equation of inertial navigation system is [4]:

$$\dot v^n = C_b^n \, f^b - (2\omega_{ie}^n + \omega_{en}^n) \times v^n - g^n$$

Take in consideration the accumulation error, the calculation and measurement err
The computed version of the velocity dynamics equation can be expressed as

$$\dot{\bar v}^n = \bar C_b^n \, \bar f^b - (2\bar\omega_{ie}^n + \bar\omega_{en}^n) \times \bar v^n - \bar g^n$$

... [3]

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$$\dot{\bar v}^n = \dot v^n + \delta \dot v^n$$

where:

$$\bar C_b^n = C_b^n + \delta C_b^n$$
$$\bar f^b = f^b + \delta f^b$$
$$\bar\omega_{ie}^n = \omega_{ie}^n + \delta\omega_{ie}^n$$
$$\bar\omega_{en}^n = \omega_{en}^n + \delta\omega_{en}^n$$
$$\bar v^n = v^n + \delta v^n$$
$$\bar g^n = g^n + \delta g^n$$

where: δv^n is the error in velocity in N-frame. δC_b^n is the error in transformation fro B-frame to N-frame. δf^b is the error in specific force measure in the B-fram $\delta\omega_{ie}^n$ is The error in projection of the rotating rate vector of the E-frame w respect to the I-frame on the N-frame. $\delta\omega_{en}^n$ is The error in projection of t rotating rate vector of the N-frame with respect to the E-frame on the N-fram δg^n is The error in gravitational acceleration.

By neglecting the second order error terms, Eq. (5) can be reduced to

$$\delta\dot v^n = -(2\delta\omega_{ie}^n + \delta\omega_{en}^n) \times v^n - (2\omega_{ie}^n + \omega_{en}^n) \times \delta v^n - \delta g^n + C_b^n \delta f^b + \delta C_b^n f^b$$

Eq. (6) can be reduced to

$$\delta\dot{v}^n = F_{VR}\,\delta r^n + F_{VV}\,\delta v^n + \delta C_b^n f^b + C_b^n\,\delta f^b$$

where: F_{VR} and F_{VV} described by the following equations [4].

محذوف: F_{VR} ,

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$$F_{VR} = \begin{bmatrix} -2v_E\omega_e\cos(\varphi) - \dfrac{v_E^2}{(N+h)\cos^2(\varphi)} & 0 & \dfrac{-v_N v_D}{(M+h)^2} + \dfrac{v_E^2\tan(\varphi)}{(N+h)^2} \\ 2\omega_e v_N\cos(\varphi) - 2\omega_e v_D\sin(\varphi) + \dfrac{v_E v_N}{(N+h)\cos^2(\varphi)} & 0 & \dfrac{-v_E v_D}{(N+h)^2} - \dfrac{v_N v_E\tan(\varphi)}{(N+h)^2} \\ \begin{gathered} 2v_E\,\omega_e\sin(\varphi) - 8.8\times10^{-9}\sin(\varphi)\cos(\varphi)h \\ - \dfrac{g_{WGS0}\,[1+E^2\sin^2(\varphi)]E^2\sin(\varphi)\cos(\varphi)}{[1-E^2\sin^2(\varphi)]^{3/2}} \\ - \dfrac{2g_{WGS0}\,g_{WGS1}\sin(\varphi)\cos(\varphi)}{[1-E^2\sin^2(\varphi)]^{1/2}} \end{gathered} & 0 & \begin{gathered} \dfrac{v_E^2}{(N+h)^2} + \dfrac{v_N^2}{(M+h)^2} \\ -4.4\times10^{-9}\sin^2(\varphi) \\ +1.44\times10^{-13}h \\ -3.0877\times10^{-6} \end{gathered} \end{bmatrix}$$

$$F_{VV} = \begin{pmatrix} \dfrac{v_D}{(M+h)} & -2\omega_e\sin(\varphi) - \dfrac{2v_E\tan(\varphi)}{(N+h)} & \dfrac{v_N}{(M+h)} \\ 2\omega_e\sin(\varphi) + \dfrac{v_E\tan(\varphi)}{(N+h)} & \dfrac{v_D + v_N\tan(\varphi)}{(N+h)} & 2\omega_e\cos(\varphi) + \dfrac{v_E}{(N+h)} \\ \dfrac{-2v_N}{(M+h)} & -2\omega_e\cos(\varphi) - \dfrac{2v_E}{(N+h)} & 0 \end{pmatrix}$$

The calculation of term $\delta C_b^n f^b$ and $C_b^n\,\delta f^b$ depends on solution approach as given table 1. and table 2.

Table 1 Calculation of term $C_b^n\,\delta f^b$

Algorithm	$C_b^n\,\delta f^b$
Quaternions	$C_b^n\,\delta f^b = G_{AVQ}\begin{pmatrix}\delta f_x \\ \delta f_y \\ \delta f_z\end{pmatrix}$ where: $G_{AVQ} = \begin{pmatrix} q_1^2+q_0^2-q_2^2-q_3^2 & 2(q_1q_2-q_3q_0) & 2(q_1q_3+q_2q_0) \\ 2(q_1q_2+q_3q_0) & q_2^2+q_0^2-q_1^2-q_3^2 & 2(q_2q_3-q_1q_0) \\ 2(q_1q_3-q_2q_0) & 2(q_2q_3+q_1q_0) & q_3^2+q_0^2-q_1^2-q_2^2 \end{pmatrix}$
Direction Cosines	$\delta C_b^n f^b = F_{VAC}\,[\delta c_{11}\ \ \delta c_{12}\ \ \delta c_{13}\ \ \delta c_{21}\ \ \delta c_{22}\ \ \delta c_{23}\ \ \delta c_{31}\ \ \delta c_{32}\ \ \delta c_{33}]^T$ where: $F_{VAC} = \begin{pmatrix} f_x & f_y & f_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_x & f_y & f_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & f_x & f_y & f_y \end{pmatrix}$

Table (2) Calculation of term $\delta C_b^n f^b$

algorithm	$\delta C_b^n f^b$
Quaternions	$\delta C_b^n f^b = F_{VAQ}\begin{pmatrix}\delta q_0\\ \delta q_1\\ \delta q_2\\ \delta q_3\end{pmatrix}$ where: $F_{VAQ} = 2\begin{pmatrix} 2q_0 f_x - q_3 f_y + q_2 f_z & 2q_1 f_x + q_2 f_y + q_3 f_z & q_1 f_y + q_0 f_z & -q_0 f_y + q_1 f_z \\ 2q_0 f_y + q_3 f_x - q_1 f_z & q_2 f_x - q_0 f_z & 2q_2 f_y + q_1 f_x + q_3 f_z & q_0 f_x + q_2 f_z \\ 2q_0 f_z - q_2 f_x + q_1 f_y & q_3 f_x + q_0 f_y & -q_0 f_x + q_3 f_y & 2q_3 f_z + q_1 f_x + q_2 f_y \end{pmatrix}$
Direction Cosines	$\delta C_b^n f^b = F_{VAC}\left[\delta c_{11}\quad \delta c_{12}\quad \delta c_{13}\quad \delta c_{21}\quad \delta c_{22}\quad \delta c_{23}\quad \delta c_{31}\quad \delta c_{32}\quad \delta c_{33}\right]^T$ where:

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4- ATTITUDE ERROR DYNAMICS

The attitude of the vehicle relative to the N-frame can be variety of set of variable the most popular being Euler angles, direction cosines, and quaternions.

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4.1- Attitude Error Dynamics Based on Quaternions

We define the quaternions error as the arithmetic difference between the quaterni estimate and the true quaternion.

محذوف: (computed)

$$\delta q_n^b = \bar{q}_n^b - q_n^b \qquad ($$

Attitude Error Dynamics can be expressed as [4] given in equation (11).

$$\delta \dot{q}_n^b = 0.5\,\Omega_{ib}^b\,\delta q_n^b + 0.5\,\delta\Omega_{ib}^b\,q_n^b \qquad ($$

where:

محذوف: $\frac{1}{2}\Omega_{ib}^b\,\delta q_n^b$, $\frac{1}{2}\delta\Omega_{ib}^b\,q_n^b$ described by the following equations.

$$\frac{1}{2}\Omega_{ib}^b\,\delta q_n^b = F_{AAQ}\begin{pmatrix}\delta q_0\\ \delta q_1\\ \delta q_2\\ \delta q_3\end{pmatrix} \qquad ($$

$$F_{AAQ} = \begin{pmatrix} 0 & -0.5\omega_x & -0.5\omega_y & -0.5\omega_z \\ 0.5\omega_x & 0 & 0.5\omega_z & -0.5\omega_y \\ 0.5\omega_y & -0.5\omega_z & 0 & 0.5\omega_x \\ 0.5\omega_z & 0.5\omega_y & -0.5\omega_x & 0 \end{pmatrix}$$

محذوف: $\frac{1}{2}\delta\Omega_{ib}^b\,q_n^b = \frac{1}{2}$ ... [4]

محذوف: ¶

$$\frac{1}{2}\delta\Omega_{ib}^{b}\,q_{n}^{b}=G_{AAQ}\begin{pmatrix}\delta\omega_x\\ \delta\omega_x\\ \delta\omega_x\end{pmatrix}$$

$$G_{AAQ}=\frac{1}{2}\begin{pmatrix}-\bar{q}_1 & -\bar{q}_2 & -\bar{q}_3\\ \bar{q}_0 & -\bar{q}_3 & \bar{q}_2\\ \bar{q}_3 & \bar{q}_0 & -\bar{q}_1\\ -\bar{q}_2 & \bar{q}_1 & \bar{q}_0\end{pmatrix}$$

4.2- Attitude Error Dynamics Based on Direction Cosines

The attitude dynamics based on direction cosines is expressed by:

$$\dot{r}^{n}=\dot{C}_{b}^{n}\,r^{b} \qquad (14)$$

Where: C_b^n is the direction cosine matrix (DCM), which represents the transformati B-frame to N-frame, r^n is the position vector in N-frame, and r^b is the position vec in B-frame as shown in the following figure.

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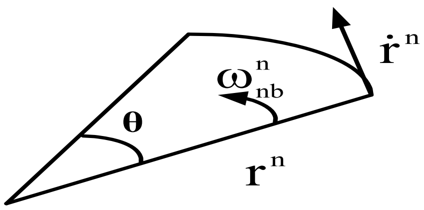

The velocity vector in N-frame can be expressed as given in equation (15) [5].

$$\begin{aligned}\dot{r}^{n}&=\omega_{nb}^{n}\times r^{n}\\ &=\Omega_{nb}^{n}\,r^{n}\\ &=C_{b}^{n}\,\Omega_{nb}^{b}\,r^{b}\end{aligned} \qquad (15)$$

Where: ω_{nb}^n is the projection of the rotating rate vector of the B-frame with respect the N-frame on the N-frame and Ω_{nb}^n is the skew-symmetric matrix, corresponding ω_{nb}^n.

محذوف: , and

Depending on the form that the derivative of DCM $\dot{C}_b^n$ can be expressed, t methods can be extracted where $\dot{C}_b^n=C_b^n\Omega_{nb}^b$ represents method one a $\dot{C}_b^n=-\Omega_{bn}^n\,C_b^n$ represents method two.

5- IMPLEMENTATION OF THE INS/GPS KALMAN FILTER

The INS/GPS Kalman Filter implementation is divided into four steps, as explained following subsections.

5.1- Continuous System Equations

$$\dot{\chi} = F\,\chi + G\,u \qquad ($$

Where: F is the dynamics matrix (state matrix), χ is the state vector, u is the forci vector function (input vector), and G is a design matrix (input matrix). The terms are described in details by the following subsection.

5.1.1- Error of Position, Velocity, and Attitude Based on Quaternions

Continuous system equations of error of position, velocity, and attitude based quaternions can be expressed as follows:

$$\chi = \begin{pmatrix} \delta r^n \\ \delta v^n \\ \delta q_b^n \end{pmatrix} = \begin{bmatrix} \delta\varphi & \delta\lambda & \delta h & \delta v_N & \delta v_E & \delta v_D & \delta q_0 & \delta q_1 & \delta q_2 & \delta q_3 \end{bmatrix}^T \qquad ($$

$$u = \begin{pmatrix} \delta f^b \\ \delta \omega_{ib}^b \end{pmatrix} = \begin{bmatrix} \delta f_x & \delta f_y & \delta f_z & \delta\omega_x & \delta\omega_y & \delta\omega_z \end{bmatrix}^T \qquad ($$

$$F = \begin{pmatrix} F_{RR} & F_{RV} & 0_{3\times4} \\ F_{VR} & F_{VV} & F_{VAQ} \\ 0_{4\times4} & 0_{4\times4} & F_{AAQ} \end{pmatrix}, \quad G = \begin{pmatrix} 0_{3\times3} & 0_{3\times3} \\ G_{AVQ} & 0_{3\times3} \\ 0_{4\times3} & G_{AAQ} \end{pmatrix} \qquad ($$

The details of matrix F and matrix G are given in [5].

5.1.2- Error of Position, Velocity, and Attitude Based on Direction Cosin Method One

Continuous system equations of error of position, velocity, and attitude based direction cosines can be expressed as follows:

$$\chi = \begin{pmatrix} \delta r^n \\ \delta v^n \\ \delta C_b^n \end{pmatrix} = \begin{bmatrix} \delta\varphi & \delta\lambda & \delta h & \delta v_N & \delta v_E & \delta v_D \\ \delta c_{11} & \delta c_{12} & \delta c_{13} & \delta c_{21} & \delta c_{22} & \delta c_{23} & \delta c_{31} & \delta c_{32} & \delta c_{33} \end{bmatrix}^T \qquad (2$$

$$u = \begin{pmatrix} \delta f^b \\ \delta \omega_{ib}^b \end{pmatrix} = \begin{bmatrix} \delta f_x & \delta f_y & \delta f_z & \delta\omega_x & \delta\omega_y & \delta\omega_z \end{bmatrix}^T \qquad (2$$

$$F = \begin{pmatrix} F_{RR} & F_{RV} & 0_{3\times9} \\ F_{VR} & F_{VV} & F_{VAC} \\ 0_{9\times3} & 0_{9\times3} & F_{AACI} \end{pmatrix}, \quad G = \begin{pmatrix} 0_{3\times3} & 0_{3\times3} \\ G_{AVC} & 0_{3\times3} \\ 0_{9\times3} & G_{AACI} \end{pmatrix} \qquad (2$$

The details of matrix F and matrix G are given in [5].

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محذوف: Continuous System Equations of

محذوف: Direction Cosines Using Alternative Algorithm

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محذوف: direction cosines using alternative algorithm

5.1.3- Error of Position, Velocity, and Attitude Based on Direction cosin method one Method two

Continuous system equations of error of position, velocity, and attitude based direction cosines method one matrix can be constructed by augmenting Eq. (1), (and خطأ! لم يتم العثور على مصدر المرجع. as follows:

$$\chi = \begin{pmatrix} \delta r^n \\ \delta v^n \\ \delta C_b^n \end{pmatrix} = \begin{bmatrix} \delta\varphi & \delta\lambda & \delta h & \delta v_N & \delta v_E & \delta v_D \\ \delta c_{11} & \delta c_{12} & \delta c_{13} & \delta c_{21} & \delta c_{22} & \delta c_{23} & \delta c_{31} & \delta c_{32} & \delta c_{33} \end{bmatrix}^T \quad ($$

$$u = \begin{pmatrix} \delta f^b \\ \delta \omega_{ib}^b \end{pmatrix} = \begin{bmatrix} \delta f_x & \delta f_y & \delta f_z & \delta\omega_x & \delta\omega_y & \delta\omega_z \end{bmatrix}^T \quad ($$

$$F = \begin{pmatrix} F_{RR} & F_{RV} & 0_{3\times 9} \\ F_{VR} & F_{VV} & F_{VAC} \\ 0_{9\times 3} & 0_{9\times 3} & F_{AAC2} \end{pmatrix}, \quad G = \begin{pmatrix} 0_{3\times 3} & 0_{3\times 3} \\ G_{AVC} & 0_{3\times 3} \\ 0_{9\times 3} & G_{AAC2} \end{pmatrix} \quad ($$

محذوف: is

The details of matrix F and matrix G are given in [5].

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5.2- CONVERT THE CONTINUOUS SYSTEM EQUATIONS TO DISCRE EQUATIONS

$$\chi(t_{k+1}) = \phi(t_{k+1}, t_k)\, \chi(t_k) + \int_{t_k}^{t_{k+1}} \phi(t_{k+1}, \tau)\, G(\tau)\, u(\tau)\, d\tau \quad ($$

or in abbreviated notation

$$\chi_{k+1} = \Phi_k\ \chi_k + w_k \quad ($$

where: χ_k is the state vector at time t_k, ϕ_k is the state transition matrix at time w_k is the vector of process noise at time t_k.

The covariance matrix associated with w_k is:

$$E\left[w_k w_i^T\right] = \begin{cases} Q & i = k \\ 0 & i \neq k \end{cases} \quad ($$

where: Q is the covariance matrix of process noise in the system state.

The numerical method to find the state transition matrix over short time inter $\Delta t = t_{k+1} - t_k$ is preferred:

$$\Phi_k = \exp(F\,\Delta t) \approx I + F\,\Delta t \quad ($$

محذوف: has the following form

The equation for calculating covariance matrix of process noise in the system state time t_k (Q_k) is given by equation (30).

$$\begin{aligned} Q_K &= E[w_K\ w_k^T] \\ &= E\left\{ \left[\int_{t_k}^{t_{k+1}} \phi(t_{k+1},\xi)\, G(\xi)\, u(\xi) d\xi\right]\left[\int_{t_k}^{t_{k+1}} \phi(t_{k+1},\xi)\, G(\xi)\, u(\xi) d\xi\right]^T \right\} \\ &= \int_{t_k}^{t_{k+1}}\int_{t_k}^{t_{k+1}} \phi(t_{k+1},\xi)\, G(\xi)\, E\left[u(\xi)\, u^T(\eta)\right] G^T(\eta)\, \phi^T(t_{k+1},\eta)\, d\xi\, d\eta \end{aligned} \quad ($$

محذوف: This estimated does not account for any of the correlations between the components of the driving noise w_k that develop over the course of a sampling period because of the integration of the continuous-time driving noise through the state dynamics Therefore, in this research

محذوف: estimated

محذوف: i.e.,

The Q_k is calculated using the first order estimation of the transition matrix, [5] expressed by equation (31).

$$Q_k \approx \phi_k G Q G^T \phi_k^T \Delta t \qquad (\text{:}$$

5.3- OBSERVATION EQUATIONS

The following observation equations

$$z_k = H_K \ \chi_k + \upsilon_k \qquad (\text{:}$$

where: z_k is the vector of measurement at time t_k, χ_k is the state vector at time H_k is the measurement matrix at time t_k, and υ_k is the vector measurement noise at time t_k.

The covariance matrices for the υ_k is given by

$$E\left[\upsilon_k \upsilon_i^T\right] = \begin{cases} R_k & i = k \\ 0 & i \neq k \end{cases} \qquad (\text{:}$$

where: R_k is the covariance matrix of noise measurement at time t_k.

The position and velocity from GPS can be considered as measurements. T straightforward formulation of the Observation equation can be written as:

$$z_K = \begin{pmatrix} r_{INS}^n - r_{GPS}^n \\ ----- \\ v_{INS}^n - v_{GPS}^n \end{pmatrix} = \begin{pmatrix} \varphi_{INS} - \varphi_{GPS} \\ \lambda_{INS} - \lambda_{GPS} \\ h_{INS} - h_{GPS} \\ ---------- \\ v_{INS}^n - v_{GPS}^n \end{pmatrix} \qquad H_k = \begin{pmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{pmatrix} \qquad (\text{:}$$

However, this approach causes numerical instabilities in calculati $\left[H_k \ P_k^- \ H_k^T + R_k\right]^{-1}$ for the Kalman gain K_k. Because $(\varphi_{INS} - \varphi_{GPS})$ and $(\lambda_{INS} - \lambda_{GI}$ are in radians and therefore they are very small values. This problem can resolved if the first and second rows are multiplied by $(M+h)$ and $((N+h)\cos(\varphi$ respectively [5]. Hence, the Observation equation will take the form:

$$z_k = \begin{pmatrix} (M+h)(\varphi_{INS} - \varphi_{GPS}) \\ [(N+h)\cos(\varphi)](\lambda_{INS} - \lambda_{GPS}) \\ (h_{INS} - h_{GPS}) \\ ---------- \\ v_{INS}^n - v_{GPS}^n \end{pmatrix} \qquad (\text{:}$$

$$H_k = \left(\begin{array}{ccc|c|c} M+h & 0 & 0 & & \\ 0 & (N+h)\cos(\varphi) & 0 & 0_{3\times3} & 0_{3\times3} \\ 0 & 0 & 1 & & \\ \hline & 0_{3\times3} & & I_{3\times3} & 0_{3\times3} \end{array}\right)$$

and the following is covariance matrix of noise measurement :

$$R_k = \mathrm{diag}(\sigma_{\varphi}^2\ \sigma_{\lambda}^2\ \sigma_h^2\ \sigma_{V_N}^2\ \sigma_{V_E}^2\ \sigma_{V_D}^2) \quad (\text{:}$$

which can be obtained from GPS processing.

5.4- Kalman Filter Algorithm

The Kalman filter can be divided into two stages, the update, and prediction. In t former, the Kalman gain, K_k is computed first, and then the state and the er covariance are updated using the prior estimate, $\hat{\chi}_k^-$ and its error covariance, P_k^- :

$$K_k = P_k^- \ H_k^T \left[H_k \ P_k^- \ H_k^T + R_k\right]^{-1} \quad (\text{:}$$

$$\hat{\chi}_k = \hat{\chi}_k^- + K_k \left[z_k - H_k \ \hat{\chi}_k^-\right] \quad (\text{:}$$

$$P_k = \left[I - K_k \ H_k\right] \ P_k^- \quad (\text{:}$$

in the prediction stage.

$$\hat{\chi}_{k+1}^- = \Phi_k \ \hat{\chi}_k \quad (\text{:}$$

$$P_{k+1}^- = \Phi_k \ P_k \ \Phi_k^T + Q_k \quad (\text{:}$$

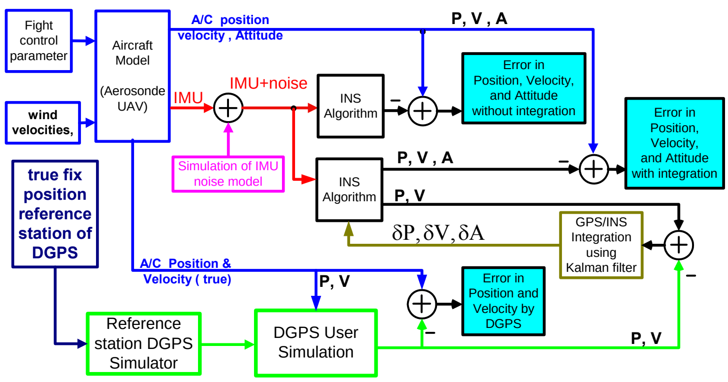
6- SIMULATION OF GPS/INS INTEGRATION USING KALMAN FILTI ALGORITHMS

Simulation for test algorithms of GPS/INS Integration using Kalman filter algorithı based on attitude by quaternions, direction cosines, and direction cosines meth one results will describe in following subsection.

6.1- Block Diagram

Chart of simulation block diagram for GPS/INS Integration using Kalman fil algorithms is shown in Fig.). It contains flight control parameters (flap, elevat aileron, rudder, throttle, mixture, and ignition), The INS algorithm, The DGPS mod and the GPS/INS Integration module using Kalman filter.

The INS algorithm is one of strapdown INS algorithms for attitude calculation. It c be calculated by either quaternions method [INSQ], direction cosines method o [INSC1].



محذوف: , and
محذوف: Integration Through Kalman
منسّق: عادي، مضبوطة، المسافة البادئة: قبل: ٠ نقطة، معلقة: ٢١,٣ نقطة
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محذوف: integration through Kalman
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محذوف: direction cosines using alternative algorithm
محذوف: integration through Kalman
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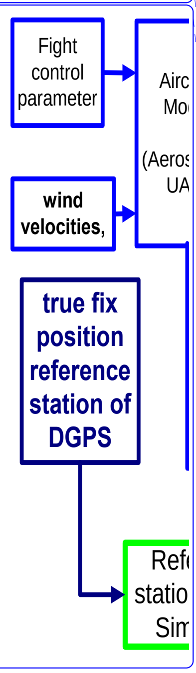

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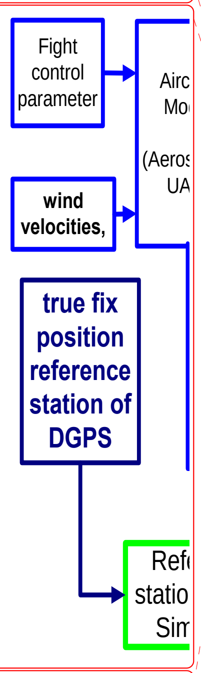
منسّق ... [5]



... [6]
محذوف: Run simulation with ... [7] d time of algorithm of
محذوف: , and
محذوف: and
محذوف: integration through Kalman
محذوف: the total time of the simulation is T = 300 sec.
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محذوف: , and
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محذوف: ¶ ... [8] t INS algorithm is using
محذوف: , and
محذوف: and Direct-Kalman [GPS-DK] methods).
محذوف: ¶ ... [9] SQ is the less error than

Fig. 2. Simulation block diagram for GPS/INS integration using Kalman

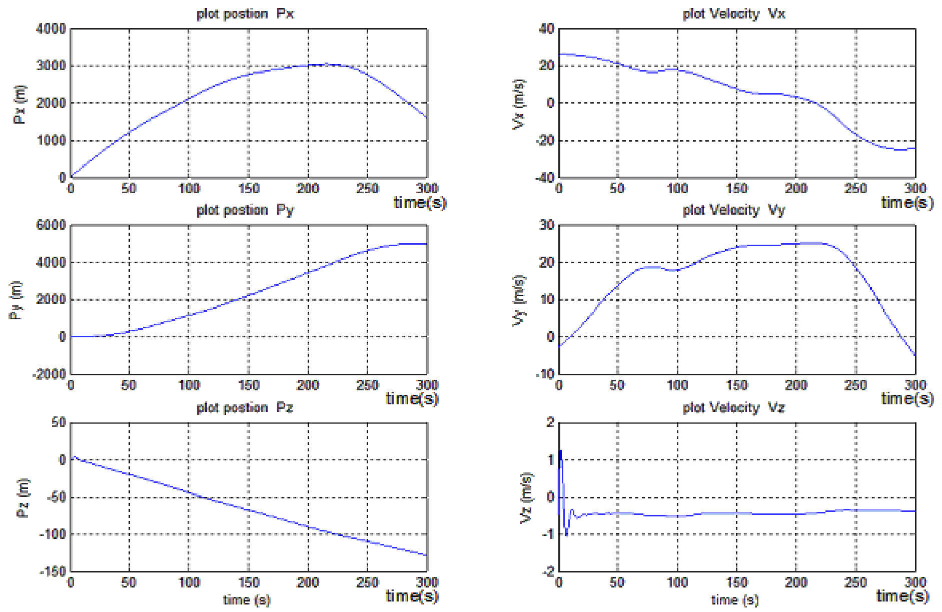
The DGPS simulation are dividing into two main parts, the first is the referen station, the second is the DGPS receiver, the calculation in receiver is done by thr methods Direct method [GPS-D], Kalman method [GPS-K], or Direct-Kalman meth [GPS-DK] . The difference between position, and velocity of outputs from DGPS a INS algorithms fed to the algorithm GPS/INS integration.

The GPS/INS Integration using Kalman filter in agreement with type of attitu calculation used for INS algorithms by three methods quaternions [INSQ], directi cosines method one [INSC1], and direction cosines method two [INSC2].
The difference between position, velocity, and attitude of aircraft model (true) a output from INS algorithm is the error in position, velocity, and attitude as shown Fig.).

6.2- SIMULATION RESULT

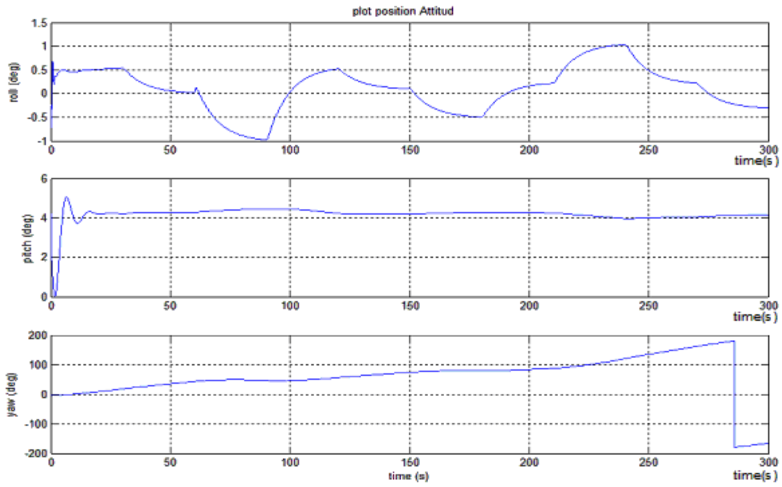
The INS data rate is taken every 0.01 sec, while the DGPS data is taken every 1 se
The sampled time of INS/GPS Integration using Kalman filter Ts (GPS/INS) = 1 sec,
The position and velocity plots from aircraft model (Aerosonde UAV) for testi GPS/INS integration algorithms is considered as the true position and velocity a shown in Fig.).
The attitude plots from aircraft model (Aerosonde UAV) is considered as the tr attitude (roll [ϕ], pitch [θ], and yaw [ψ]) are shown in Fig.).

The error in position, velocity, and attitude based on previous assumptions usi different methods of solution (Direct [GPS-D], Kalman [GPS-K], or Direct-Kalm [GPS-DK] methods) are shown in the Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, and Fig. 1(



محذوف: integration through Kalman

Fig.1. Position and velocity of Aerosonde UAV for test INS/GPS Integration using Kalman filter



محذوف: integration through Kalman

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Fig.2. Attitude of Aerosonde UAV for test INS/GPS Integration using Kalman filter

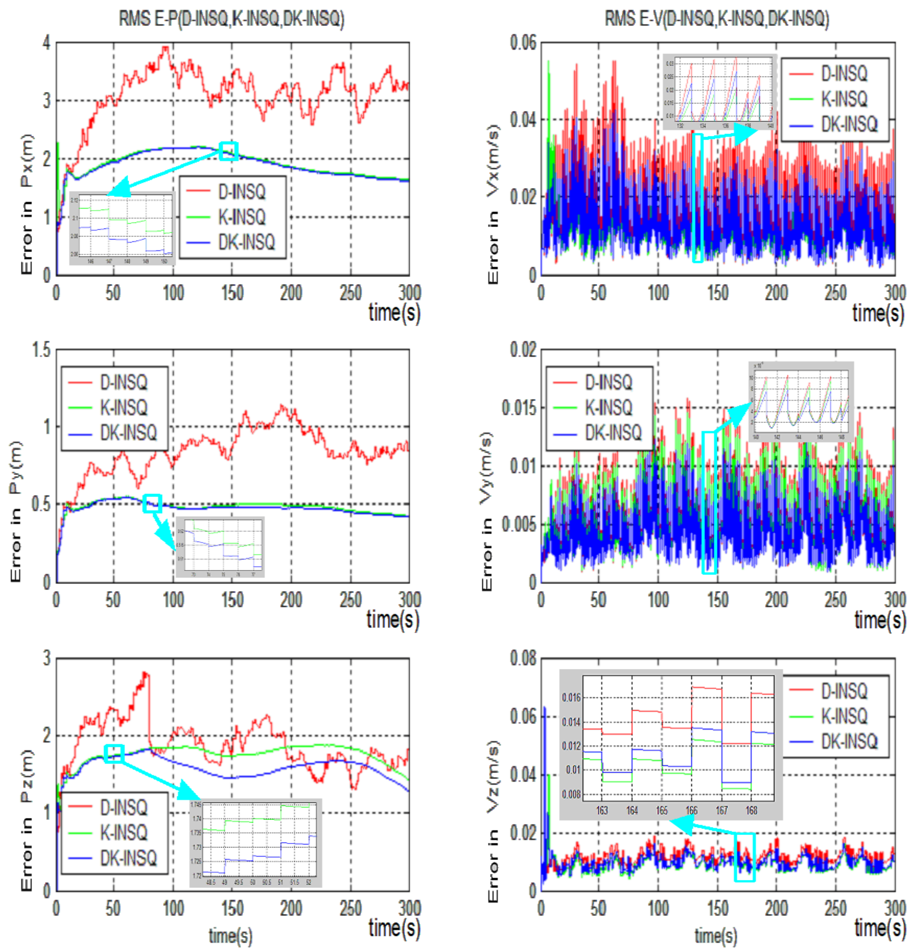

Fig.3. The RMS error in position and velocity of INS algorithm using quaternions

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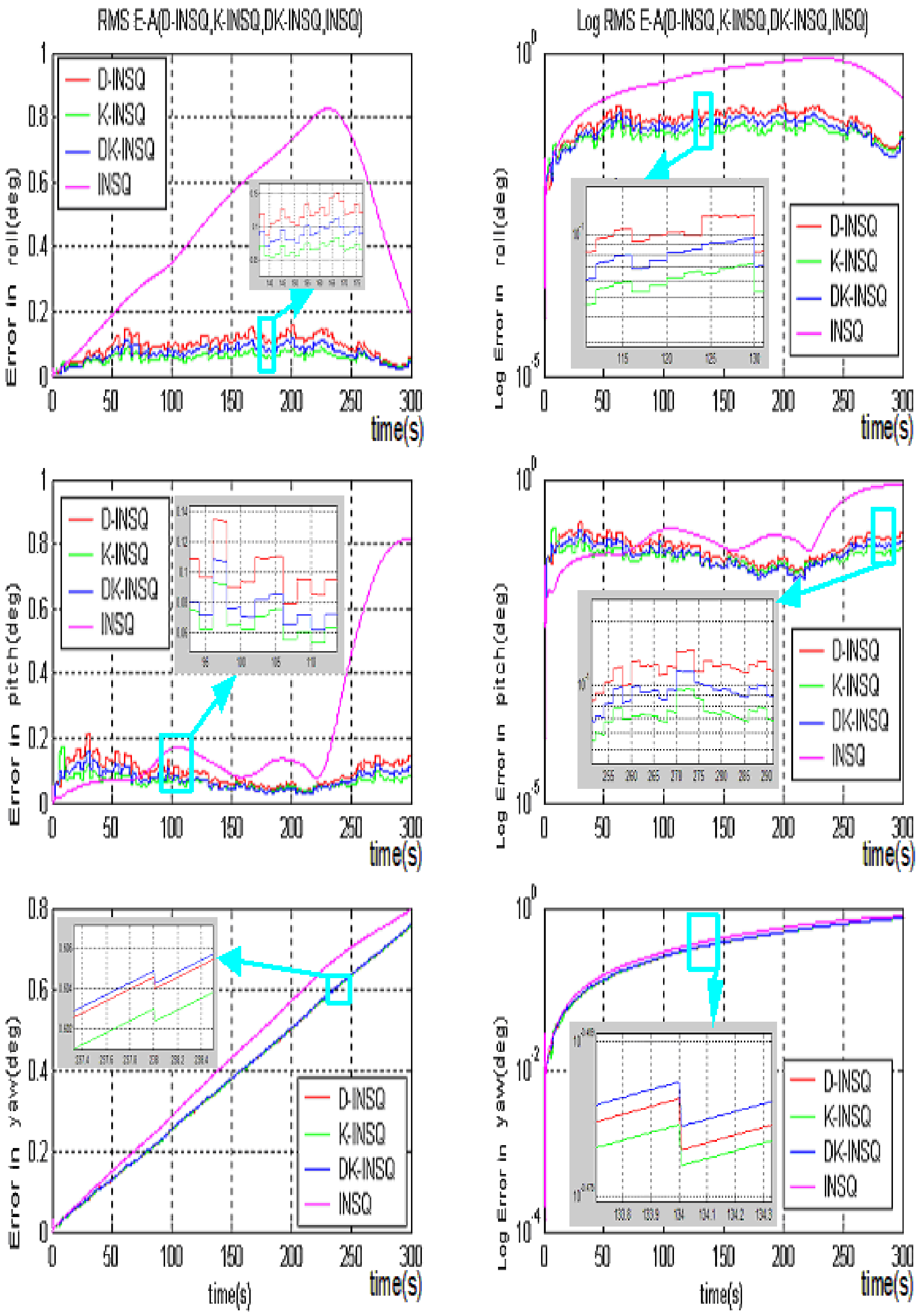

Fig.4. The RMS and Log RMS error in attitude of INS algorithm using quaternions

منسّق:متوسط

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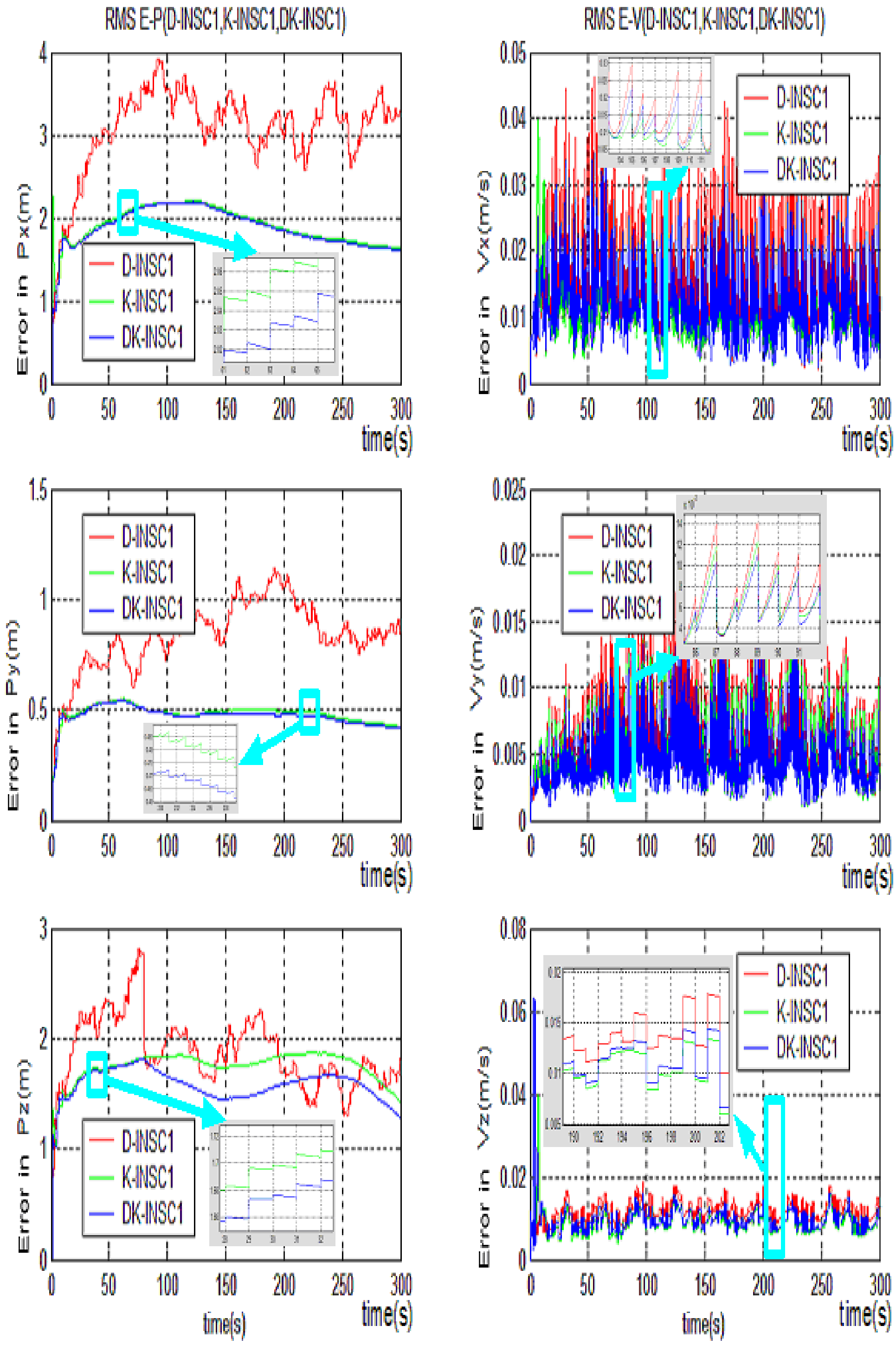
محذوف: The INSC1 is the less error than anther algorithm in roll (ϕ) less error until 2.0687 sec from start, pitch (θ) less error until 80.0016 sec from start, yaw (ψ) less error until 34.0023 sec from start.¶



محذوف: is

محذوف: direction cosines using alternative algorithm

Fig.5. The RMS error in position and velocity of INS algorithm using direction cosines method one

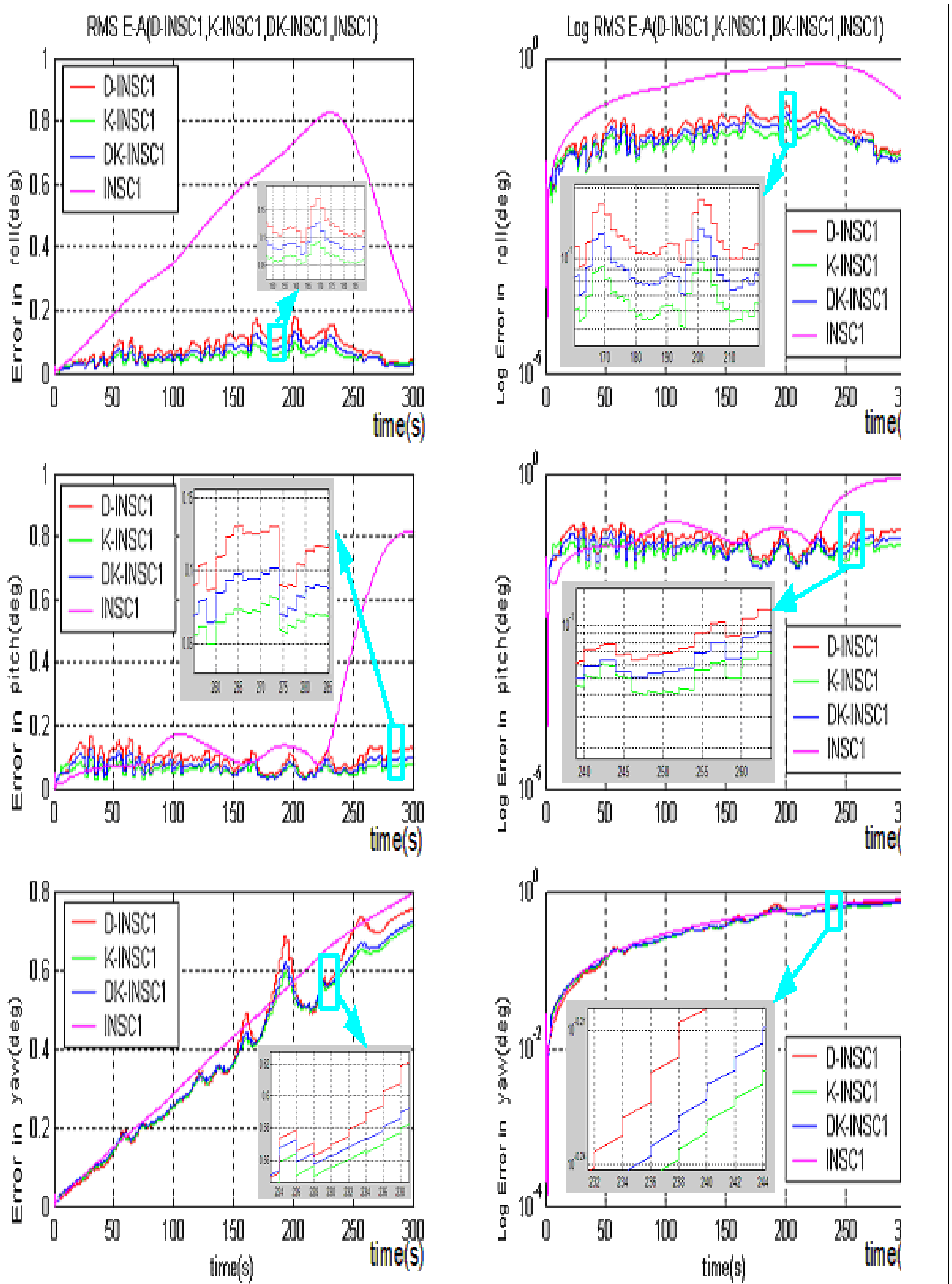


Fig.6. The RMS and Log RMS error in attitude of INS algorithm using direction cosines method one

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محذوف: direction cosines using alternative algorithm

منسّق:متوسط

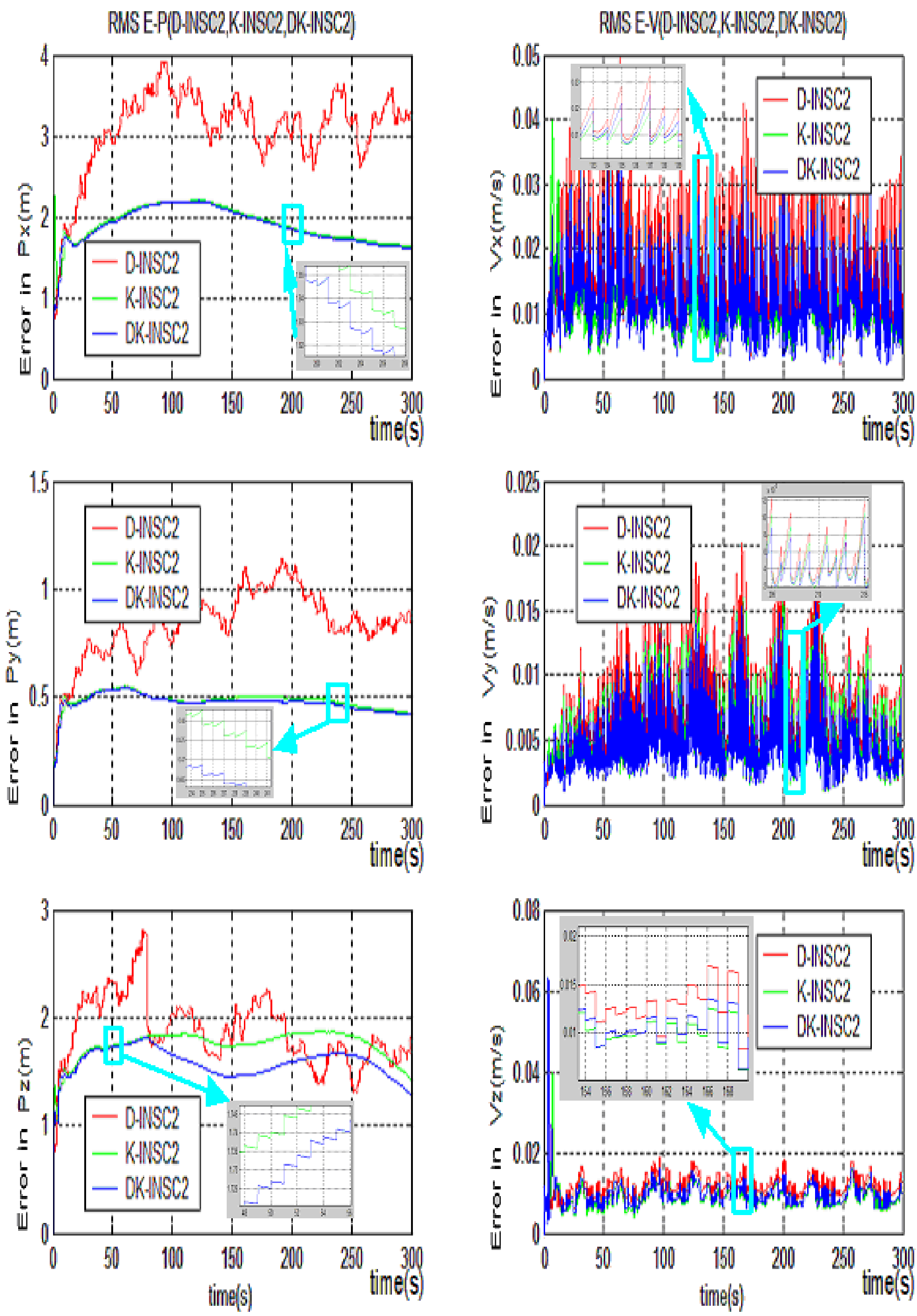
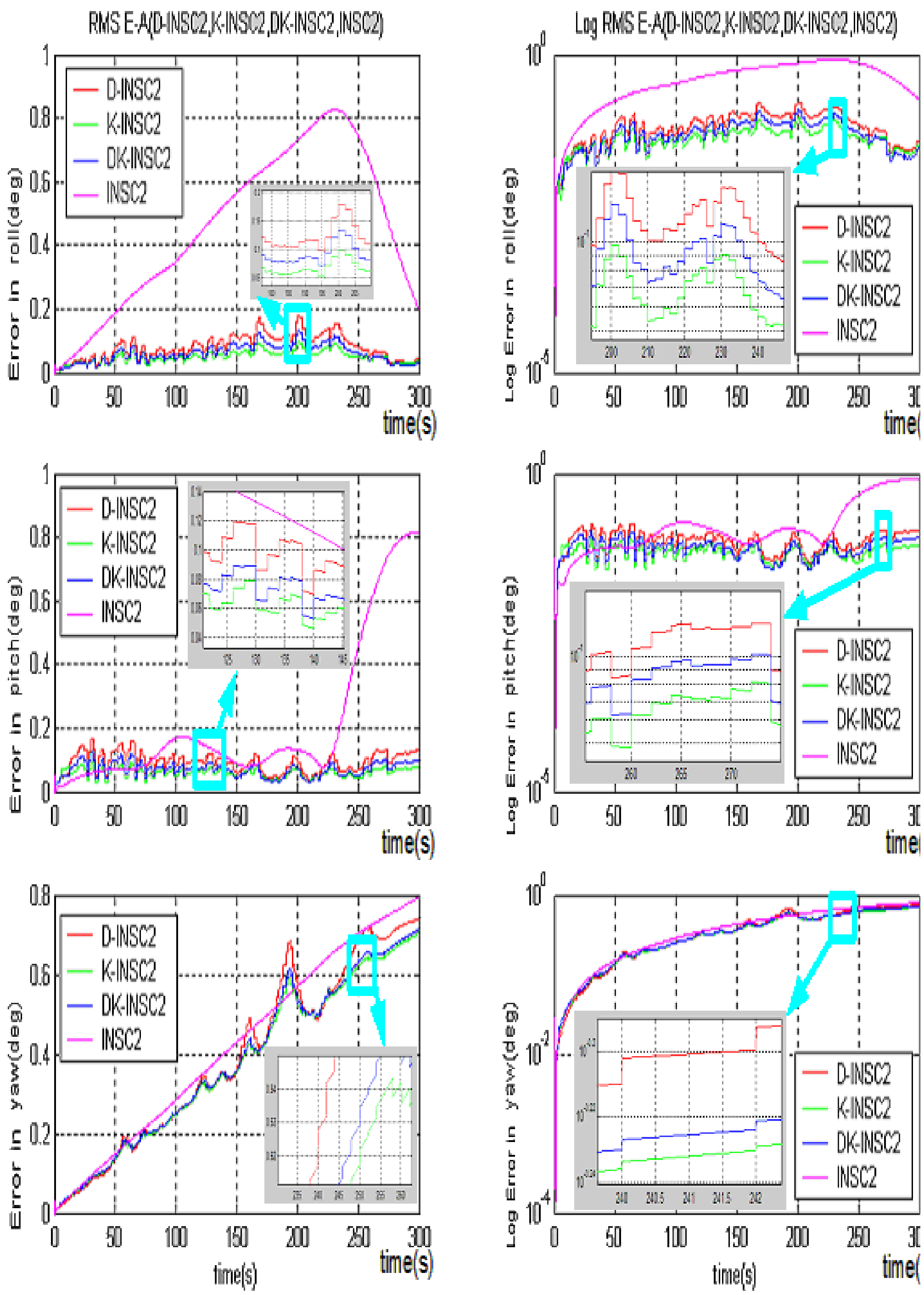


Fig.7. The RMS error in position and velocity of INS algorithm using direction cosin method two

منسّق:متوسط



محذوف: , and
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Fig.8. The RMS and Log RMS error in attitude of INS algorithm using direction cosines method two

CONCLUSION

As GPS / INS integration methods have become increasingly sophisticated, it is oft no longer possible to test the sub-systems individually and extrapolate the combin performance from the separate results; they must be tested together. Evaluating t performance of an integrated INS/GPS system requires the stimulation of both t GPS and inertial sub-systems simultaneously. In this paper, simulation of differe GPS/INS integration methods is applied. The GPS motion data is used in a tim manner. In the implementation used, the simulated IMU data is subjected to an er model, which adds representative errors to the IMU. The simulation results explo that the integration of GPS using Direct-Kalman and INS algorithm based quaternions is the most reliable algorithm.

محذوف: The lowest RMS error in position is INS algorithm is using quaternions for attitude integration with GPS using Direct-Kalman to solve navigation equation. ¶
The lowest RMS error in velocity and attitude is INS algorithm is using direction cosines for attitude integration with GPS using Kalman filter to solve navigation equation. ¶

محذوف: is

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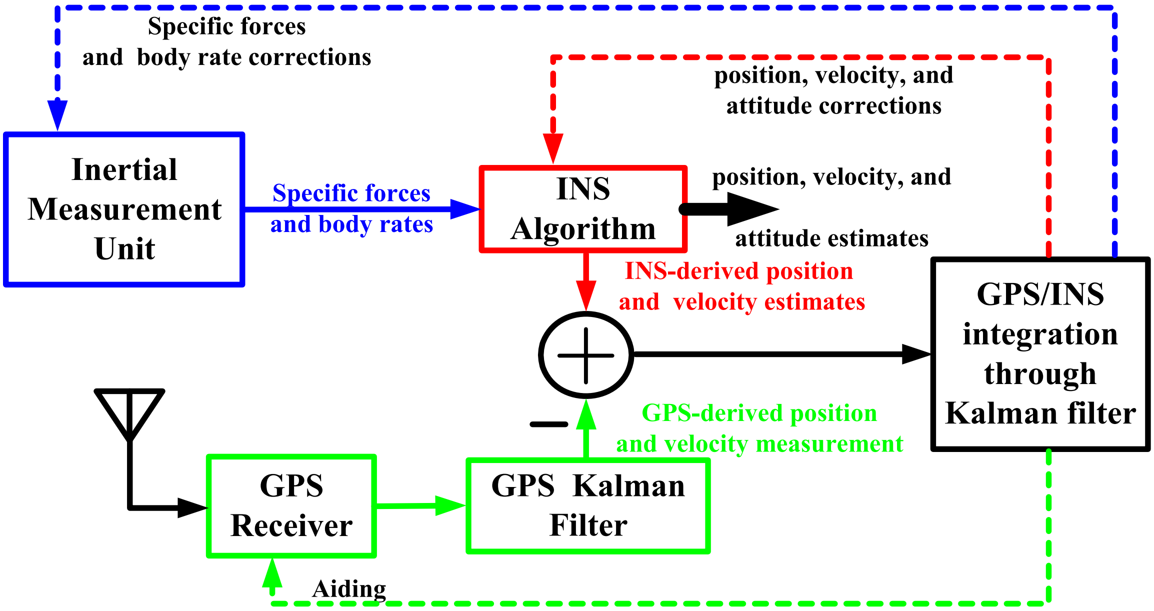
[5] Eun-Hwan Shin, "Accuracy Improvement of Low Cost INS/GPS for Land Application Msc. Thesis, The University of Calgary, Alberta, December, (2001).

الصفحة ٣: [١] منسّق	Ahmed	٢٠٠٧/٠١/٠٨ ٠٢:١٨:٠٠ م
نص أساسي، تباعد الأسطر: مفرد، السماح بعلامات التنقيط المعلقة، ضبط المسافة بين النص اللاتيني والآسيوي، ضبط المسافة بين النص والأرقام الآسيوية، محاذاة الخط: تلقائي

الصفحة ٣: [٢] محذوف	Ahmed	٢٠٠٧/٠١/٠٨ ٠١:١٧:٠٠ م



Fig(4.1): GPS/INS system with loose integration

الصفحة ٤: [٣] محذوف	Ahmed	٢٠٠٧/٠١/٠٨ ٠٣:٢١:٠٠ م

الصفحة ٧: [٤] محذوف	Ahmed	٢٠٠٧/٠١/٠٥ ٠١:٥٨:٠٠ ص

$$\frac{1}{2}\delta\Omega_{ib}^{b}\, q_n^b = \frac{1}{2}\begin{pmatrix}0 & -\delta\omega_x & -\delta\omega_y & -\delta\omega_z\\ \delta\omega_x & 0 & \delta\omega_z & -\delta\omega_y \\ \delta\omega_y & -\delta\omega_z & 0 & \delta\omega_x\\ \delta\omega_z & \delta\omega_y & -\delta\omega_x & 0\end{pmatrix}\begin{pmatrix}\bar q_0\\ \bar q_1\\ \bar q_2\\ \bar q_3\end{pmatrix} = G_{AAQ}\begin{pmatrix}\delta\omega_x\\ \delta\omega_x\\ \delta\omega_x\end{pmatrix} \qquad (4.38)$$

الصفحة ١٣: [٥] منسّق	Ahmed	٢٠٠٧/٠١/٠٤ ١١:٢٧:٠٠ م
Style (Latin) 14 pt (Latin) Bold Black Justified Line spacing ، ... المسافة البادئة: قبل: ٠ نقطة، السطر الأول: ٠ نقطة

الصفحة ١٣: [٦] محذوف	Ahmed	٢٠٠٧/٠١/٠٥ ٠٣:١١:٠٠ ص

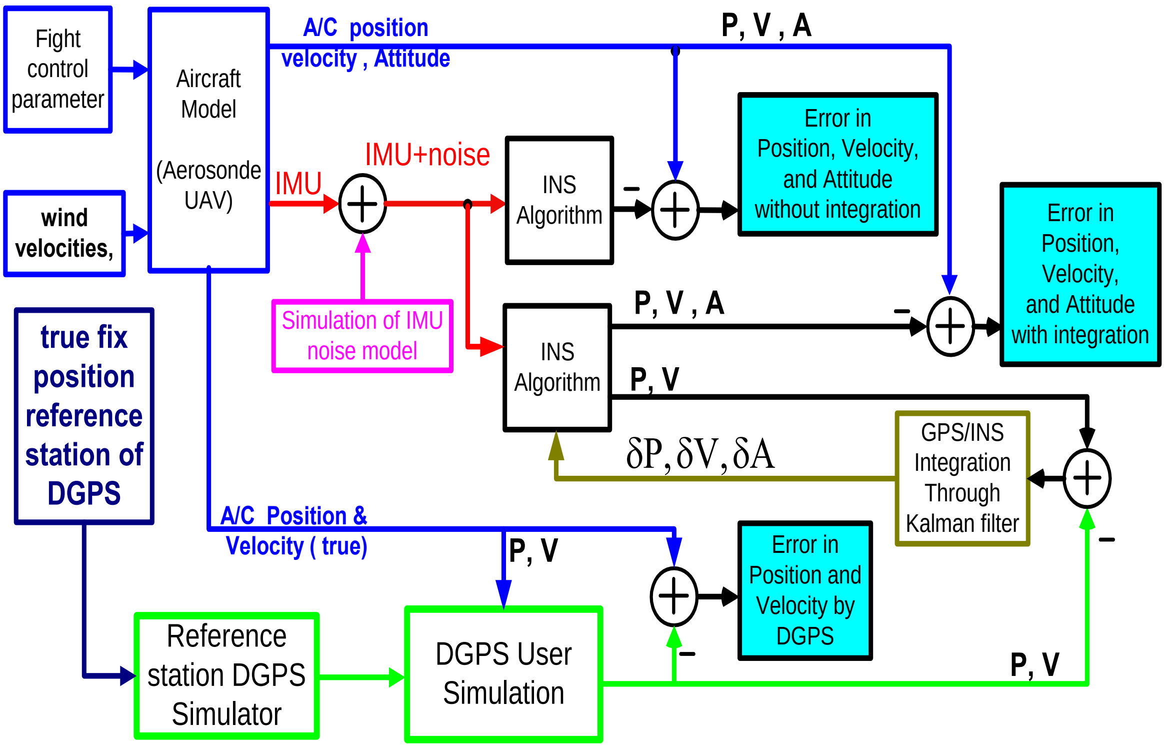


Fig. (4.4): Chart of simulation block diagram for test GPS/INS through Kalman integration

الصفحة ١٣: [٧] محذوف Ahmed ٢٠٠٧/٠١/٠٩ ٠٧:٣٩:٠٠ ص

Run simulation with sampled time of algorithm of INS Ts (INS) = 0.01 sec, sampled time of DGPS Ts (GPS) = 1 sec,

الصفحة ١٣: [٨] محذوف Ahmed ٢٠٠٧/٠١/٠٩ ٠٨:٣٦:٠٠ ص

For test INS algorithm is using quaternions for attitude integration with GPS using different methods to solve navigation equation (Direct [GPS-D], Kalman [GPS-K],

الصفحة ١٣: [٩] محذوف Ahmed ٢٠٠٧/٠١/٠٩ ٠٨:٤٧:٠٠ ص

The INSQ is the less error than anther algorithm in roll (ϕ) less error until 10.0042 sec from start, pitch (θ) less error until 78.0068 sec from start, yaw (ψ) less error until 10.0048 sec from start.