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AEROELASTIC ANALYSIS OF A WIND BELT ENERGY HARVESTER

Mohammed A. Al-Rahman, Ayman Ali El-Badawy, Al-Adl A. El Kady and M. Sameh
Mechanical Engineering Department, Al-Azhar University

إن تكنولوجيا حزام الهواء المرن هي من طرق استخلاص الطاقة التي تحول طاقة الرياح الى طاقة ميكانيكية ومن ثم الى طاقة كهربائية. فحزام الهواء المرن يعتمد علي ظاهرة معروفة برفرفة المرونة الهوائية. حيث أن حزام الهواء المرن يستخدم حزام مشدود لكي يستخرج الطاقة من الرياح مثبت علية زوج من المغناطيس التى تتحرك مع الحزام مقترنين بملف كهربي ثابت. تتسبب رفرفة الحزام تغييرًا في شدة المجال المغناطيسي المطبق على الملف الكهربي وبالتالي يتولد الكهرباء . إن الغرض من هذا البحث هو تحليل حركة حزام الهواء المرن التي هي ازدواج بين حركه عرضيه وحركة دورانية لتقدير الترددات الطبيعية والسرعة التي يحدث عندها الرفرفة. جهاز حزام الهواء المرن المطور ينتج 1 ملي وات من التيار المستمر عندما تكون سرعة الهواء 3 م/ث.

ABSTRACT

Wind belt technology is a method of energy harvesting that converts wind energy into mechanical energy, and then into electrical energy. The wind belt relies on an aerodynamic phenomenon known as aeroelastic flutter. The wind belt uses a tensioned belt undergoing flutter to extract energy from the wind and pair of magnets are attached to belt are coupled with fixed electric coils. The flutter of belt causes a change in the strength of the magnetic field applied to the electric coil, thus generating electricity. The objective of this paper is analyze of wind belt motion which is coupled between transverse and rotational motion to estimate flutter speed and natural frequencies. Developed wind belt device produce power about 1 mW DC at wind speed 3 m/s.

Key words: **Wind belt, Aeroelastic flutter, coupled vibration analysis**

1 INTRODUCTION

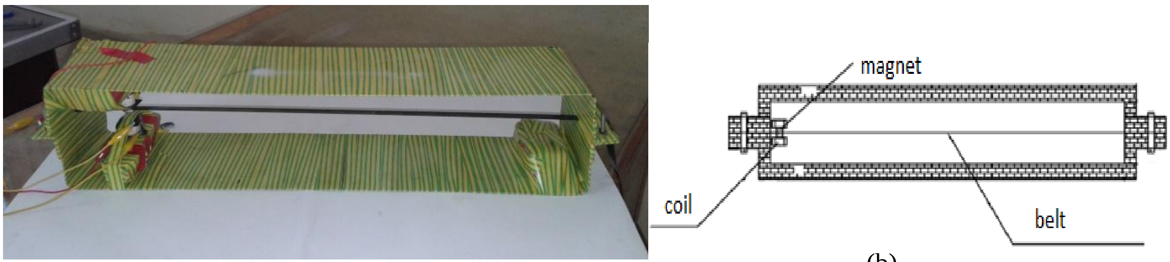
Renewable energy is the energy that comes from resources which are continually replenished on a human time scale. There are lots of renewable energy resources such as sunlight, wind, rain, tides, waves and geothermal heat [1,2]. Energy harvesting has been an active research area as demands for renewable energy sources increase. Energy harvesting, a term that was originally used in a publication about the photosynthesis of light in1966 .Energy harvesting, known as energy scavenging, is a term used for a system which captures or hunts the available ambient energy, and converts it into electrical energy [3].

As is well known flutter is a destructive aeroelastic phenomenon that must be avoided in aeronautical and industrial structure. The present study, however, deals with a power generation system which extracts wind energy from flutter phenomenon by utilizing it. The idea of extracting wind energy from flutter phenomenon is not new. Flutter Engine by Duncan (1948) might be the first machine although it was originally built for explaining the flutter phenomenon, and not for the wind power generator. Mckinney and DeLaurier (1981) proposed the “Wing mill” which utilized a harmonically oscillating wing to extract wind energy [4]. Isogai et al. [5] also proposed an elastic wing system driven by electric motor to optimize the efficiency. Shawn [6, 7] and his group provided an idea to create a new type of wind generator (wind belt) based on belt-flutter phenomenon, which could be considered as a good progress in utilizing elastic structures to harvest energy from air flows.

Wind belts are type of wind energy harvester which used aeroelastic flutter phenomena to produce frequency which converted to electricity. Energy harvesting through conversion of ambient vibrations or oscillation to electrical power has been the subject of many studies over the past two decades. The energy conversion methods that have been used for transforming mechanical (mostly vibrational and kinetic) energy into electrical energy are the electromagnetic, electrostatic and piezoelectric [8].
The objective of this work is to develop a device which uses electromagnetic induction to converts flutter or oscillation to electrical power. Also to analyze of wind belt oscillation to estimate flutter speed and natural frequencies.

2 CONSTRUCTION OF WIND BELT

As shown in figure1. The wind belt composed of fixed-fixed belt glued on it a pair of magnets that vibrate between a pair of fixed coils and a frame to hold these equipment. The belt made of 20 □m thick from Mylar tape, PE (Polyethylene) film or video tape whose properties are shown in table 1. The belt vibrates when subjected to a fluid flow. The vibration of the belt caused by the fluid flow causes a relative movement between the fixed electrical coils and the applied magnetic field according to Faraday's law of electromagnetic induction the e.m.f. will generating in the electrical coils.
The dimension of the wind belt is = 60cm × 1.25cm ×20 □m



(a) (b)

Figure 1: (a) Wind belt device
(b)Schematic of the wind belt model

Table 1: AVERAGE PROPERTY OF WIND BELT AND DIMENSIONS

Property	Average value
Density	1420 Kg/m^3
Young modulus**[9]**	0.7 GPa
Poisson ratio	0.4
Mass	0.371 g/m
Length	0.6 m
Tension	2 N
Eccentricity	0.001 m

3 BELT AEROELASTIC MODEL

Consider a belt with length, L as shown in figure 2 that is vibrating due to its flexibility in flapwise (out of plane) and twisting about the shear axis due to its torsional flexibility.

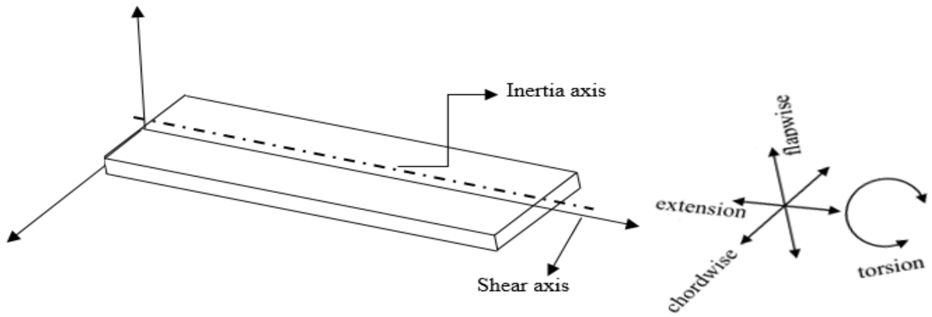


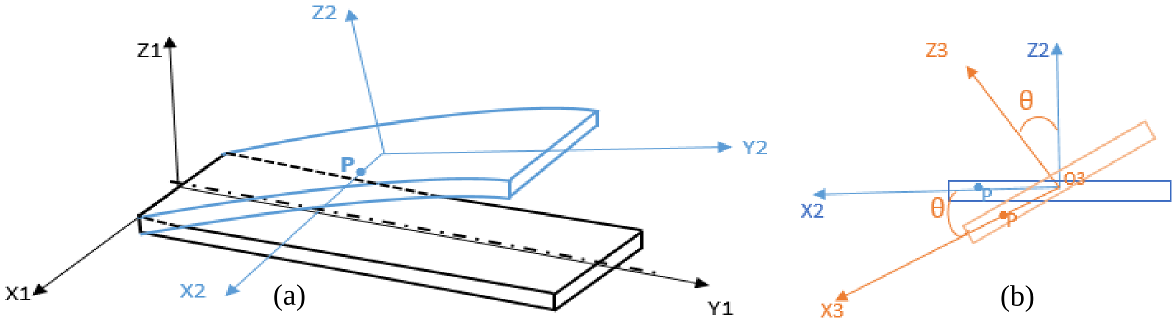
Figure 2: Uniform belt under flapwise and torsional vibrations

The equations of motion governing the belt's flapwise vibration and torsional motions. First, kinematic analysis of the belt was performed to compute the displacement and velocity fields of any generic point on the belt's section relative to fixed frame. Second, expressions for the kinetic, strain energies and work done by aerodynamic loading per unit length were derived. And finally, the extended Lagrange's equation was applied and the equations of motion and the associated boundary conditions were directly obtained.

3.1 KINEMATIC ANALYSIS

Consider the inertial fixed frame $[XYZ]_1$ with an origin O_1 is located at the shear center of a wind belt. Now, let P be a point on the belt whose position is described by ${}^1r_p = [x_p, y_p, z_p, 1]^T$ relative to frame $[XYZ]_1$. Also, let the coordinate system $[XYZ]_2$ be displaced by a distance y_p away from O_1 along the y_1 axis. The origin point O_2 is attached to the shear center at that cross section even after deformation and the y_2 axis extends along the deformed elastic axis at that cross section. Using the notation x for x_p, y for y_p and z for z_p for the sake of generality and convenience, the position of point P is described as ${}^2r_p = [x\ 0\ z\ 1]$ relative to $[XYZ]_2$ in the undeformed state of the belt as shown in figure 3a. Also, let the coordinate system $[XYZ]_3$ be fixed such that O_2 coincides with O_3 and the Y_2 axis is collinear with the Y_3 axis. The X_3 axis is rotated by an angle from the X_2 axis about the Y_2 axis as shown in figure3b. The resultant homogenous transformation matrix due to transverse motion from the frame 1 to the frame 2 is thus.

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -w'(y,t) & 0 \\ 0 & w'(y,t) & 1 & w(y,t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$



(a) (b)

Figure 3 (a) Wind belt cross section deformation due to transverse motion.
(b) The wind belt cross-section twisted by the torsional angle θ about the $y_2$axis.

Also the transformation matrix ${}^2T_3$ due to rotation about Y axis expressed as:

$$ {}^{2}T_{3} = \begin{bmatrix} 1 & 0 & \theta(y,t) & 0 \\ 0 & 1 & 0 & 0 \\ -\theta(y,t) & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{therefore, } {}^{1}T_{3} = {}^{1}T_{2}\,{}^{2}T_{3} = \begin{bmatrix} 1 & 0 & \theta & 0 \\ -\theta w' & 1 & -w' & y \\ -\theta & w' & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

The value of product $\theta w'$ is very small and can be neglected. So, the position $\overrightarrow{r_P}$ of any point on belt after the rotating and transverse deformation relative to fixed frame 1 expressed as:

$$ {}^{1}\overrightarrow{r_P} = \begin{bmatrix} 1 & 0 & \theta & 0 \\ 0 & 1 & -w' & y \\ -\theta & w' & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + z\theta \\ y - zw' \\ z + w - x\theta \\ 1 \end{bmatrix} \tag{3} $$

The displacement vector $\vec{u}$ due to the elastic deformations and the velocity vector ${}^{1}\overrightarrow{\dot{r}_P}$ are get as:

$$ \vec{u} = \begin{bmatrix} z\theta \\ -zw' \\ w - x\theta \end{bmatrix} \quad and \quad {}^{1}\overrightarrow{\dot{r}_P} = \begin{bmatrix} z\dot{\theta} \\ -z\dot{w}' \\ \dot{w} - x\dot{\theta} \end{bmatrix} $$

The strain energy density expression as:

$$ U_0 = \int_0^{\varepsilon_y} \sigma_y \, d\varepsilon_y + 2G\left[\int_0^{\gamma_{xy}} \gamma_{xy}\, d\gamma_{xy} + \int_0^{\gamma_{yx}} \gamma_{yx}\, d\gamma_{yx} + \int_0^{\gamma_{yz}} \gamma_{yz}\, d\gamma_{yz} + \int_0^{\gamma_{zy}} \gamma_{zy}\, d\gamma_{zy} + \int_0^{\gamma_{xz}} \gamma_{xz}\, d\gamma_{xz} + \int_0^{\gamma_{zx}} \gamma_{zx}\, d\gamma_{zx}\right] $$

Where $\int_0^{\varepsilon_y} \sigma_y \, d\varepsilon_y$ is strain energy density from tendency of tension T to restore the belt to equilibrium position, therefore the potential energy of tensile force T applied on the belt can be expressed as $\frac{1}{2}\int_0^L T\ w'^2 dy$ [10], and the shear strain γ_{ij} is as given in [11]

$$ \gamma_{yx} = \gamma_{xy} = \frac{1}{2}\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right] = \frac{1}{2} z\theta' \,, \gamma_{yz} = \gamma_{zy} = \frac{1}{2}\left[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right] = -\frac{1}{2} x\theta' \ and $$

$$ \gamma_{zx} = \gamma_{xz} = \frac{1}{2}\left[\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right] = 0 $$

The potential energy is integrated with respect to the volume of belt to get the elastic potential energy.

$$ P.E = \iiint U_0 d(vol) = \frac{1}{2}\int_0^L (T\ w'^2 + G\, J_y\, \theta'^2) dy \tag{4} $$

The kinetic energy expression is

$$ K.E = \frac{1}{2}\iiint \rho\, \dot{r}_C{}^T\, \dot{r}_c\, d(vol) $$

Where $\dot{r}_c$ is velocity of mass center and $= {}^{1}\dot{r}_P + r_x \dot{\theta}\, \vec{k} = \begin{bmatrix} z\dot{\theta} \\ -z\dot{w}' \\ \dot{w} - x\dot{\theta} \end{bmatrix} + r_x\dot{\theta}\,\vec{k} = \begin{bmatrix} z\dot{\theta} \\ -z\dot{w}' \\ \dot{w} - (x - r_x)\dot{\theta} \end{bmatrix}$

The model is dealing with small deflections and is not interested in the higher orders of vibration, thus the term $z\,\dot{w}'$ can be ignored. The kinetic energy can thus be written as.

$$ K.E = \frac{1}{2}\int_0^L \left(m\dot{w}^2 + 2mr_x\dot{w}\dot{\theta} + I_{y_{zm}}\dot{\theta}^2\right) dy \tag{6} $$

Virtual work due to distributed force $f(y,t)$ and torque $\tau(y,t)$ is given by:

$$ W = \int_0^L [f(y,t)w(y,t) + \tau(y,t)\theta(y,t)] dy \tag{7} $$

3.2 AEROELASTIC ANALYSIS

The assumed modes method is a procedure for discretizing distributed parameter systems that is closely related to the Rayleigh-Ritz method. It is basically a series discretization technique where the solution is assumed to be a linear combination of a set of *n* trial functions. The assumed modes method aims at developing the discretized modal equations of motion by first discretizing the potential energy, the kinetic energy and virtual work expressions Then, Lagrange's equations are employed to produce the modal equations of motion from the discretized energy expressions [10]. A distributed system and approximate vibrational displacements are expressed as the product of two functions in space and time as

$$w(y,t) = \sum_{i=1}^{n_w} \psi_i(y)\,\eta_i(t) \quad and \quad \theta(y,t) = \sum_{i=1}^{n_\theta} \theta_i(y)\,\emptyset_i(t) \qquad (8)$$

Where $\psi_i(y)$ and $\theta_i(y)$ are known trial functions used to approximate the spatial mode shapes. $\eta_i(t)$ and $\emptyset_i(t)$ are the generalized coordinate used to describe the time response of the belt vibrations. It is important to note that the utilized trial functions in the assumed modes approach satisfy the problem boundary conditions. To simplify the aeroelastic analysis the following mode shapes $\theta_i(y) = \psi_i(y) = \sqrt{2}\,sin(\alpha_i y)$, where: $\alpha_i = \frac{i\pi}{L}$.

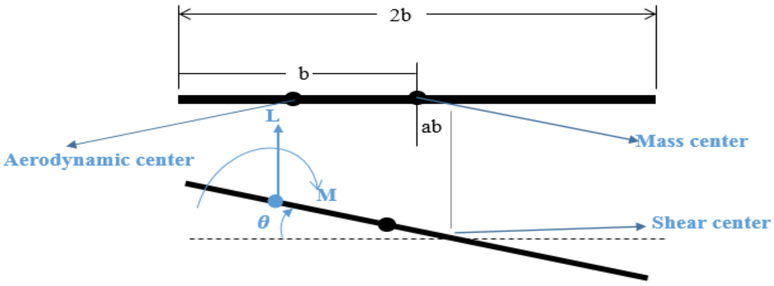


Figure 4: Schematic show aerodynamic model of the wind belt

The potential and kinetic energies of a fixed-fixed uniform belt are discretized as follows:

$$P.E = \frac{1}{2}\left[\frac{T}{L}\sum_{i=1}^{n_w}(\alpha_i L)^2\,\eta_i^{\,2} + \frac{GJ_y}{L}\sum_{j=1}^{n_\theta}(\alpha_i L)^2\,\emptyset_i^{\,2}\right]$$

$$K.E = \frac{mL}{2}\left[\sum_{i=1}^{n_w}\dot{\eta}_i^{\,2} + b^2 r^2\sum_{i=1}^{n_\theta}\dot{\emptyset}_i^{\,2} - 2bx_\theta\sum_{i=1}^{n_\theta}\sum_{j=1}^{n_w}\dot{\emptyset}_i\dot{\eta}_j\right] \qquad (9)$$

The virtual work due to the aerodynamic forces in figure (4) is given by:

$$\delta W = \int_0^L (L'\delta w + (M'_{ac} + eL')\delta\theta)\,dy$$

where: $e = \left(\frac{1}{2} + a\right)b$, b is the semi chord of wind belt, $L'\ and\ M'_{ac} = M'_{1/4}$ are the distributed lift and pitching moment per unit length at quarter the chord of the belt. Let $r_x = -bx_\theta$ and $r^2 = \frac{I_y}{mb^2}$. Discretization of virtual work is thus.

$$\delta W = \sum_{i=1}^{n_w} E_{w_i}\delta\eta_i + \sum_{i=1}^{n_\theta} E_{\theta_i}\delta\emptyset_i \qquad (11)$$

where E_{w_i} and E_{θ_i} are the generalized forces, given by:

$$E_{w_i} = \int_0^L \psi_i(y) L'\,dy \qquad (12)$$

$$E_{\theta_i} = \int_0^L \theta_i(y)\left(M'_{1/4} + \left(\frac{1}{2}+a\right) bL'\right) dy$$

The discretized energy expressions are substituted in Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{q}_k}\right) - \frac{\partial K.E}{\partial q_k} + \frac{\partial P.E}{\partial q_k} = Q_k \qquad (k = 1,2,\dots,n)$$

Where: Q_k are the generalized forces,

k is number of generalized coordinate.

The equations of motion expressed as:

$$mL\begin{bmatrix} [I] & -bx_\theta[I] \\ -bx_\theta[I] & b^2r^2[I] \end{bmatrix}\begin{bmatrix} \ddot{\eta} \\ \ddot{\emptyset} \end{bmatrix} + \begin{bmatrix} \frac{T}{L}[B] & [0] \\ [0] & \frac{GJ}{L}[B] \end{bmatrix}\begin{bmatrix} \eta \\ \emptyset \end{bmatrix} = \begin{bmatrix} E_w \\ E_\theta \end{bmatrix} \tag{13}$$

Where $[I]$ denotes an identity matrix and [0] denotes a matrix of zeros, element of the diagonal matrices $[B]$ is given by $B_{ii} = (\alpha_{ii}L)^2$.

For free vibrations E_w and E_θ are equal to zero. Mathematically, this type of motion can be represented by $\eta_j(t) = \emptyset(t) = e^{i\omega t}$, therefore:

$$-mL\omega^2\begin{bmatrix} [I] & -bx_\theta[I] \\ -bx_\theta[I] & b^2r^2[I] \end{bmatrix} + \begin{bmatrix} \frac{T}{L}[B] & [0] \\ [0] & \frac{GJ}{L}[B] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For first mode $i = 1$ the frequencies are $\omega_1 = 382.rad/s$, $\omega_2 = 2172\,rad/s$

3.2.1 STEADY FLOW

For the aerodynamics, the steady-flow of thin airfoil theory states that [13]:

$$L' = 2\pi\rho_\infty bU^2\theta(y,t) \qquad and \qquad M'_{1/4} = 0 \tag{14}$$

Substituting equation 14 into equation 12, therefore the generalized force is:

$$E_{w_i} = \int_0^L \psi_i(y)2\pi\rho_\infty bU^2\theta(y,t)dy = 2\pi\rho_\infty bU^2L[I]\emptyset$$

$$E_{\theta_i} = \int_0^L \theta_i(y)\left(0 + \left(\frac{1}{2}+a\right)2\pi\rho_\infty b^2U^2\theta(y,t)\right) dy = \left(\frac{1}{2}+a\right)2\pi\rho_\infty b^2U^2L[I]\emptyset$$

There are now more degrees of freedom if either n_w or n_θ exceeds unity. Let us consider the case in which $n_w = n_\theta =1$, therefore $[B] = (\alpha L)^2 = \pi^2$ and the fundamental bending and torsion frequencies are $\omega_w = \frac{\pi}{L}\sqrt{\frac{T}{m}}$ and $\omega_\theta = \frac{\pi}{L}\sqrt{\frac{GJ}{mb^2r^2}}$ [14]. Therefore Equation 13 write as

$$\begin{bmatrix} mb^2 & -b^2x_\theta \\ -b^2x_\theta & b^2r^2 \end{bmatrix}\begin{bmatrix} \frac{\ddot{\eta}}{b} \\ \ddot{\emptyset} \end{bmatrix} + \begin{bmatrix} mb^2{\omega_w}^2 & -2\pi\rho_\infty b^2U^2 \\ 0 & mb^2r^2{\omega_\theta}^2 - \left(\frac{1}{2}+a\right)2\pi\rho_\infty b^2U^2 \end{bmatrix}\begin{bmatrix} \frac{\eta}{b} \\ \emptyset \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In P-method substituting $\eta = \bar{\eta}e^{vt} = \bar{\eta}e^{s\omega_\theta t}$ and $\theta = \bar{\theta}e^{vt} = \bar{\theta}e^{s\omega_\theta t}$, where $s = \frac{v}{w_\theta}$ therefore:

$$\begin{bmatrix} (s^2+\sigma^2) & -(x_\theta s^2 + \frac{2V^2}{\mu}) \\ -x_\theta s^2 & r^2s^2 + r^2 - \frac{2V^2}{\mu}\left(\frac{1}{2}+a\right) \end{bmatrix}\begin{bmatrix} \frac{\bar{\eta}}{b} \\ \bar{\emptyset} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{16}$$

In order to simplify the problem, the following dimensionless parameters will be introduced

$r^2 = \frac{I_{y2}}{mb^2}$ (mass radius of gyration about shear center)

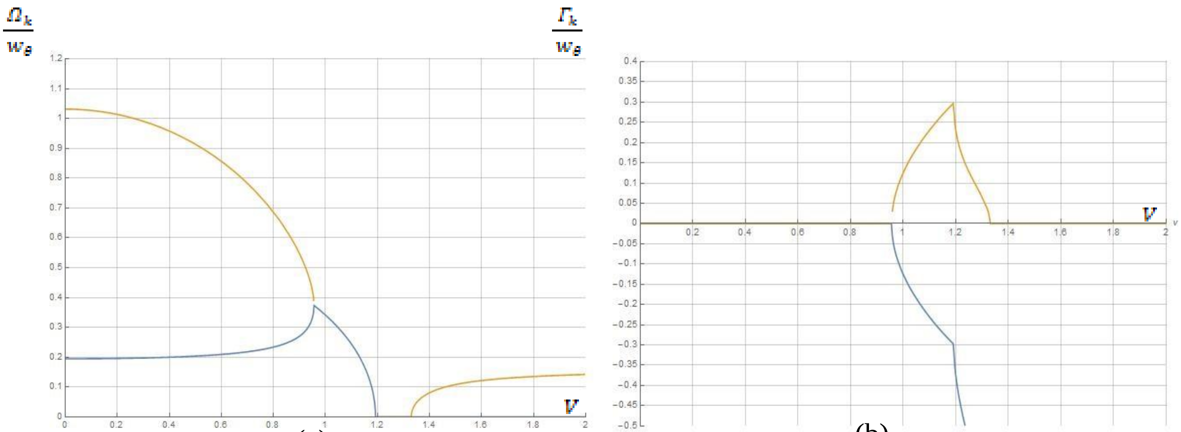
$\sigma = \frac{w_h}{w_\theta}$ (ratio of uncoupled bending to torsional frequencies)

$\mu = \frac{m}{\pi \rho_\infty b^2}$ $\left(mass\ ratio\ = \frac{airfoil\ mass}{air\ volume\ mass} \right)$

$V = \frac{U}{bw_\theta}$ (reduce velocity or dimensionless air free stream speed of air)

The problem becomes a regular eigenvalue problem, in order to get the non-trivial solution the determinant of the matrix must be equal to zero. The roots of the equation are two complex conjugate pairs $s_1 = \frac{\Gamma_1}{w_\theta} \pm i \frac{\Omega_1}{w_\theta}\ and\ s_2 = \frac{\Gamma_2}{w_\theta} \pm i \frac{\Omega_2}{w_\theta}$ where $\frac{\Gamma_2}{w_\theta}$ and $\frac{\Omega_2}{w_\theta}$ are frequency and damping model respectively. It is required to find the flutter speed. Set trial values of "V" say from 0 to 2 and then plot the values of $\frac{\Omega_k}{w_\theta}$ and $\frac{\Gamma_k}{w_\theta}$. Until the value of $\frac{\Gamma_k}{w_\theta}$ is not equal to zero. From "Figure (5a, 5b)", we can deduce that critical flutter occurs at the value of wind belt is:

$V_f = 0.97$



(a) (b)

Figure 5 (a) Plot of $\frac{\Omega_k}{w_\theta}$ vs. V, (b) Plot of $\frac{\Gamma_k}{w_\theta}$ vs. V

From "Figure 5b", flutter occurs when the value of $\frac{\Gamma_k}{w_\theta}$ is not equal to zero. After obtaining the value of "V_f" at which flutter occur, therefore the flutter start at air speed is obtained from:

$$U_f = V_f b w_\theta = 1.9\, m/s$$

3.2.2 UNSTEADY FLOW

For the unsteady aerodynamics, Theodorsen's unsteady thin-airfoil theory states that [12]

$$L' = 2\pi \rho_\infty UbC(k) \left[U\theta - \frac{\partial w}{\partial t} + b\left(\frac{1}{2} - a\right)\frac{\partial \theta}{\partial t} \right] + \pi \rho_\infty b^2 \left(U\frac{\partial \theta}{\partial t} - \frac{\partial^2 w}{\partial t^2} - ba\frac{\partial^2 \theta}{\partial t^2} \right)$$

$$M'_{1/4} = -\pi \rho_\infty b^3 \left(U\frac{\partial \theta}{\partial t} - \frac{1}{2}\frac{\partial^2 w}{\partial t^2} + b\left(\frac{1}{8} - \frac{a}{2}\right)\frac{\partial^2 \theta}{\partial t^2} \right)$$

Therefor the generalized forces are simplified and can be written as:

$$\begin{bmatrix} E_w \\ E_\theta \end{bmatrix} = -\pi \rho_\infty L b^2 \begin{bmatrix} [I] & ba[I] \\ ba[I] & b^2\left(a^2 + \frac{1}{8}\right)[I] \end{bmatrix} \begin{bmatrix} \ddot{\eta} \\ \ddot{\emptyset} \end{bmatrix}$$
$$-\pi \rho_\infty UbL \begin{bmatrix} 2C(k)[I] & -b\left(1 + 2\left(\frac{1}{2} - a\right)C(k)\right)[I] \\ 2b\left(\frac{1}{2} + a\right)C(k)[I] & b^2\left(\frac{1}{2} - a\right)\left[1 - 2\left(\frac{1}{2} + a\right)C(k)\right][I] \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \dot{\emptyset} \end{bmatrix} \tag{18}$$
$$-\pi \rho_\infty U^2 bL \begin{bmatrix} [0] & -2C(k)[I] \\ [0] & -b(1 + 2a)C(k)[I] \end{bmatrix} \begin{bmatrix} \eta \\ \emptyset \end{bmatrix}$$

In classical flutter analysis it is presumed that the motion is simple harmonic i.e. $\eta = \bar{\eta} e^{i\omega t}$ and $\theta = \bar{\theta} e^{i\omega t}$, where ω is the frequency of the simple harmonic motion. Let us consider the case in which $n_w = n_\theta = 1$. Substituting $\eta = \bar{\eta} e^{i\omega t}$ and $\theta = \bar{\theta} e^{i\omega t}$ into equation (13) and (18), thus the generalized forces matrix can be write as:

$$\begin{bmatrix} E_w \\ E_\theta \end{bmatrix} = \omega^2 \pi \rho_\infty L b^3 \begin{bmatrix} 1 - \frac{2iC(K)U}{b\omega} & a + iU\frac{1+2\left(\frac{1}{2}-a\right)C(k)}{b\omega} + \frac{2C(k)U^2}{b^2\omega^2} \\ b\left(a - \frac{2iU\left(\frac{1}{2}+a\right)C(k)}{b\omega}\right) & b\left(\left(a^2+\frac{1}{8}\right) - iU\frac{\left(\frac{1}{2}-a\right)\left[1-2\left(\frac{1}{2}+a\right)C(k)\right]}{b\omega} + \frac{U^2(1+2a)C(k)}{b^2\omega^2}\right) \end{bmatrix} \begin{bmatrix} \frac{\bar{\eta}}{b} \\ \bar{\emptyset} \end{bmatrix} \quad (19)$$

Therefore may be introduce the dimensionless aerodynamic coefficients as functions of the Theodorsen function and the reduced frequency. Their expressions are:

$$l_h = 1 - \frac{2i\,C(k)}{k} \qquad l_\theta = a + \frac{i}{k} + \frac{2\,C(k)}{k^2} + \frac{2i\left(\frac{1}{2}-a\right)C(k)}{k}$$

$$m_h = a - \frac{2i\left(\frac{1}{2}+a\right)C(k)}{k} \qquad m_\theta = \frac{1}{8} + a^2 - \frac{i\left(\frac{1}{2}-a\right)}{k} + \frac{2\left(\frac{1}{2}+a\right)C(k)}{k^2} + \frac{2i\left(\frac{1}{4}-a^2\right)C(k)}{k}$$

$C(k) = \dfrac{0.01365+0.2808i*k-\frac{k^2}{2}}{0.01365+0.3455i*k-k^2}$ and $k = \dfrac{b\omega}{U}$ (reduce frequency)

Substituting equation (19) into equation (13) to get the equations of motions as:

$$\begin{bmatrix} \mu\left(1-\sigma^2\left(\frac{\omega_\theta}{\omega}\right)^2\right) + l_h & -\mu x_\theta + l_\theta \\ -\mu x_\theta + m_h & \mu r^2\left[1-\left(\frac{\omega_\theta}{\omega}\right)^2\right] + m_\theta \end{bmatrix} \begin{bmatrix} \frac{\bar{\eta}}{b} \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (20)$$

Since the equations of motions are homogenous and linear, so the determinant of the matrix should be equal to zero in order to get a non-trivial solution. The determinant in this case is called the "flutter determinant". The expansion of the determinant yields a quadratic polynomial in the unknown $\left(\frac{\omega_\theta}{\omega}\right)^2$. To determine the flutter boundary of equation (20), it must be recognized that two unknowns remain: ω_θ/ω and $k=b\,\omega/U$. Then following the procedure of classical flutter to plotting the imaginary parts of both roots versus k, so that the value of k at which one of the imaginary parts crosses the zero axis can be determined. This value of k is an approximation of k_f, making the value of $\left(\frac{\omega_\theta}{\omega}\right)^2$ real when $k = k_f$, therefore from figure 6.

$$k_f = 0.45$$

hence

$$U_f = \frac{b\omega_f}{k_f} = 2.9\ m/s$$

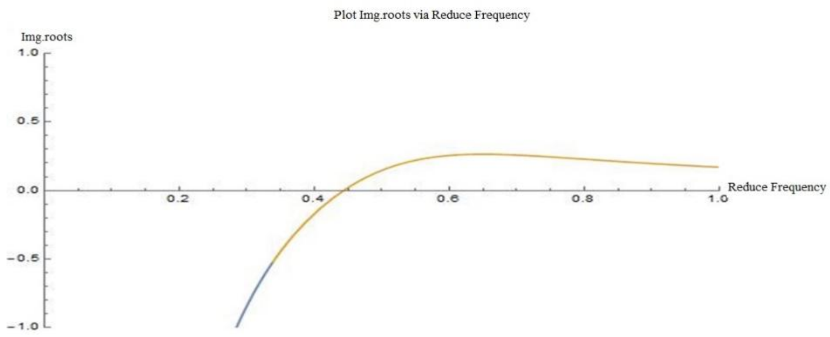


Figure 6: Plot reduce frequency (*k*) via *Img*. Roots

4 Experimental results

From previous analysis the flutter occurs at low speed. We may use this flutter to produce power by electromagnetic induction. Wind speed was measured by Anemometer. At air speed of 3 m/s the wind belt produces 4 volt AC as shown in figure 7.

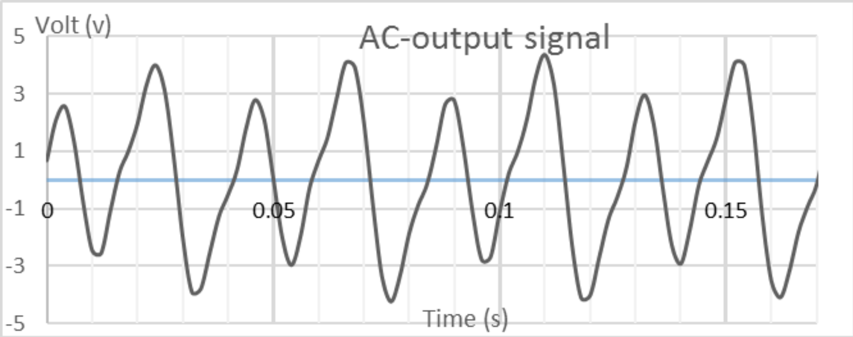


Figure 7: Wind belt output signal

The AC voltage is rectified by using rectifier circuit (DB107) as shown in figure 8. The capacitor (100 μF) is added to the rectifier circuit to obtain stable DC output as shown in figure 9.

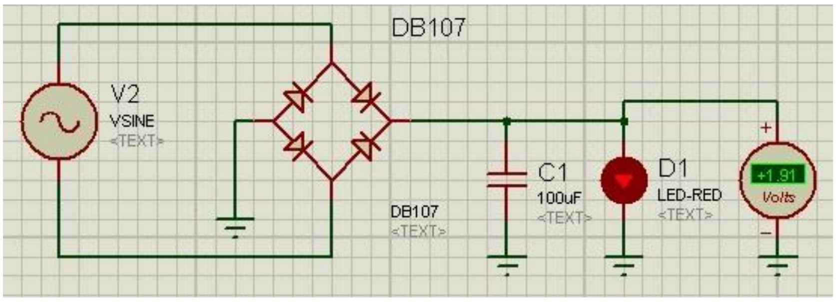


Figure 8: Schematic of rectifier circuit

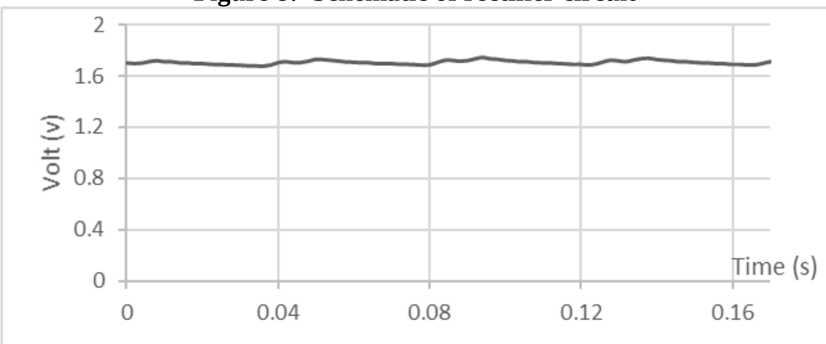


Figure 9: Wind belt output DC signal with capacitor (100 μF)

CONCLUSION

In this study, analysis investigation of the wind belt system showed the following.

- The wind belt natural frequency was calculated to be $60Hz$.
- The wind belt start fluttering at low speed for steady flow $\approx 1.9\ m/s$ but for unsteady flow the fluttering speed $\approx 2.9\ m/s$
- The wind belt device can produce 4 Volt AC at wind speed 3m/s.

Nomenclature

Variable	Meaning	Unit
a	Dimensionless offset between shear center and mass center	——
b	Semi chord length	M
G	Modulus of rigidity of the blade	N/m^2
γ_{ij}	Shear strain in i and j directions	——
I_y	polar mass moment of inertia about y (shear axis)	$Kg.m^2$
J_y	polar moment of area about y axis (shear axis)	m^4
m	Mass per unit length	Kg/m
r_x	Eccentricity between mass and shear center	M
ρ_∞	Density of air	Kg/m^3
$T(y)$	Tension exerted on belt	N
$\theta(y,t)$	Elastic axis torsional displacement	rad/s
U	Air velocity	m/s
$w(y,t)$	Elastic axis flapwise displacement	m
x_θ	Dimensionless express static-unbalance parameter	——

REFERENCES:

[1] Sherief M. Fathy, *"Development and Analysis of a Vortex-Induced Vibrations energy harvester"*, M.s. Thesis, Department of Mechatronics Engineering, G UC in Cairo 2014

[2] Miao Shi, "*Energy Harvesting from Wind-Induced Vibration of Suspension Bridges*", M.S. Thesis, Department of Civil and Environmental Engineering Massachusetts Institute of Technology Jun 2013

[3] Mehmet Yilmaz *"Passive Full-Wave MOSFET Rectifiers for Electromagnetic Harvesting"*, M.S. Thesis, Department of Electrical and Computer Engineering, Waterloo University,Ontario, Canada 2013.

[4] Eriko Shimizu, *"MULTI-OBJECTIVE DESIGN STUDY OF A FLAPPING WING GENERATOR"* ICAS 2004 24th international congress of the aeronautical sciences.

[5] M. Isogai, K. Yamasaki, Matsubara, M., and Asaoka, T., " *Design Study of Elastically Supported Flapping Wind Power Generator*" International Forum on Aeroelasticity and Structural Dynamics, Amsterdam, 2003.

[6] Wind belt invention, June 2012. URL http://www.appropedia.org/Samoa_Hostel_Windbelt.

[7] http://www.humdingerwind.com

[8] A. Abdelkefi·A.H. Nayfeh·M.R. Hajj," Modeling and analysis of piezoaeroelastic energy harvesters", Springer (2012) 67:925–939

[9] Ed. Frank Kreith, " *Mechanical Engineering Handbook* ", CRC Press LLC, 1999.

[10] L.Meirovitch. *"Fundamentals of Vibration* " McGraw-Hill, 2001.

[11] A. H. Nayfeh and P. F. Pai. "*Linear and Nonlinear Structural Mechanics*." John Wiley and Sons, 2004.

[12] Dewey H. Hodges and G. Alvin Pierce *"Introduction to Structural Dynamics and Aeroelasticity* " Cambridge University Press, 2011.

[13] John D. Anderson, " *Fundamentals of Aerodynamics* ", McGraw-Hill, 2001.

[14] Singiresu S. Rao "Mechanical Vibrations", Pearson, 2011.