Journal of the Egyptian Mathematical Society
Volume (26) - Issue (1) - 2018
ISSN: 1110-256x
DOI: : 10.21608/JOEMS.2018.9459

EFFECT OF BOUNDARY CONDITIONS ON ELECTROTHERMAL CONVECTION IN A POROUS MEDIUM WITH VARIABLE VISCOSITY

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Received 21/1/2017 Revised 4/6/2017 Accepted 14/9/2017

Abstract: The effect of thermal and velocity boundary conditions on electrothermal convection in a dielectric fluid-saturated layer of Brinkman porous medium with temperature dependent viscosity (TDV) has been studied. The lower rigid/free boundary is either fixed temperature or fixed heat flux with respect to temperature perturbations, while at the upper rigid/free boundary the Robin type of thermal boundary condition is invoked. The eigenvalue problem is solved numerically using the Galerkin-type of weighted residual method. The instability threshold depends significantly on boundary conditions and dimensionless physical parameters namely, the Biot number, temperature dependent viscosity and permeability parameters. The critical gravity thermal or electric thermal Rayleigh number making the onset of electrothermal convection is found to be higher for fixed temperature conditions and also for both boundaries rigid while it is lower for fixed heat flux conditions and also for both boundaries free. Some known results are recovered as special cases from the present study.

Keywords: *Electrohydrodynamic; Dielectric fluid; Electrothermal convection; porous medium; Galerkin technique*

Math Subject Classification: 76E06 (convection); 80A20 (Heat and mass Transfer, heat flow); 85A30 (hydrodynamics and hydromagnetics problems)

1. Introduction

Electrohydrodynamics (EHD) is the branch of fluid mechanics which deals with more complex interactions among fluid, heat and electric fields [1]. Electrohydrodynamics has wide applications in flow and heat transfer control, enhancement of heat and mass transfer, micro-electromechanical systems (MEMS) and some other industrial processes [2]. Recently, two conceptual designs were established with applications in the cooling system of laptops and devices on a flight in space [3,4]. The relative backwardness of such applications with the EHD technique is attributed to the lack of complete mastering of the characteristics of flow motion.

The study of convective instability in the presence of electric field and buoyancy effects is called the Rayleigh-Bénard-electroconvection or electrothermal convection (see Taylor [5]). Turnbull and Melcher [6] studied the natural convective instability problem of electroconvection under an applied AC or DC electric field and also successfully carried an experiment to verify the theoretical prediction. Roberts [7] studied theoretically the onset of electroconvection in an insulating fluid layer subject to temperature gradients and electrical potential differences across the fluid layer. Turnbull [8] analyzed the effect of dielectrophoretic force on the onset of natural convection by considering electrical conductivity is temperature dependent. Many researchers investigated Rayleigh–Bénard electroconvection considering various effects [9-17]. Shivakumara et al. [18] investigated the problem of electrothermal convection in a rotating dielectric fluid layer by taking different boundary conditions while the effect of couple stresses on the aforementioned problem has been presented by Shivakumara et al. [19].

Thermal convection in a layer of dielectric fluid-saturated porous medium under a uniform vertical AC electric field has also attracted significant attention in the literature due to its importance in geophysics and porous materials modeling. Moreno et al. [20] studied on fluid flow in a porous medium subjected to an external electric field, particular importance in view of its possibility of reduction of fluid viscosity in enhancing petroleum production. Rio and Whitaker [21] developed the frequency-dependent governing equations for EHD in saturated porous medium. Rudraiah and Gayathri [22] investigated the temperature modulation effect on electroconvection in a dielectric fluid-saturated porous medium in the presence of a uniform vertical AC electric field. Shivakumara et al. [23] analyzed the onset of electrothermal convection in a dielectric fluid-saturated Brinkman porous layer for different velocity boundary conditions while the additional effect of rotation on the above study is considered by Shivakumara et al. [24]. In all of these investigations, the fluid viscosity is considered to be a constant. In reality, the viscosity is a strong function of temperature and it affects the stability of the system significantly.

The aim of the present paper is to determine analytically the effect of temperature dependent viscosity (TDV) on thermal convection in a dielectric fluid-saturated porous medium in the presence of a uniform vertical AC electric field for different temperature and velocity boundary conditions. The lower rigid/free surface is considered to be either conducting (constant temperature) or insulating with respect to temperature perturbations (constant heat flux) while at the upper rigid/free surface the Robin type of thermal boundary condition is utilized. The outline of the present paper is as follows. The mathematical formulation of the problem is described in section 2. The basic state equations are derived in section 3. The linear instability analysis using normal mode expansion procedure is also handled in this section. In the same section the eigenvalue problem involving the system of differential equations and the boundary conditions are also specified in this section. The numerical results obtained using the ninth-order Galerkin weighted residual method are discussed and explained in detail in section 4. Finally, some conclusions are documented in section 5.

2. Mathematical Formulation

The schematic geometry of the problem considered is presented in Fig. 1. We consider an incompressible dielectric fluid-saturated Brinkman porous medium under a uniform AC electric field acting perpendicular to the horizontal porous layer bounded by the surfaces $z = 0$ and $z = d$. The lower and upper surfaces are maintained at temperatures $T = T_0$ and $T = T_1 = T_0 - \Delta T$ ($\Delta T > 0$), respectively. A Cartesian coordinate system (*x*, *y*, *z*) is chosen such that the origin is at the bottom of the porous layer. Gravity is acting in the negative vertical z-direction. The viscosity μ is considered to be varying linearly with temperature: $\mu = \mu_0[1 - \Gamma(T - T_0)]$

where μ_0 and Γ are positive constants.

The governing equations in the relevant context are:

Mass conservation: $\nabla \cdot \vec{q} = 0$, (1) Momentum conservation:

$$\rho_0 \frac{1}{\phi}\frac{d\vec{q}}{dt} = -\nabla p + \rho_0\left[1-\alpha\left(T-T_0\right)\right]\vec{g} + 2\nabla\cdot[\mu\bar{\bar{D}}] - \frac{\mu}{k}\vec{q} + \vec{f}_e \quad (2)$$

Energy conservation: $$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa\nabla^2 T \qquad (3)$$

Electrical equation: $$\nabla\cdot\varepsilon_0\left[1-\eta\left(T-T_0\right)\right]\vec{E} = 0 \qquad (4)$$

Maxwell equation: $\nabla\times\vec{E} = 0$ or $\vec{E} = -\nabla V.$ (5)

The last term in Eq. (2) is the electric force induced by the electrical field which is of the form:

$$\vec{f}_e = \rho_e\vec{E} - E^2\frac{\nabla\varepsilon}{2} + \nabla\left[E^2\frac{\rho}{2}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]. \qquad (6)$$

In Eq. (6), the first term stands for Coulomb force exerted by an electric field upon the free charge within the bulk liquid. The second term is dielectrophoretic (or dielectric) force and the third term is electrostrictive force which is a conservative vector and can be conveniently combined with static pressure. The additional quantities appeared in the above set of governing equations are defined in the nomenclature. The standard linear stability analysis procedure leads to (for details see [8] and [17])

$$\rho_0\frac{\partial}{\partial t}\left(\nabla^2 w\right) - \mu(z)\nabla^4 w = 2\frac{\partial\mu(z)}{\partial z}\nabla^2\left(\frac{\partial w}{\partial z}\right) + \nabla_h^2\left[\varepsilon_0 E_0\eta\beta\left(E_0\eta T + \frac{\partial V}{\partial z}\right) + \rho_0\alpha g\,T\right] + \frac{\mu(z)}{k}\nabla^2 w, \qquad (7)$$

$$\left(\frac{\partial}{\partial t} - \kappa\nabla^2\right)T = \frac{\Delta T}{d}w, \qquad (8)$$

$$\nabla^2 V = -\eta E_0\frac{\partial T}{\partial z}, \qquad (9)$$

where $\mu(z) = \mu_0\left(1+\Gamma\,\beta\,z\right)$ is the temperature dependent viscosity function and $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator. We employ the normal mode expansion in the form

$$\{w, T, V\}(x, y, z, t) = \{W, \Theta, \Phi\}(z)\, e^{i\,(a_x x + a_y y) + \omega t} \qquad (10)$$

where a_x and a_y are wave numbers in x and y directions, respectively, ω is the growth factor which is complex, in general, while W, Θ and Φ are the amplitudes of perturbed velocity, temperature and electric potential, respectively.

Using Eq. (10) in Eqs. (7)-(9) and non-dimensionalizing the resulting equations by applying the definitions:

$(x, y, z) = (x^*, y^*, z^*)d$, $W=(\kappa/d)W^*$, $t=(d^2/\kappa)\,t^*$ (11a)

$\Theta = \Delta T\,\Theta^*$, $\Phi = \eta E_0\,\Delta T\, d\,\Phi^*$, $\Gamma = \beta d\,\Gamma^*$, $\omega = \left(\kappa/d^2\right)\omega^*$ (11b)

we obtain the stability equations (after ignoring the asterisks) in the form

$$f\left[\left(D^2-a^2\right)^2 - \left(\sigma^2+\omega\right)\left(D^2-a^2\right)\right]W - \sigma^2 Df\,DW + 2\,Df\left(D^3-a^2D\right)W - a^2\left[R_t\,\Theta + R_e\left(D\Phi+\Theta\right)\right] = 0 \qquad (12a)$$

$$\left(D^2-a^2-\omega\Pr\right)\Theta + W = 0 \qquad (12b)$$

$$\left(D^2-a^2\right)\Phi + D\Theta = 0 \qquad (12c)$$

where $f = 1 + z\Gamma$. By performing qualitative analysis on the oscillatory instability, Shivakumara et al. [23] have shown that the principle of exchange of stability is valid for the onset of Darcy-Brinkman electrothermal convection irrespective of the nature of velocity boundary conditions. This is expected as there are no physical mechanisms to set up oscillatory motions when a dielectric fluid-saturated porous medium under a uniform vertical AC electric field is heated from below. Here, the motion, temperature and electric fields are all in phase and no restoring force exists and hence oscillatory convection is not possible. Similar is the situation in the present paper and of course the variation in viscosity with respect to temperature does not introduce oscillatory motions. Therefore, the principle of exchange of stability is considered to be valid in the present case as well and take $\omega = 0$ in Eqs. (12a-c) to get

$$f\left[\left(D^2-a^2\right)^2 - \sigma^2\left(D^2-a^2\right)\right]W - \sigma^2 Df\,DW + 2\,Df\left(D^3-a^2D\right)W - a^2\left[R_t\,\Theta + R_e\left(D\Phi+\Theta\right)\right] = 0 \qquad (13a)$$

$$\left(D^2-a^2\right)\Theta + W = 0 \qquad (13b)$$

$$\left(D^2-a^2\right)\Phi + D\Theta = 0\,. \qquad (13c)$$

The above equations are solved by imposing the following types of boundary conditions:

(i) Both boundaries rigid (R-R boundaries):

$W = DW = \Phi = 0$ at $z = 0, 1$

$\Theta = 0$ or $D\Theta = 0$ at $z = 0$;

$D\Theta + Bi\,\Theta = 0$ at $z = 1$;

(14)

(ii) Lower rigid and upper free boundaries (R-F boundaries):

$W = DW = \Phi = 0,\ \Theta = 0$ or $D\Theta = 0$ at $z = 0$ (15a)

$W = D^2W = D\Phi = 0,\ D\Theta + Bi\,\Theta = 0$ at $z = 1$. (15b)

(iii) Both boundaries free (F-F boundaries):

$W = D^2W = D\Phi = 0$ at $z = 0, 1$

$\Theta = 0$ or $D\Theta = 0$ at $z = 0$;

$D\Theta + Bi\,\Theta = 0$ at $z = 1$. (16)

Here, the Biot number $Bi = h_t\, d / k$ is the ratio of rate of heat from the interface to the environment to the rate of heat supply to the interface from the bulk of a fluid due to the thermal conduction at the upper boundary. Increase in Bi from 0 to ∞ means change in the thermal condition at the upper boundary from “fixed heat flux condition” or “insulating case" (i.e., $D\Theta = 0$) to the "constant temperature” or “conducting case" (i.e., $\Theta = 0$).

3. Method of solution

Equations (13a-c) together with the chosen boundary conditions constitute an eigenvalue problem which has been solved numerically using the Galerkin technique. Accordingly, the unknown variables are written in series of trial (basis) functions as

$$W(z) = \sum_{m=1}^{9} A_m W_m(z),$$

$$\Theta(z) = \sum_{m=1}^{9} B_m \Theta_m(z),$$

$$\Phi(z) = \sum_{m=1}^{9} C_m \Phi_m(z), \quad (17)$$

where A_m, B_m and C_m are constants and W_m, Θ_m and Φ_m represent the basis functions. On substituting Eq.(17) into Eqs. (13a-c), multiplying both sides of resulting Eq.(13a) by $W_n(z)$, Eq.(13b) by $\Theta_n(z)$ and Eq.(13c) by $\Phi_n(z)$; and integrating the resulting relations over the region $V = \{0 \le z \le 1\}$ and using the boundary conditions, we obtain a system of algebraic equations which can be written in the form

$$\begin{bmatrix} E_{nm} & F_{nm} & G_{nm} \\ I_{nm} & J_{nm} & 0 \\ 0 & K_{nm} & L_{nm} \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

where,

$$E_{nm} = < f\,[D^2W_n D^2W_m + (2a^2 + \sigma^2) DW_n DW_m + a^2(a^2 + \sigma^2) W_n W_m] >,$$

$$F_{nm} = -a^2(R_t + R_e) < W_n \Theta_m >,$$

$$G_{nm} = -a^2 R_e < DW_n \Phi_m >,$$

$$I_{nm} = -< \Theta_n W_m >,$$

$$J_{nm} = < D\Theta_n D\Theta_m + a^2\,\Theta_n\,\Theta_m > - Bi\,\Theta_n(1)\,\Theta_m(1),$$

$$K_{nm} = < D\Theta_n\,\Phi_m >,$$

$$L_{nm} = < D\Phi_n D\Phi_m + a^2 \Phi_n \Phi_m >.$$

and $<\bullet> = \int_0^1 (\bullet)\, dz$. The system of equations given by Eq. (18) has a non-trivial solution if and only if the determinant of the coefficient matrix is zero. That is,

$$\begin{vmatrix} E_{nm} & F_{nm} & G_{nm} \\ I_{nm} & J_{nm} & 0 \\ 0 & K_{nm} & L_{nm} \end{vmatrix} = 0. \quad (19)$$

The eigenvalue has to be extracted from Eq. (19). For this, we select the following trial functions satisfying the boundary conditions:

(i) R-R boundaries:

$$W_m = (z^{m+3} - 2z^{m+2} + z^{m+1})T_m^*,\ \Theta_m = (z^m - z^{m+1}/2)T_m^*,$$

$$\Phi_m = (z^m - z^{m+1})T_m^* \quad (20)$$

(ii) R-F boundaries:

$$W_m = (2\,z^{m+3} - 5\,z^{m+2} + 3\,z^{m+1})T_m^*,\ \Theta_m = (z^m - z^{m+1}/2)T_m^*,$$

$$\Phi_m = (z^m - z^{m+1}/2)T_m^* \quad (21)$$

(iii) F-F boundaries:

$$W_m = (z^{m+3} - 2\,z^{m+2} + z^m)T_m^*,\qquad \Theta_m = (z^m - z^{m+1}/2)T_m^*,$$

$$\Phi_m = (z^{m+2} - 3z^{m+1}/2)T_m^* \quad (22)$$

where T_m^*'s are Chebyshev polynomials ($m = 0, 1, 2, \ldots\ldots$) of the second kind. It is seen that the trial functions chosen satisfy the respective boundary conditions except the thermal boundary condition $D\Theta + Bi\,\Theta = 0$ at $z = 1$ but the residue is included from the differential Eqs.(13a-c). On substituting Eqs. (20- 22) into Eq. (19) leads to an equation of the form:

$$f\left(R_t,\ R_e,\ Bi,\ \sigma^2,\ \Gamma,\ a\right) = 0. \quad (23)$$

Equation (23) allows to determine the critical values of R_{tc} or R_{ec} and a_{ec} with respect to other physical parameters Bi, Γ and σ^2.

4. Results and Discussion

The Galerkin-type of weighted residual method is used to obtain the critical values of R_{tc} or R_{ec} and a_{ec} for various values of physical parameters Bi, Γ and σ^2 using Eq. (23). The overall trend of R_{ec} and R_{tc} for various boundary conditions (R-R, R-F and F-F), temperature dependent viscosity (TDV) and porous parameter are presented in Tables 1-3 and also shown graphically in Figs. 2-8.

To validate the results obtained by applying the numerical procedure, a comparison with some existing results is made (see Tables 1 and 2). The results obtained for several values of Bi with $R_e = 0$, $\sigma^2 = 0$ and $\Gamma = 0$ compare very well with those of Sparrow et al. [25]. Besides, the critical stability parameters (R_{ec}, a_c) are compared with those of Roberts [7] in Table 3 for selected values of R_t with $\Gamma = 0$, $\sigma^2 = 0$ and $Bi = 0$. The results are found to be in good agreement.

The variation of R_{tc} as a function of Bi is shown in Fig. 2 for fixed values R_e = 0, 10, 50, 100, 200, $\sigma^2 = 0$ and $\Gamma = 0$ for R-F boundaries with $\Theta(0) = 0 = D\Theta(1)$. In Fig. 2, the results of Char and Chiang [26] are also presented and note that there is an excellent agreement between the present results and those of [26]. Here, we find that R_{tc} decreases with increasing AC electric field strength.

In Figs.3-9, the plots of R_{ec} and a_{ec} against Bi are illustrated. In these figures, the solid curves correspond to temperature boundary condition of the type $\Theta(0) = 0$ while the dotted curves correspond to the condition of the type $D\Theta(0) = 0$. Figure 3 shows the results for $\sigma^2 = 10$, $\Gamma = 0.2$ and $R_t = 50$. It shows that the results are bridging the space between the fixed heat flux and constant temperature at the upper surface with increasing *Bi*. Clearly, imposing fixed heat flux condition at the lower surface advances the electrothermal convection compared to constant temperature condition. Figure 3 also reveals that the system with R-R surfaces is stable compared to F-F surfaces. The values of R_{ec} initially increases slowly with *Bi* and then increases quickly and approaches an asymptotic value $R_{ec} = 2872.38$, 2656.78 and 2450.40 with further increasing *Bi* for R-R, R-F and F-F boundaries, respectively when the lower surface is held at constant temperature while the asymptotic values for the said boundaries are found to be $R_{ec} = 1271.99$, 928.858 and 814.86 when the lower surface is held at fixed heat flux condition. It is also evident that the dielectric fluid layer under an AC electric field becomes more stable with increasing Bi. Besides, increase in the value of heat transfer coefficient Bi is to increase the critical electrical thermal Rayleigh number and thus its effect is to delay the onset of electrothermal convection. This may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable. On the upper free surface, for small values of Bi, these perturbations are very prone to heat transfer coefficient and for large values of Bi, these can be regarded as an imposed constant temperature that causes R_{ec} to approach this asymptotic value. The critical wave number reported in Fig.4 for various values of Bi reveals that $(a_{ec})_{Bi=0} > (a_{ec})_{Bi\,=\text{finite}} > (a_{ec})_{Bi\to\infty}$.

In Fig.5, we have depicted the variation of R_{ec} verses σ^2 for three types of boundaries when $Bi = 2$, $\Gamma = 0.2$ and $R_t = 50$. It is seen that increase in the value of permeability parameter σ^2 is to delay the onset of electrothermal convection. Here, we note that the results for two types of temperature boundary conditions differ only quantitatively and the system is found to be more stable when the lower surface is fixed at constant temperature as expected. From Fig.6, it is seen that the critical electric wave number a_{ec} increases as σ^2 increases. Therefore, increase in the porous parameter is to reduce the size of convection cells.

The effect of temperature dependent viscosity parameter Γ on the onset of electrothermal convection in a dielectric fluid saturated porous medium is presented in Fig.7 for fixed values $Bi = 2$, $\sigma^2 = 10$ and $R_t = 50$. It is observed that R_{ec} increases with increasing Γ indicating its effect is stabilizing on the system. That is, the effect of increasing Γ is to delay electrothermal convection in the presence of AC electric field. Whereas Fig.8 reveals that the variation in a_{ec} with Γ is insignificant. In Fig.9, the variation of R_{ec} and R_{tc} is plotted for fixed values of $Bi = 2$, $\sigma^2 = 10$ and $\Gamma = 0.2$. Here, we focus on the strengths between buoyancy and electric forces on the stability of the system. If there is an increase in the strength of one, then there is a decrease in the other. Thus the strength of AC electric field leads to destabilizing effect on the system. This result is true for all the boundary conditions considered. A closer inspection of the figures further reveals that $(a_{ec})_{\text{R-R}} > (a_{ec})_{\text{R-F}} > (a_{ec})_{\text{F-F}}$ and $(a_{ec})_{\text{lower surface at }\Theta=0} > (a_{ec})_{\text{lower surface at }D\Theta=0}$

5. Conclusion

The effect of variable viscosity on the onset of electrothermal convection in a porous medium under a uniform vertical AC electric field has been studied for different types of velocity and temperature boundary conditions. It is observed that:

1. The onset of electrothermal convection is to be delayed with increasing Bi

2. The effect of increasing AC electric field is to hasten the onset of convection.

3. The system is more stable for R-R surfaces while for F-F surfaces it is least stable. Also, $(R_{tc} \text{ and } R_{ec})_{\Theta=0} > (R_{tc} \text{ and } R_{ec})_{D\Theta=0}$

4. The critical electric wave number a_{ec} increases with increasing Bi and σ^2 Thus their effect is to contract the size of convection cells. Also, $(a_{ec})_{\text{constant temperature}} > (a_{ec})_{\text{fixed heat flux}}$.

Nomenclature

$a = \left(a_x + a_y\right)^{1/2}$, over all dimensionless wave number
$Bi = h_t\, d / k$, Biot number
$d =$ horizontal fluid layer of thickness
$D \equiv d/dz$, ordinary differential operator
$\vec{E} = (0,\ 0,\ E_z)$, applied AC electric field
$\vec{g} = (0,\ 0,\ -g)$, gravitational acceleration
h_t = heat transfer co-efficient
k = permeability of the porous medium
p = pressure
$\vec{q} = (u,\ v,\ w)$, velocity vector of the fluid
$\Pr = \gamma/\kappa$, Prandtl number
$R_e = \eta^2 \varepsilon_0 E_0^2 (\Delta T)^2 d^2 / \mu\kappa$, electric thermal Rayleigh number
$R_t = \alpha\, g\, \Delta T d^3 / \nu\kappa$, gravity thermal Rayleigh number
T = temperature
T_o = temperature at lower surface
T_1 = temperature at upper surface
V = root mean square velocity of the electric potential
W = amplitude of perturbed vertical velocity component
x, y, z = Cartesian co-ordinates

Greek symbols

$\alpha (> 0)$ = thermal expansion coefficient
$\beta = \Delta T / d$, temperature gradient
$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, Laplacian operator
$\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, horizontal Laplacian operator
ε = dielectric constant
ε_0 = reference temperature for dielectric constant
η (>0)= analog for dielectric constant of thermal expansion coefficient
ϕ = porosity
Φ = amplitude of perturbed electric potential
Γ = temperature dependent viscosity
κ = effective thermal diffusivity
μ_0. = constant viscosity
$\nu = \mu / \rho_0$, kinematic viscosity
ρ = density of the fluid

ρ_e = free charge density
$\rho_0 = \rho(T_o)$, density at reference temperature
$\sigma = d / \sqrt{k}$, porous parameter
Θ = amplitude of perturbed temperature

Acknowledgements

The authors wish to thank the reviewers for their useful comments and suggestions which have helped in improving the paper. Also, the authors gratefully acknowledged the financial support received in the form of a "Research Fund for Talented Teacher Scheme" (No.KSTePS/VGST/06/RFTT/2015-16) from Vision Group of Science & Technology, Government of Karnataka, Bengaluru-560001, India.

Reference

[1] A. Castellanos ed, Electrohydrodynamics, Springer-Verlag, New York, 1999.
[2] A.M. Thoma, M. Morari, Control of Fluid Flow, Springer-Verlag, Berlin Heidelberg, 2006.
[3] N.E. Jewell-Larsen, H. Ran, Y. Zhan, Electrodynamic (EHD) cooled laptop, Semiconductor Thermal Measurement and Management Symposium, 25th Annual IEEE, 2009.
[4] Shaun Tassavur, Super Cool 'EHD Pump' by NASA Cools Devices in Space, 31st May 2011, http://onlygizmos.com/super-cool-ehd-pump-by-nasa-cools-devices-in-space/2011/05.
[5] G.I. Taylor, Studies in electrohydrodynamics, Circulation produced in a drop by an electric field, Proc. R. Soc. Lond. A 291 (1966) 159–166.
[6] R.J. Turnbull, J.R. Melcher, Electrodynamic Rayleigh-Taylor bulk instability, Phys. Fluids 12 (1969) 1160-1166.
[7] P.H. Roberts, Electrohydrodynamic convection, Qly. J. Mech. Appl. Math. 22 (1969) 211- 220.
[8] R.J. Turnbull, Effect of dielectrophoretic forces on the Benard instability, Phys. Fluids 12 (1969) 1809–1815.
[9] M. Takashima, Effect of rotation on electrohydrodynamic instability, Can. J. Phys. 54 (1976) 342-347.
[10] M. Takashima, H. Hamabata, The stability of natural convection in a vertical layer of dielectric fluid in the presence of a horizontal electric field, J. Phys Soc. Jpn. 53 (1984) 1728-1736.
[11] S. Oliveri, P. Atten, On electroconvection between non-parallel electrodes and its possible use to model general atmospheric circulation, IEEE Trans. Indus. Appl. 21, (1985) 699-704.
[12] P.J. Stiles, Electrothermal convection in dielectric liquids, Chem. Phys. Lett. 179 (1991) 311-315.
[13] T. Maekawa, K. Abe, I. Tanasawa, Onset of natural convection under an electric field, Int. J. Heat Mass Trans. 35 (1992) 613-621.
[14] P.J. Stiles, F. Lin, P.J. Blennerhassett, Convective heat transfer through polarized dielectric liquids, Phys. Fluids 5 (1993) 3273-3279.
[15] E.I. Adawi, S.F. Shehawey, S.A. Shalaby, M.I.A. Othman, The stability of natural convection in an inclined fluid layer in the presence of ac electric field, J. Phys. Soc. Japn. 65 (1996) 2479-2484.
[16] M.I.A. Othman, S. Zaki, The effect of thermal relaxation time on electrohydrodynamic viscoelastic fluid layer heated from below, Can. J. Phys. 81 (2003) 779-787.
[17] I.S. Shivakumara, M.S. Nagashree, K. Hemalatha, 2007, Electrothermoconvective instability in a heat generating dielectric fluid layer, Int. Comm. Heat Mass Trans. 34 (2007) 1041–1047.
[18] I.S. Shivakumara, Lee Jinho, K. Vejravelu, M. Akkanagamma, Electrothermal convection in a rotating dielectric fluid layer: effect of velocity and temperature boundary conditions, Int. J. Heat Mass Trans. 55 (2012) 2984– 2991.
[19] I.S. Shivakumara, M. Akkanagamma, Ng Chiu-On, Electrohydrodynamic instability of a rotating couple stress dielectric fluid layer, Int. J. Heat Mass Trans. 62 (2013) 761–771.
[20] R.Z. Moreno, E.J. Bonet, O.V. Trevisan, Electric alternating current effects on flow of oil and water in porous media. In K. Vafai & P. N. Shivakumar (Eds.), Proceedings of the International Conference on Porous Media and Their Applications in Science (1996) 147–172. Hawaiir: Engineering and Industry.
[21] J.A. Del Rio, S. Whitaker, Electrohydrodynamics in porous media, Trans. Porous Media 44 (2001) 385–405.
[22] N. Rudraiah, M.S. Gayathri, Effect of thermal modulation on the onset of electrothermoconvection in a dielectric fluid saturated porous medium. ASME J. Heat Transfer 131 (2009) 101009–101015.
[23] I.S. Shivakumara, N. Rudraiah, J. Lee, K. Hemalatha, The onset of Darcy–Brinkman electroconvection in a dielectric fluid saturated porous layer, Trans. Porous Media, 90 (2011) 509-528
[24] I.S. Shivakumara, Ng. Chiu-On, M.S. Nagashree, The onset of electrothermoconvection in a

rotating Brinkman porous layer, Int. J. Engg. Sci. 9 (2001) 646–663.
[25] E.M. Sparrow, R.J. Goldstein, U.K. Jonson, Thermal instability in a horizontal fluid layer: effect of boundary conditions in a horizontal fluid layer: effect of boundary conditions and non-linear temperature profiles. J. Fluid Mech. 18 (1964) 513-528.
[26] Char Ming-I & Ko-Ta Chiang, 1994, Boundary effects on the Bénard-Marangoni instability under an electric field, Applied Scientific Research 52 (1994) 331-354.

Table 1 Comparison of R_{tc} and a_c for lower boundary rigid at fixed temperature with different values of Bi in the absence of AC electric field strength with $\sigma^2 = 0 = \Gamma$

B_i	Upper boundary free				Upper boundary rigid			
	Sparrow et al. [25]		Present study		Sparrow et al. [25]		Present study	
	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c
0	669.001	2.09	668.98	2.086	1295.781	2.55	1295.78	2.552
0.1	682.361	2.115	682.36	2.116	1309.545	2.58	1309.54	2.582
0.3	706.365	2.17	706.39	2.169	1334.149	2.64	1334.08	2.632
1	770.569	2.3	770.57	2.293	1398.508	2.75	1398.51	2.751
3	872.506	2.46	872.53	2.452	1497.594	2.90	1497.57	2.901
30	1055.345	2.65	1055.5	2.648	1667.102	3.08	16671	3.084
100	1085.893	2.67	1085.9	2.672	1694.573	3.11	1694.57	3.106
∞	1100.657	2.68	1100.5	2.682	1707.765	3.12	1707.76	3.116

Table 2 Comparison of R_{tc} and a_c for lower boundary rigid at fixed heat flux with different values of Bi in the absence of AC electric field strength with $\sigma^2 = 0 = \Gamma$

B_i	Upper boundary free				Upper boundary rigid			
	Sparrow et al. [25]		Present study		Sparrow et al. [25]		Present study	
	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c
0	320.00	0	320	0.648	720.000	0.71	720	0.679
0.1	381.665	1.015	381.665	1.015	807.676	1.23	807.676	1.228
0.3	428.29	1.3	428.29	1.299	869.231	1.57	869.208	1.557
1	513.792	1.64	513.79	1.644	974.173	1.94	974.172	1.943
3	619.666	1.92	619.666	1.921	1093.744	2.24	1093.74	2.242
30	780.240	2.18	780.237	2.176	1259.884	2.51	1259.91	2.511
100	804.973	2.2	804.972	2.203	1284.263	2.53	1284.28	2.539
∞	816.748	2.21	816.744	2.215	1295.781	2.55	1295.781	2.552

Table 3 Comparison of R_{ec} and a_{ec} for different values of R_t for $\sigma^2 = 0 = \Gamma = Bi$

Roberts [7]			Present study	
R_t	R_{ec}	a_{ec}	R_{ec}	a_{ec}
-1000	3370.077	3.2945	3370.077	3.29446
-500	2749.868	3.2598	2749.868	3.25983
0	2128.696	3.2260	2128.696	3.22596
500	1506.573	3.1929	1506.573	3.19287
1000	883.517	3.1606	883.517	3.16059
1707.762	0	3.1162	0	3.11621

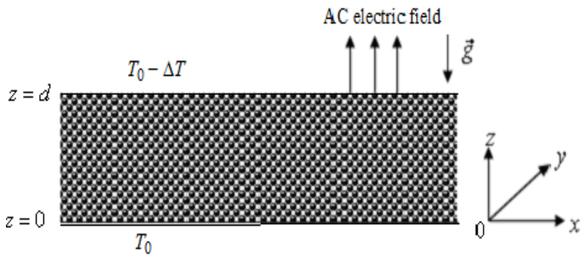

Figure 1 Physical Configuration

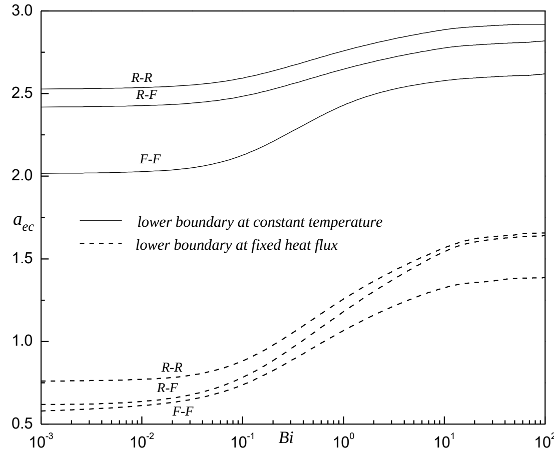

Figure 4 Variation of a_{ec} as a function of Bi when $R_t = 50$, $\Gamma = 0.2$ and $\sigma^2 = 10$

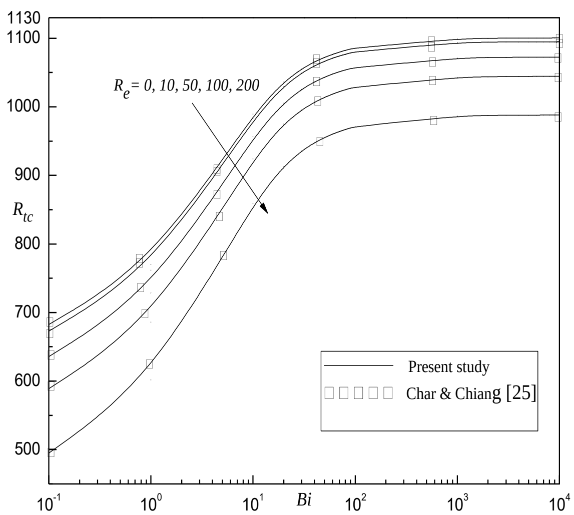

Figure 2 Variation of R_{tc} as a function of Bi when $\sigma^2 = 0 = \Gamma$

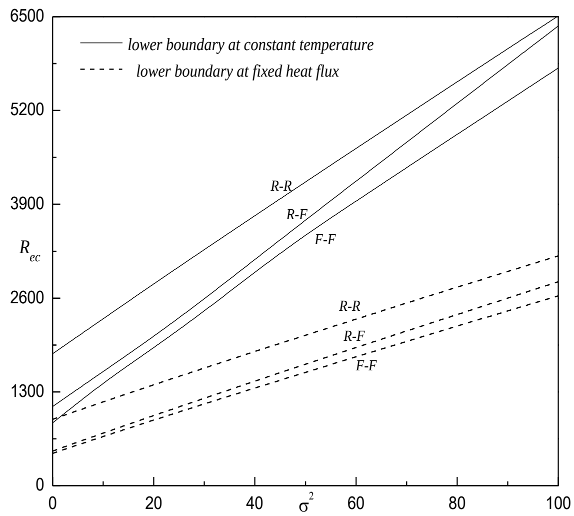

Figure 5 Variation of R_{ec} as a function of σ^2 when $R_t = 50$, $Bi = 2$ and $\Gamma = 0.2$

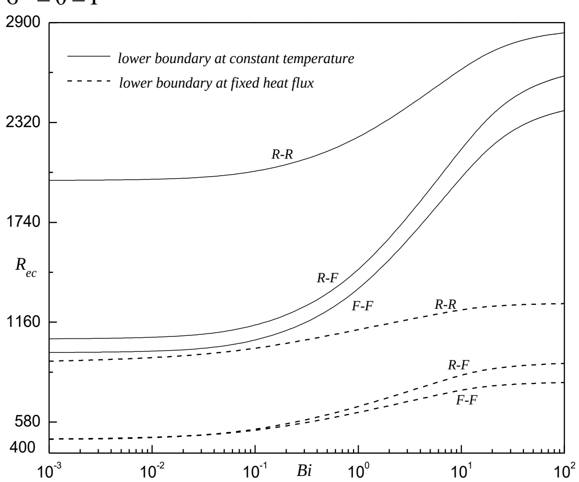

Figure 3 Variation of R_{ec} as a function of Bi when $R_t = 50$, $\sigma^2 = 10$ and $\Gamma = 0.2$

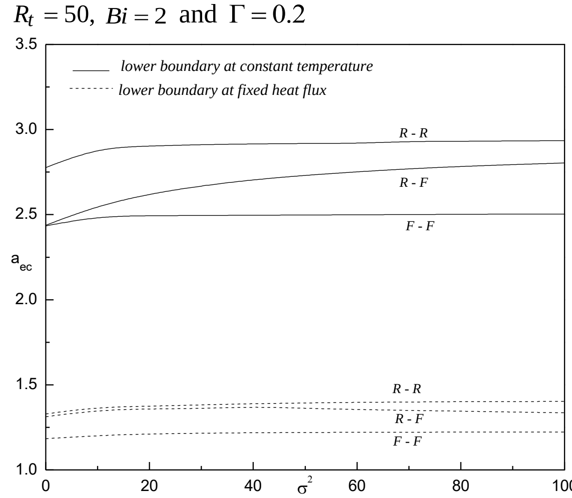

Figure 6 Variation of a_{ec} as a function of σ^2 for $R_t = 50$, $Bi = 2$ and $\Gamma = 0.2$

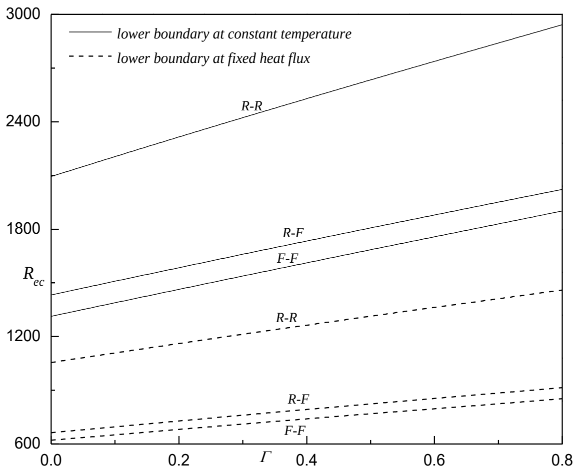

Figure 7 Variation of R_{ec} as a function of Γ when $R_t = 50$, $Bi = 2$ and $\sigma^2 = 10$

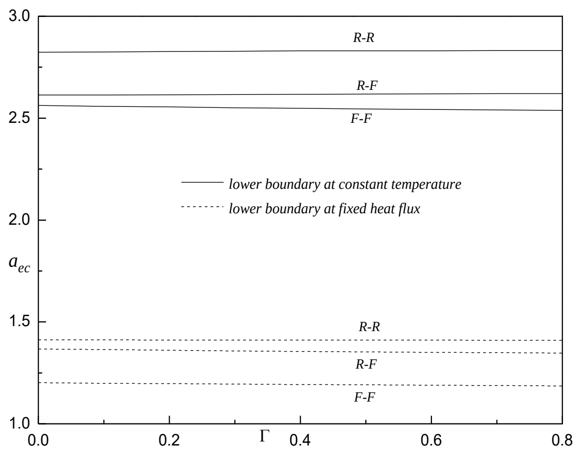

Figure 8 Variation of a_{ec} as a function of Γ when $R_t = 50$, $Bi = 2$ and $\sigma^2 = 10$

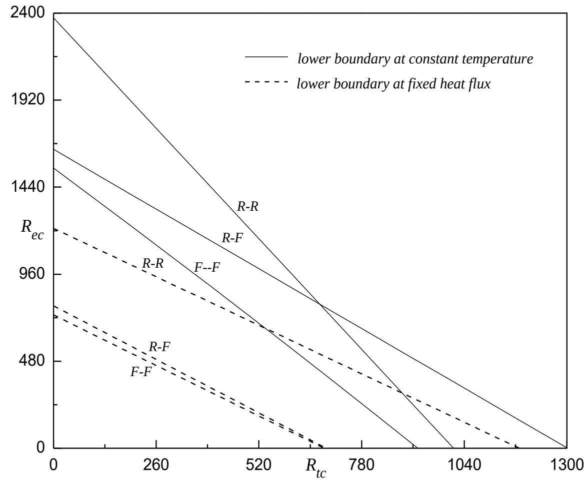

Figure 9 Locus of R_{ec} as a function of R_{tc} when $\Gamma = 0.2$, $Bi = 2$ and $\sigma^2 = 10$