

NANO ALMOST I-OPENNESS AND NANO ALMOST I-CONTINUITY

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Abstract

Our purpose is to present the nano almost I-open and nano almost I-closed sets. Utilizing these new concepts the nano almost I-continuous functions have been obtained. We give a diagram that well illustrates the relations.

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1 Introduction

Topological spaces with ideals have been considered since 1930 by Kuratowski [1]. The paper of Vaidyanathaswamy [2] in 1945 gave the subject great importance.

"A non-empty collection of subsets of X with heredity and finite additivity conditions is called as an ideal or a dual filter on X. Namely a non-empty family $I \subseteq P(X)(P(X))$ is the set of all subsets of X) is nammed an ideal if and only if: i) $A \in I$ gives $P(A) \subseteq I$ (heredity).

ii) $A, B \in I$ gives $A \cup B \in I$ (finite additivity).

Given X carries topology τ with an ideal I on X, a set operator()^{*} : $P(X) \to P(X)$, named a local function [2] of A with respect to τ and I is defined as follows: for $A \subseteq X, A^*(I, \tau) = \{x \in X : G_x \cap A \notin I \text{ for every } G_x \in \tau(x)\}$ where $\tau(x) = \{G \in \tau : x \in G\}$. A Kuratowski closure operator $Cl^*()$ for a topology $\tau^*(I, \tau)$, named the *-topology finer than τ is defined by $Cl^*(A) = A \cup A^*(I, \tau)$ [2]. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X, then the space (X, τ, I) is called an ideal topological space".

"The concept of nano topology was introduced by Lellis Thivagar and Carmel Richard [3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also they defined nano closed sets, nono interior and nano-closure".

The concept of nano ideal topological spaces was introduced by Parimala et al. [4] and studied its properties and characterizations.

The basic object of this paper is to present the nano almost *I*-open and nano almost *I*-closed sets. Utilizing these new concepts the nano almost *I*-continuous functions have been obtained. Nano almost *I*-openness and nano almost *I*-continuity are considered as a generalization of nano *I*-openness and nano *I*-continuity which are known before. Numerous nano topological properties of these new notions have been discussed.

2 Preliminaries

Before entering our working, we have compiled some basic facts on rough sets.

Definition 2.1. "(see[5]) Let U be a non-empty finite set of objects named the cosmos and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the some equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. The lower approximation of X with respect to R is denoted by $\underline{apr}(X)$. That is, $\underline{apr}(X) = \bigcup \{R(x) : R(x) \subseteq X; x \in U\}$, The upper approximation of X with respect to R is denoted by $\overline{apr}(X)$. That is, $\overline{apr}(X) = \bigcup \{R(x) : R(x) \cap X \neq \phi; x \in U\}$ and the boundary region of X with respect to R is denoted by $B_R(X)$. That is, $B_R(X) = \overline{apr}(X) - apr(X)$ as Figure 1".



Figure 1: Rough set.

Proposition 2.2. "([5], Proposition 2.2) If (U, R) is an approximation space and $X, Y \subseteq U$, then: (i) $\underline{apr}(X) \subseteq X \subseteq \overline{apr}(X)$.

- (*ii*) $apr(\phi) = \overline{apr}(\phi) = \phi$.
- (iii) $apr(U) = \overline{apr}(U) = U$.

 $(iv) \ \overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y).$

 $(v)\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y).$

 $(vi) \ \underline{apr}(X \cup Y) \supseteq \underline{apr}(X) \cup \underline{apr}(Y).$

(vii) $apr(X \cap Y) = apr(X) \cap apr(Y)$.

(viii) $apr(X) \subseteq \overline{apr}(Y)$ and $\overline{apr}(X) \subseteq \overline{apr}(Y)$, whenever $X \subseteq Y$.

(ix) $\overline{apr}(X^c) = [apr(X)]^c$ and $apr(X^c) = [\overline{apr}(X)]^c$.

 $(x) \ \overline{apr}[\overline{apr}(X)]] = [apr[\overline{apr}(X)]] = \overline{apr}(X).$

(xi) $apr[apr(X)] = \overline{apr}[apr(X)] = apr(X)$ ".

Definition 2.3. "(see[3]) Let U be the cosmos, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, \underline{apr}(X), \overline{apr}(X), B_R(X)\},\$ where $X \subseteq U$. Then by Proposition 2.2, $\tau_R(X)$ satisfies the condition of topology on U.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and the pair $(U, \tau_R(X))$ is called a nono

topological space. The elements of $\tau_R(X)$ are called nano open sets in U and the complement of a nano open set is called a nano closed set. Elements of $[\tau_R(X)]^c$ being called duel nano topology of $\tau_R(X)$ ".

Remark 2.4. "Let $(U, \tau_R(X))$ be a nano topological space with respect to X where $X \subseteq U$ and R be an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R".

Definition 2.5. "Let ([3], Definition 3.1) $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be: (i) nano semi-open if $A \subseteq nCl(nInt(A))$,

(ii) nano preopen if $A \subseteq nInt(nCl(A))$,

(iii) nano β -open if $A \subseteq nCl(nInt(nCl(A)))$ ".

Definition 2.6. "Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. A mapping $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called

(i) nano continuous [6] if $f^{-1}(B)$ is nano open in U for every nano open set B in V.

(ii) nano semi-continuous [6] if $f^{-1}(B)$ is nano semi-open in U for every nano open set B in V. (iii) nano precontinuous [6] if $f^{-1}(B)$ is nano preopen in U for every nano open set B in V.

(iv) nano β -continuous [7] if $f^{-1}(B)$ is nano β -open in U for every nano open set B in V".

3 Nano Ideal Topological Spaces

Recently in 2016, Thivagar and Devi [8] have considered the nano local function in nano ideal topological space and they have obtained a new topology. Before starting the discussion we shall consider the following concepts.

Definition 3.1. "([8], Definition 3.2) Let $(U, \tau_R(X), I)$ be a nano ideal topological space. A set operator $\binom{n}{2} : P(U) \to P(U)$ is named the nano local function. And for a subset $A \subseteq U$. $A_n^*(I, \tau_R(X)) = \{x \in U : G_x \cap A \notin I, \text{ for every } G_x \in \tau_R(X)\}$ is named the nano local function of A with respect to I and $\tau_R(X)$. We will simply write A_n^* for $A_n^*(I, \tau_R(X))^*$.

Example 3.2. "Let $(U, \tau_R(X))$ be a nano topological space with an ideal I on U and for every $A \subseteq U$. (i) If $I = \{\phi\}$, then $A_n^* = nCl(A)$, (*ii*) If I = P(U), then $A_n^* = \phi^*$.

Theorem 3.3. "([8], Theorem 3.3) Let $(U, \tau_R(X))$ be a nano topological space with ideals I, J on U and A, B be subsets of U. Then the following statements are true:

(i) $A \subseteq B \Rightarrow A_n^* \subseteq B_n^*$, (*ii*) $I \subseteq J \Rightarrow A_n^*(J) \subseteq A_n^*(I)$, (iii) $A_n^* = nCl(A_n^*) \subseteq nCl(A)$ (A_n^* is a nano closed subset of nCl(A)), $\begin{array}{l} (iv) \ (A_n^*)_n^* \subseteq A_n^*, \\ (v) \ A_n^* \cup B_n^* = (A \cup B)_n^*, \end{array}$ (vi) $A_n^* - B_n^* = (A - B)_n^* - B_n^* \subseteq (A - B)_n^*$, (vii) $V \in \tau_R(X) \Rightarrow V \cap A_n^* = V \cap (V \cap A)_n^* \subseteq (V \cap A)_n^*$ and (viii) $E \in I \Rightarrow (A \cup E)_n^* = A_n^* = (A - E)_n^*$ ".

The converse implications of (i), (ii) and (iii) of Theorem 3.3 do not hold in general, as seen from the next instance

Example 3.4. Let $U = \{1, 2, 3, 4\}$ be the universe.

(i) If $X = \{1,2\} \subseteq U; U/R = \{\{1\}, \{3\}, \{2,4\}\}$. One can deduce that $apr(X) = \{1\}, \overline{apr}(X) = \{1,2,4\}, B_R(X) = \{2,4\}, B_R($ then $\tau_R(X) = \{U, \phi, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$. Let $I = \{\phi, \{1\}\}$ for $A = \{1, 3\}$ and $B = \{1, 4\}$, we have $A_n^* = \{3\}, B_n^* = \{2, 3, 4\}$. that is $A_n^* \subseteq B_n^*$ but A not subset from B. Also, let $I = \{\phi, \{1\}\}$ and $J = \{\phi, \{2\}\}$. It is easily seen that, for $A = \{1, 3, 4\}$, $A_n^*(I) = \{2, 3, 4\}, A_n^*(J) = \{1, 2, 3, 4\} = U$, that is $A_n^*(I) \subseteq A_n^*(J)$, while I not a subset from J.

(ii) Let $X = \{1,4\}, U/R = \{\{2\}, \{4\}, \{1,3\}\}$. One can deduce that $apr(X) = \{4\}, \overline{apr}(X) = \{1,3,4\}, B_R(X) = \{1,3\}, A_R(X) = \{1,3\}, A_R(X)$ then $\tau_R(X) = \{U, \phi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. Let $I = \{\phi, \{4\}\}$ for $A = \{2, 4\}$, we have $nCl(A) = nCl(\{2, 4\}) = \{2, 4\}$. $A_n^* = \{2, 4\}_n^* = \{2\}$ and $nCl(A_n^*) = nCl(\{2\}) = \{2\}$. Therefore, $nCl(A) \not\subseteq A_n^* = nCl(A_n^*)$.

Definition 3.5. "([4], Definition 2.6) Let $(U, \tau_R(X))$ be a nano topological space with an ideal I on U. The set operator nCl^* is named a nano *-closure and is defined as: $nCl^*(A) = A \cup A_n^*$, for $A \subseteq X^{"}$.

Theorem 3.6. ([4], Theorem 2.7.) The set operator nCl^* satisfies the following conditions : (i) $A \subseteq nCl^*(A)$, (ii) $nCl^*(\phi) = \phi$ and $nCl^*(U) = U$, (iii) If $A \subseteq B$, then $nCl^*(A) \subseteq nCl^*(B)$, (iv) $nCl^*(A) \cup nCl^*(B) = nCl^*(A \cup B)$. (v) $nCl^*(nCl^*(A)) = nCl^*(A)$.

Proof. It is clear from Definition 3.6 and Theorem 3.4.

4 Nano Almost *I*-open Sets

The fourth section we have interpreted the properties of nano almost *I*-open sets in terms of its approximations.

Definition 4.1. In a nano ideal topological space $(U, \tau_R(X), I), W \subseteq U$ is called nano almost *I*-open if $W \subseteq nCl(nInt(A_n^*)), (U-W)$ is called nano almost *I*-closed.

When there is no chance of confusion, the collection of all nano almost *I*-open sets of $(U, \tau_R(X), I)$ will be symbolized by $NAIO(U, \tau_R(X))$. Also, NAIO(U, x) means the class of all nano almost *I*-open sets containing $x \in U$. "Recall that, a subset *A* of a nano ideal topological space $(U, \tau_R(X), I)$ is named nano *I*-open [9] if $A \subseteq nInt(A_n^*)$."

Proposition 4.2. Arbitrary union of nano almost I-open sets is also nano almost I-open.

Proof. Let $(U, \tau_R(X), I)$ be any nano ideal topological space and $W_i \in NAIO(U, \tau_R(X))$ for $i \in \nabla$. This means that for each $i \in \nabla, W_i \subseteq nCl(nInt(W_i)_n^*)$ and so, $\bigcup_{i \in \nabla} W_i \subseteq \bigcup_{i \in \nabla} nCl(nInt(W_i)_n^* \subseteq nCl(nInt(\bigcup_{i \in \nabla} W_i)_n^*))$. Hence $\bigcup_{i \in \nabla} W_i \in NAIO(U, \tau_R(X))$.

Remark 4.3. A finite intersection of nano almost I-open sets need not in general be nano almost I-open as shown in the next example.

Example 4.4. Let $U = \{1, 2, 3, 4\}$ be the universe, $U/R = \{\{2\}, \{4\}, \{1, 3\}\}, X = \{1, 4\}$. Then $\tau_R(X) = \{U, \phi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}$. For $I = \{\phi, \{4\}\}$, we deduce that the two sets $A = \{1, 2\}$ and $B = \{2, 3\}$ are nano almost *I*-open while their intersection $C = \{2\}$ does not nano almost *I*-open.

The connections between nano almost *I*-openness with some other corresponding types have been given throughout the following implication in Figure 2.



The above relationship can not be reversible as the following examples illustrate.



Example 4.5. Let $U = \{1, 2, 3, 4\}$ be the universe, $U/R = \{\{2\}, \{4\}, \{1, 3\}\}$ be the family of the equivalence classes of U by the equivalence relation R and $X = \{1, 4\}$. Then one can deduce that:

 $\frac{apr(X) = \{4\}, \overline{apr}(X) = \{1, 3, 4\} \text{ and } B_R(X) = \{1, 3\}. \text{ Therefore, the nano topology } \tau_R(X) = \{U, \phi, \{4\}, \{1, 3\}, \{1, 3, 4\}\}.$ For $I = \{\phi, \{4\}\}, we \ can \ show \ the \ following:$

(i) The set A = {1,2} is nano almost I-open but is not nano I-open.
(ii) The set B = {1,2,4} is nano peopen but not nano I-open.

Example 4.6. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\}$. Then one can deduce that $\underline{apr}(X) = \{1\}, \overline{apr}(X) = \{1, 2, 4\}$ and $\tau_R(X) = \{U, \phi, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$. If $A = \{2, 3\}$, then A is nano β -open but neither nano preopen nor nano semi-open.

Example 4.7. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{3\}, \{4\}, \{1, 2\}\}$ and $X = \{1, 3\}$. Then one can deduce that $\underline{apr}(X) = \{3\}, \overline{apr}(X) = \{1, 2, 3\}, B_R(X) = \{1, 2\}$. Therefore $\tau_R(X) = \{U, \phi, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$. Now $A = \{1, 4\}$ is nano $\overline{\beta}$ -open in U but not nano almost I-open in U, where $I = \{\phi, \{1\}\}$.

Proposition 4.8. In a nano ideal topological space $(U, \tau_R(X), I)$, if $G \in \tau_R(X)$ and $H \in NAIO(U, \tau_R(X), I)$, then $G \cap H$ is nano almost I-open.

Proof. By assumption and the fact that $G \cap nCl(H) \subseteq nCl(G \cap H)$, we have, $G \cap H \subseteq G \cap nCl(nInt(H)_n^*) \subseteq nCl(G \cap nInt(H)_n^*)$. Then we have $G \cap H \subseteq nCl(nInt(G \cap (H)_n^*)) \subseteq nCl(nInt((G \cap H)_n^*))$. Hence the result. \Box

Proposition 4.9. The following statements are hold.

(i) For $(U, \tau_R(X), \{\phi\})$, then $NAIO(U, \tau_R(X)) = N\beta O(U, \tau_R(X))$.

(ii) For $(U, \tau_R(X), P(U))$, then $NAIO(U, \tau_R(X)) = NIO(U, \tau_R(X))$.

(iii) For any nano ideal topological space $(U, \tau_R(X), I)$, each nano almost I-open which it is nano *-closed is nano semi-open. (iv) For any nano ideal topological space $(U, \tau_R(X), I)$, each nano semi-open which it is nano *-dense in itself is nano almost I-open.

5 Nano Almost *I*-Continuous Functions

First we introduce a weak form of nano I-continuous function (cf. Definition 5.5) called nano almost I-continuous (Definition 5.1 below)

Definition 5.1. A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(X))$ is named nano almost *I*-continuous (briefly, naI-continuous) if for every $G \in \tau_{R'}(X), f^{-1}(G) \in NAIO(U, \tau_R(X))$.

The next theorem gives several characterizations of nano almost *I*-continuous functions.

Theorem 5.2. For $f: (U, \tau_R(X), I) \to (V, \tau_{R'}(X))$ be a function, the next are equivalent:

(i) f is naI-continuous.

(ii) The reverse image of each nano closed set in $(V, \tau_{R'}(X))$ is a nano almost I-closed in $(U, \tau_R(X), I)$.

(iii) For each $x \in U$ and each $G \in \tau_{R'}(X)$ including f(x), there exists $W \in NAIO(U)$ containing x such that $f(W) \subseteq G$.

(iv) For each $x \in U$ and each $G \in \tau_{R'}(X)$ including $f(x), (f^{-1}(G))_n^*$ is a nano neighbourhood of x.

Proof. (i) \Leftrightarrow (ii): Obvious

(i) \Rightarrow (iii): Let x in X and G be a nano almost I-open set of Y including f(x). By (i), $f^{-1}(G)$ is a nano almost I-open set. Set $W = f^{-1}(G)$, we have $f(W) \subseteq G$.

(iii) \Rightarrow (i): Let A be a nano open set in Y. If $f^{-1}(A) = \phi$, then $f^{-1}(A)$ is clearly a nano almost I-open set. Assume that $f^{-1}(A) \neq \phi$. Let $x \in f^{-1}(A)$. Then $f(x) \in A$, which implies that there exists a nano almost I-open set W including x such that $f(W) \subseteq A$. Thus, $W \subseteq f^{-1}(A)$. Since W is nano almost I-open, $x \in W \subseteq nInt(W_n^*) \subseteq nInt(f^{-1}(A)_n^*)$ and so $f^{-1}(A) \subseteq nInt(f^{-1}(A)_n^*)$. Hence $f^{-1}(A)$ is nano almost I-open set and so f is nano almost I-continuous.

(iii) \Rightarrow (iv): Let $x \in X$ and G be a nano open set of Y including f(x). Then there exist a nano almost I-open set W including x such that $f(W) \subseteq G$. It follows that $W \subseteq f^{-1}(f(W)) \subseteq f^{-1}(G)$. Since W is nano almost I-open, $x \in W \subseteq nInt(W_n^*) \subseteq nInt(f^{-1}(G)_n^*) \subseteq f^{-1}(G)_n^*$. Hence $f^{-1}(G)_n^*$ is nano almost I-continuous of x. (iv) \Rightarrow (i): Obvious **Proposition 5.3.** The next equivalents are verify:

(i) $f: (U, \tau_R(X), \{\phi\}) \to (V, \tau_{R'}(X))$ is nano almost I-continuous if and only if it is nano β -continuous. (ii) $f: (U, \tau_R(X), P(X)) \to (V, \tau_{R'}(X))$ is nano almost I-continuous if and only if it is nano I-continuous. (iii) Nano almost I-continuity of a function $f: (U, \tau_R(X), I) \to (V, \tau_{R'}(X))$ concides with nano semi-continuity if for each $G \in \tau_{R'}(X), f^{-1}(G)$ is nano *-perfect or $f^{-1}(G)$ is both nano *-dense-in-itself and nano *-closed.

Definition 5.4. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. A mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano β -irresolute if $f^{-1}(B)$ is nano β -open in U for every nano β -open set B in V.

Proposition 5.5. For the function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(X), J)$ and $g : (V, \tau_{R'}(X), J) \to (W, \tau_{R'}(X))$, the following are hold:

(i) If f nano almost I-continuous and g is nano continuous, then the composition $(g \circ f)$ is nano almost I-continuous. (ii) $(g \circ f)$ is nano β -continuous, if f is nano β -irresolute and g is nano almost I-continuous.

Definition 5.6. Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $(V, \tau_{R'}(Y))$ be a nano topological space. A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called nano *I*-continuous [7] if the reverse image for every nano open set *B* in *V* is nano *I*-open in *U*.

Relationship of several functions defined in this research, from the Figure 3.



Figure 3: Relationship between some weak forms of nano continuity.

The reverses need not be true in general as shown in the next examples.

Example 5.7. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{4\}, \{2, 3\}\}$ and $X = \{1, 4\}$. Then one can deduce that $\tau_R(X) = \{U, \phi, \{1, 4\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as: f(1) = y = f(2), f(3) = z, f(4) = w. It is clear that f is both nano semi-continuous and nano precontinuous but not nano continuous.

Example 5.8. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\} \subseteq U$. Then one can deduce that $\tau_R(X) = \{U, \phi, \{1\}, \{2, 4\}, \{1, 2, 4\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\} \subseteq V$. Then $\sigma_{R'}(Y) = \{V, \phi, \{x, z\}, \{x, y, z\}\}$. Define $f : (U, \tau_R(X)) \to (V, \sigma_{R'}(Y))$ as: f(1) = z, f(2) = x, f(3) = w, f(4) = y. It is clear that f is nano β -continuous but not nano semi-continuous.

Example 5.9. Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{b\}, \{d\}, \{e\}, \{a, c\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then one can deduce that $\tau_R(X) = \{U, \phi, \{a, b, c\}\}$. Let $V = \{1, 2, 3, 4, 5\}$ with $V/R' = \{\{1\}, \{2, 5\}, \{3, 4\}\}$ and $Y = \{1, 2, 5\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \phi, \{1, 2, 5\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as: f(a) = 5, f(b) = 1, f(c) = 2, f(d) = 3, f(c) = 4. It is clear that for $I = \{\phi, \{b\}, \{c\}, \{c\}, \{b, c\}\}, f$ is nano β -continuous but not nano almost I-continuous.

6 Conclusion

It is shown that we are interested in finding the notion of nano almost *I*-open sets and nano almost-*I*-continuous functions in nano ideal topological spaces and some of their properties are studied. It is to be expected that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of nano ideal topology in the field of mathematics.

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