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NANO $\delta\beta$-OPEN SETS AND NANO $\delta\beta$-CONTINUITY

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Abstract

This paper mainly concerns with generalizing nano near open sets by proposing new sort of sets called nano $\delta\beta$-open sets. These sets are stronger than any type of the other nano near open sets such as, nano regular open, nano α-open, nano semi-open, nano pre-open, nano γ-open and nano β-open sets. The main properties and the relationships among of these sets are studied. Moreover, various forms of nano $\delta\beta$-open sets corresponding to different cases of approximations are also derived. Finally, the notion of nano $\delta\beta$-continuous functions is introduced and compared to the other types of nano continuous functions.

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1 Introduction

The study of nano topological spaces was initiated by Thivagar and Richard [1], which was defied in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. The subset generated by lower approximations is characterized by certain objects that definitely form part of an interest subset, whereas the upper approximation is characterized by uncertain objects that possibly form part of an interest subset. The elements of a nano topological space are called the nano open sets. They also defined nano closed sets, nano interior and nano closure. Moreover, they introduced the weak forms of nano open sets namely nano α-open sets, nano semi-open sets, nano pre-open sets and nano regular open sets. Revathy and Ilango [2] generalized the previous work [1] by introducing the concept of nano β-open sets. In 2016, Nasef et al. [3] studied some properties for near nano open (closed) sets. Thivagar and Richard [4] presented the notion of nano continuity in nano topological spaces. Thereafter, Mary and Arockiarani [5,6] introduced nano α-continuous, nano semi-continuous, nano pre-continuous, and nano γ-continuous. The nano β-continuity was investigated in [3].

The current work concentrates on generalization the previous near nano open sets and nano continuous functions. The remainder of this paper is arranged as follows. The fundamental concepts of nano topological spaces are introduced in Section 2. The goal of Section 3 is to suggest new near nano open sets namely, nano $\delta\beta$-open sets and study their properties. Additionally, these sets are compared to the previous one [1,2] and shown to be more general. Various forms of nano $\delta\beta$-open

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sets corresponding to different cases of approximations are investigated in Section 4. In Section 5, we propose a new class of functions on nano topological spaces called nano $\delta\beta$-continuous functions. Moreover, the concepts of nano $\delta\beta$-closure and nano $\delta\beta$-interior are presented. Furthermore, nano $\delta\beta$-continuous functions and their characterizations are studied in terms of nano closed sets, nano closure, nano interior, nano $\delta\beta$-closed sets, nano $\delta\beta$-closure and nano $\delta\beta$-interior. Relationships between the current functions and the previous one [3–6] are obtained. Finally, this paper concludes in Section 6.

2 Preliminaries

This section contains the basic concepts of nano topological spaces, nano near open sets, nano near continuous and their properties.

Definition 2.1. *[7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$:*

1. *the lower approximation of X with respect to R is denoted by $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.*

2. *the upper approximation of X with respect to R is denoted by $U_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *the boundary region of X with respect to R is denoted by $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[1] Let U be the universe, R be an equivalence relation on U and $X \subseteq U$. The nano topology on U with respect to X is defined by $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano open sets and the complement of nano open set is called nano closed set.*

Definition 2.3. *[1] Let $(U, \tau_R(X))$ be a nano topological space with respect to X, where $X \subseteq U$. For a subset $A \subseteq U$:*

1. *the nano interior of A is defined as the union of all nano open subsets contained in A, and is denoted by $nint(A)$.*

2. *the nano closure of A is defined as the intersection of all nano closed subsets containing A, and is denoted by $ncl(A)$.*

Definition 2.4. *[1, 2] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The set A is said to be:*

1. *nano regular open, if $A = nint(ncl(A))$.*

2. *nano α-open, if $A \subseteq nint[ncl(nint(A))]$.*

3. *nano semi open, if $A \subseteq ncl(nint(A))$.*

4. *nano pre-open, if $A \subseteq nint(ncl(A))$.*

5. *nano γ-open, if $A \subseteq nint(ncl(A)) \cup ncl(nint(A))$.*

6. *nano β-open (nano semi pre-open), if $A \subseteq ncl[nint(ncl(A))]$.*

The relationships between nano near open sets are presented in Figure 1 [3].

The family of all nano regular open (respectively, nano α-open, nano semi-open, nano pre-open, nano γ-open and nano β-open) sets in a nano topological space $(U, \tau_R(X))$ is denoted by $NRO(U, X)$ (respectively, $N_\alpha(U, X), NSO(U, X), NPO(U, X), N_\gamma(U, X$ and $N_\beta(U, X)$).

Definition 2.5. *[1, 2] A subset K of a nano topological space $(U, \tau_R(X))$ is called nano regular closed (respectively, nano α-closed, nano semi-closed, nano preclosed, nano γ-closed and nano β-closed) if its complement is nano regular open (respectively, nano α-open, nano semi-open, nano pre-open, nano γ-open and nano β-open).*

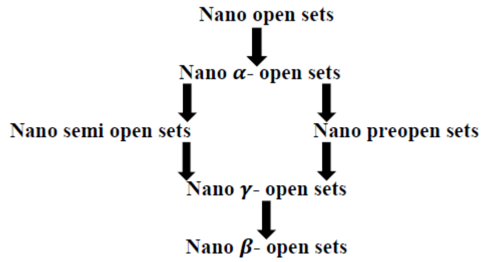

Figure 1: The relationships between nano near open sets.

Definition 2.6. *Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces. A mapping $f : (U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ is said to be:*

1. *nano continuous [4] if $f^{-1}(B)$ is nano open set in U for every nano open set B in V.*
2. *nano α-continuous [5] if $f^{-1}(B)$ is nano α-open set in U for every nano open set B in V.*
3. *nano semi-continuous [5] if $f^{-1}(B)$ is nano semi-open set in U for every nano open set B in V.*
4. *nano pre-continuous [5] if $f^{-1}(B)$ is nano pre-open set in U for every nano open set B in V.*
5. *nano γ-continuous [6] if $f^{-1}(B)$ is nano γ-open set in U for every nano open set B in V.*
6. *nano β-continuous [3] if $f^{-1}(B)$ is nano β-open set in U for every nano open set B in V.*

Nasef et al. [3] introduced the relationships between the different types of near nano continuous functions as shown in the following diagram in Figure 2.

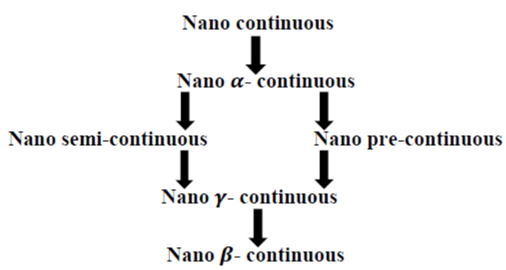

Figure 2: The relationships between near nano continuous functions.

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, and U/R denotes the family of equivalence classes of U by R.

3 Nano $\delta\beta$-open sets

In this section, the concept of nano $\delta\beta$-open sets and their properties are presented. It is showed that this concept is an extension of the previous concepts of nano near open sets [1–3]. Several nano topological properties of this concept are studied.

Definition 3.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The nano δ-closure of A is defined by $ncl^\delta(A) = \{x \in U : A \cap nint(ncl(G)) \neq \phi, G \in \tau_R(X)$ and $x \in G\}$. A set A is called nano δ-closed if $A = ncl^\delta(A)$. The complement of a nano δ-closed set is nano δ-open. Notice that $nint^\delta(A) = U - ncl^\delta(U - A)$, where $U - A = A'$ is the complement of A.*

Proposition 3.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then, $ncl(A) \subseteq ncl^\delta(A)$.*

Proof. Let $x \in ncl(A)$. Then, $G \cap A \neq \phi, \forall\ G, x \in G, G \in \tau_R(X)$. Therefore, $A \cap G = A \cap nint(nint(G)) \subseteq A \cap nint(ncl(G))$. Thus, $A \cap nint(ncl(G)) \neq \phi$. Hence, $ncl(A) \subseteq ncl^\delta(A)$.

Remark 3.1. *The inclusion in Proposition 3.1 can not be replaced by equality relation as shown in the following example.*

Example 3.1. *Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{c, d\}$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$. If $A = \{a\}$, then $ncl(A) = \{a, b\}$ and $ncl^\delta(A) = U$. Hence, $ncl^\delta(A) \nsubseteq ncl(A)$.*

The main properties of nano δ-closure are studied in the following theorem.

Theorem 3.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A, B \subseteq U$. Then the following properties hold:*

1. *$A \subseteq ncl^\delta(A)$.*
2. *If $A \subseteq B$, then $ncl^\delta(A) \subseteq ncl^\delta(B)$.*
3. *$ncl^\delta(A \cap B) \subseteq ncl^\delta(A) \cap ncl^\delta(B)$.*
4. *$ncl^\delta(A \cup B) = ncl^\delta(A) \cup ncl^\delta(B)$.*

Proof. Obvious.

Remark 3.2. *Example 3.1 shows that*

1. *the inclusion in Theorem 3.1 parts 1 and 3 can not be replaced by equality relation:*
   (i) *for part 1, if $A = \{a\}, ncl^\delta(A) = U$, then $ncl^\delta(A) \nsubseteq A$.*
   (ii) *for part 3, if $A = \{d\}, B = \{a, c\}, A \cap B = \phi, ncl^\delta(A) = U, nclt^\delta(B) = U, ncl^\delta(A) \cap ncl^\delta(B) = U \nsubseteq ncl^\delta(A \cap B) = \phi$.*
2. *the converse of part 2 is not necessarily true. If $A = \{a\}, B = \{b\}$, then $ncl^\delta(A) = ncl^\delta(B) = U$. Therefore, $ncl^\delta(A) \subseteq ncl^\delta(B)$, but $A \nsubseteq B$.*

Definition 3.2. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The set A is called nano $\delta\beta$-open, if $A \subseteq ncl[nint(ncl^\delta(A))]$. The complement of a nano $\delta\beta$-open set is a nano $\delta\beta$-closed set. The family of all nano $\delta\beta$-open is denoted by $N_{\delta\beta}(U, X)$.*

The following proposition shows that $\delta\beta$-open sets are generalization of nano β-open sets [2]. Consequently, it is stronger than all the previous nano near open sets [1] as shown in Figure 3.

Proposition 3.2. *Every nano β-open is nano $\delta\beta$-open.*

Proof. By using the properties of nano interior, nano closure and Proposition 3.1, the proof is obvious.

Remark 3.3. *The converse of Proposition 3.2 is not necessarily true as shown in Example 3.1, $A = \{a\}$ is a nano $\delta\beta$-open set, but it is not a nano β-open set.*

Remark 3.4. *The family of all nano $\delta\beta$-open sets in U does not form a topology, as the intersection of two nano $\delta\beta$-open sets need not be nano $\delta\beta$-open set as shown in the following example.*

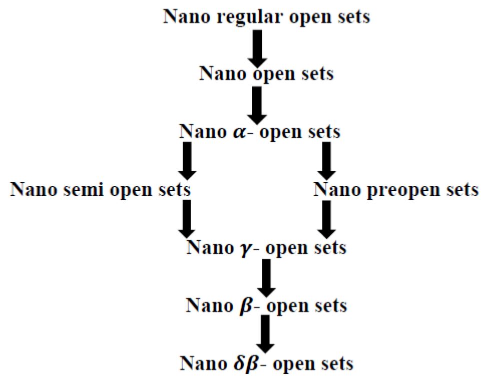

Figure 3: The relationships between nano $\delta\beta$-open sets and the other nano near sets.

Example 3.2. *In Example 3.1, let $X = \{a, c\}$. Then, $\tau_R(X) = \{U, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$. If $A = \{c, d\}, B = \{a, b, d\}$, then A, B are nano $\delta\beta$-open sets, but $A \cap B = \{d\}$ is not a nano $\delta\beta$-open set.*

Proposition 3.3. *The union of nano $\delta\beta$-open sets is also nano $\delta\beta$-open sets.*

Proof. Let A and B be nano $\delta\beta$-open sets. Then, $A \subseteq ncl(nint(ncl^{\delta}(A)))$ and $B \subseteq ncl(nint(ncl^{\delta}(B)))$. Thus,

$$\begin{aligned} A \cup B &\subseteq ncl(nint(ncl^{\delta}(A))) \cup ncl(nint(ncl^{\delta}(B))), \\ &= ncl[nint(ncl^{\delta}(A)) \cup nint(ncl^{\delta}(B))], \\ &\subseteq ncl[nint[ncl^{\delta}(A) \cup ncl^{\delta}(B)]], \\ &= ncl(nint(ncl^{\delta}(A \cup B)))(\text{by Theorem 3.1 (4)}). \end{aligned} \tag{1}$$

Therefore, $A \cup B \subseteq ncl(nint(ncl^{\delta}(A \cup B)))$. Hence, $A \cup B$ is also nano $\delta\beta$-open in U.

Corollary 3.1. *Let $(U, \tau_R(X))$ be a nano topological space. Then, $NSO(U, X) \cup NPO(U, X) \subseteq N_{\delta\beta}O(U, X)$.*

Remark 3.5. *The equality in Corollary 3.1 does not hold in general. In Example 3.1, the set $A = \{b\}$ is nano $\delta\beta$-open but not in $NSO(U, X) \cup NPO(U, X)$.*

Remark 3.6. *The arbitrary intersection of nano $\delta\beta$-closed sets is nano $\delta\beta$-closed, but the union of two nano $\delta\beta$-closed sets may not be a nano $\delta\beta$-closed set. This is clearly by Example 3.2 as the subsets $A = \{c\}$ and $B = \{a, b\}$ are nano $\delta\beta$-closed sets, but $A \cup B = \{a, b, c\}$ is not a nano $\delta\beta$-closed set.*

Proposition 3.4. *The intersection of nano open and nano $\delta\beta$-open is nano $\delta\beta$-open.*

Proof. Let A be nano open and B be nano $\delta\beta$-open. Then,

$$\begin{aligned} A \cap B &\subseteq A \cap ncl(nint(ncl^{\delta}(B))), \\ &\subseteq ncl(A \cap nint(ncl^{\delta}(B))), \\ &\subseteq ncl(nint(ncl^{\delta}(A \cap B))). \end{aligned} \tag{2}$$

Therefore, $A \cap B$ is nano $\delta\beta$-open.

Corollary 3.2. *The union of nano closed and nano $\delta\beta$-closed is nano $\delta\beta$-closed.*

Proposition 3.5. *If A and B are nano subsets of U such that $A \subseteq B \subseteq ncl(nint(A))$, then B is nano $\delta\beta$-open in U.*

Proof. Since,

$$
\begin{aligned}
nint(A) &\subseteq ncl(A), \\
\Rightarrow nint(nint(A)) &\subseteq nint(ncl(A)), \\
\Rightarrow nint(A) &\subseteq nint(ncl(A)), \\
\Rightarrow ncl(nint(A)) &\subseteq ncl(nint(ncl(A))), \\
&\subseteq ncl(nint(ncl^{\delta}(A)))(\text{by Proposition 3.1}).
\end{aligned} \tag{3}
$$

Then,

$$
\begin{aligned}
B &\subseteq ncl(nint(A)), \\
&\subseteq ncl(nint(ncl^{\delta}(A)))(\text{ by Theorem 3.1 (1)}), \\
&\subseteq ncl(nint(ncl^{\delta}(B)))(\text{ by Theorem 3.1 (2)}).
\end{aligned} \tag{4}
$$

Hence, $B \subseteq ncl(nint(ncl^{\delta}(B)))$. Therefore, B is nano $\delta\beta$-open in U.

Definition 3.3. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The set A is said to be:*

1. *nano δ-regular open, if $A = nint(ncl^{\delta}(A))$.*
2. *nano $\delta\alpha$-open, if $A \subseteq nint[ncl(nint^{\delta}(A))]$.*
3. *nano δ-semi open, if $A \subseteq ncl(nint^{\delta}(A))$.*
4. *nano δ-pre-open, if $A \subseteq nint(ncl^{\delta}(A))$.*

Definition 3.4. *A subset K of a nano topological space $(U, \tau_R(X))$ is called nano δ-regular closed (respectively, nano $\delta\alpha$-closed, nano δ-semi-closed and nano δ-pre-closed) if its complement is nano δ-regular open (respectively, nano $\delta\alpha$-open, nano δ-semi-open and nano δ-pre-open).*

Remark 3.7. *Each nano δ-regular open (respectively, nano $\delta\alpha$-open, nano δ-semi-open and nano δ-pre-open) is a nano $\delta\beta$-open set (as shown in Figure 4).*

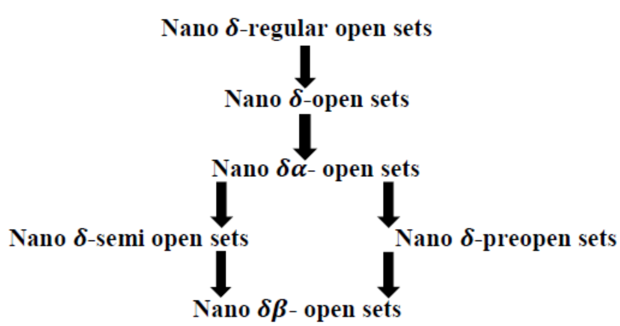

Figure 4: The relationships between nano δ near sets.

The converse implications in Remark 3.7 are not necessarily true as shown in

1. Example 3.1
   - $A = \{b\}$ is a nano $\delta\beta$-open set, but it is not nano δ-semi-open.

2. Example 3.2

   - $A = \{c, d\}$ is a nano $\delta\beta$-open set, but it is neither nano δ-regular open nor nano $\delta\alpha$-open.
   - $A = \{a, b, d\}$ is a nano $\delta\beta$-open set, but it is not nano δ-pre-open.
   - $A = \{c, d\}$ is a nano $\delta\beta$-open set, but it is not nano δ-open.

Proposition 3.6. *Each nano $\delta\beta$-open set which is*

1. *nano δ-semi-closed is nano semi-open.*
2. *nano $\delta\alpha$-closed is nano closed.*
3. *nano $\delta\alpha$-closed set is nano regular closed.*

Proof.

1. Let A be nano $\delta\beta$-open and nano δ-semi-closed. Then, $A \subseteq ncl(nint(ncl^\delta(A)))$ and $nint(ncl^\delta(A)) \subseteq A$. Hence, $nint(ncl^\delta(A)) \subseteq A \subseteq ncl(nint(ncl^\delta(A)))$. Since, $nint(nint(ncl^\delta(A))) = nint(ncl^\delta(A)) \subseteq nint(A)$, thus $ncl(nint(ncl^\delta(A))) \subseteq ncl(nint(A))$. Therefore, $A \subseteq ncl(nint(ncl^\delta(A))) \subseteq ncl(nint(A))$. Hence, $A \subseteq ncl(nint(A))$ and thus A is a nano semi-open set.

2. Let $A \subseteq U$ be nano $\delta\beta$-open set and nano $\delta\alpha$-closed set. Then, $A \subseteq ncl(nint(ncl^\delta(A)))$ and $ncl(nint(ncl^\delta(A))) \subseteq A$. Hence, $ncl(nint(ncl^\delta(A))) \subseteq A \subseteq ncl(nint(ncl^\delta(A)))$ So, $A = ncl(nint(ncl^\delta(A)))$. This means that A is nano closed.

3. Obvious.

Corollary 3.3. *Each nano $\delta\beta$-closed which is*

1. *nano δ-semi-open is nano semi-closed.*
2. *nano $\delta\alpha$-open is nano open.*
3. *nano $\delta\alpha$-open set is nano regular open.*

4 Nano $\delta\beta$-open sets and lower & upper approximations

The object of this section is to investigate various forms of nano $\delta\beta$-open sets corresponding to different cases of approximations.

Proposition 4.1. *If $U_R(X) = U$ in a nano topological space, then $N_{\delta\beta}(U, X)$ is $P(U)$.*

Proof. Let $U_R(X) = U$.

1. If $L_R(X) = \phi$, then $B_R(X) = U$ and $\tau_R(X) = \{U, \phi\}$. Hence, $ncl(nint(ncl^\delta(A))) = U \ \forall A \subseteq U$. So, $A \subseteq ncl(nint(ncl^\delta(A))) \ \forall A \subseteq U$. Therefore, A is nano $\delta\beta$-open in U. Hence, $N_{\delta\beta}(U, X)$ is $P(U)$.

2. If $U_R(X) = U, L_R(X) \neq \phi$, then $\tau_R(X) = \{U, \phi, L_R(X), [L_R(X)]^{'}\}$.

   - $A \subseteq L_R(X)$, then $ncl^\delta(A) = L_R(X)$, and $ncl(nint(ncl^\delta(A))) = L_R(X)$. Therefore, A is nano $\delta\beta$-open in U.
   - $A \subseteq [L_R(X)]^{'}$, then $ncl^\delta(A) = [L_R(X)]^{'}$ and $ncl(nint(ncl^\delta(A))) = [L_R(X)]^{'}$. Therefore, A is nano $\delta\beta$-open in U.
   - If $A \cap L_R(X) \neq \phi$ and $A \cap [L_R(X)]^{'} \neq \phi$, then $ncl^\delta(A) = U, ncl(nint(ncl^\delta(A))) = U$. Therefore, A is nano $\delta\beta$-open in U.

Proposition 4.2. *If $U_R(X) \neq U$, and ($L_R(X) = \phi$ or $L_R(X) = U_R(X)$) in a nano topological space, then $N_{\delta\beta}(U, X)$ is $P(U)$.*

Proof. Let $U_R(X) \neq U$, and ($L_R(X) = \phi$ or $L_R(X) = U_R(X)$). In both cases $\tau_R(X) = \{U, \phi, U_R(X)\}$, and hence $ncl(nint(ncl^\delta(A))) = U \ \forall A \subseteq U$. Therefore, $N_{\delta\beta}(U, X)$ is $P(U)$.

Proposition 4.3. *If $U_R(X) \neq U, L_R(X) \neq \phi$ in a nano topological space, then U, ϕ and any set which intersects $U_R(X)$ are nano $\delta\beta$-open set in U.*

Proof. Let $U_R(X) \neq U$, $L_R(X) \neq \phi$. Then, $\tau_R(X) = \{U, L_R(X), U_R(X), B_R(X)\}$.

(1) If $A \subseteq U_R(X)$.

- If $A \subseteq L_R(X)$, then $ncl^\delta(A) = L_R(X), ncl(nint(ncl^\delta(A))) = [B_R(X)]'$ hence $A \subseteq L_R(X) \subseteq [B_R(X)]' = ncl(nint(ncl^\delta(A)))$. Therefore, A is nano $\delta\beta$-open in U.
- If $A \subseteq B_R(X)$, then $ncl^\delta(A) = B_R(X), ncl(nint(ncl^\delta(A))) = [L_R(X)]'$ hence $A \subseteq B_R(X) \subseteq [LR(X)]' = ncl(nint(ncl^\delta(A)))$. Therefore, A is nano $\delta\beta$-open in U.
- If $A \cap L_R(X) \neq \phi$ and $A \cap B_R(X) \neq \phi$, then $ncl^\delta(A) = U, ncl(nint(ncl^\delta(A))) = U$. Therefore, A is nano $\delta\beta$-open in U.

(2) If $A \subseteq [U_R(X)]'$, then $ncl^\delta(A) = [U_R(X)]', ncl(nint(ncl^\delta(A))) = \phi$. Therefore, A is not nano $\delta\beta$-open in U.

(3) If $A \cap U_R(X) \neq \phi$ and $A \cap [U_R(X)]' \neq \phi$.

- If $A \cap L_R(X) \neq \phi$ and $A \cap [U_R(X)]' \neq \phi$, then $ncl^\delta(A) = [B_R(X)]', nint(ncl^\delta(A)) = L_R(X)$, $ncl(nint(ncl^\delta(A))) = [B_R(X)]' = [U_R(X)]' \cup L_R(X) \supseteq A$. Therefore, A is nano $\delta\beta$-open in U.
- If $A \cap B_R(X) \neq \phi$ and $A \cap [U_R(X)]' \neq \phi$, then $ncl^\delta(A) = [L_R(X)]', nint(ncl^\delta(A)) = B_R(X), ncl(nint(ncl^\delta(A))) = [L_R(X)]' = [U_R(X)]' \cup B_R(X) \supseteq A$. Therefore, A is nano $\delta\beta$-open in U.
- If $A \cap L_R(X) \neq \phi, A \cap B_R(X) \neq \phi$ and $A \cap [U_R(X)]' \neq \phi$, then $ncl^\delta(A) = U$ hence $ncl(nint(ncl^\delta(A))) = U$. Therefore, A is nano $\delta\beta$-open in U.

5 Nano $\delta\beta$-continuity

The notion of nano $\delta\beta$-continues functions is introduced in this section. It is a generalization of the other types of nano near continues functions [3–6]. The main properties and characterizations of this new type of nano near continues functions are studied. Many important result are also investigated in this section.

Definition 5.1. *Let $(U, \tau_R(X))$ and $(V, \tau^*_{R'}(Y))$ be nano topological spaces. A mapping $f : (U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ is said to be a nano $\delta\beta$-continuous function if $f^{-1}(B)$ is a nano $\delta\beta$-open set in U for every a nano open set B in V.*

The relationships between nano β-continuous and nano $\delta\beta$-continuous functions are clearly by the following remark.

Remark 5.1. *Every nano β-continuous is nano $\delta\beta$-continuous.*

The converse of Remark 5.1 is not necessarily true as shown in the following example.

Example 5.1. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Then, $\tau_R(X) = \{U, \phi, \{a, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}, Y = \{x, y\}$. Then, $\tau^*_R(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define $f : U \to V$ as $f(a) = f(b) = x, f(c) = z, f(d) = w$, then f is a nano $\delta\beta$-continuous, but it is not nano β-continuous.*

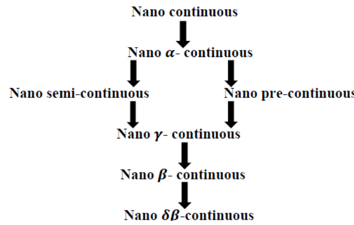

Figure 5: The relationships between the different types of nano near continuous functions.

It should be noted that from Remark 5.1 and Figure 2 we present Figure 5 which shows that the current Definition 5.1 is a generalization of the previous Definition 2.6 in [3–6].

Definition 5.2. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The nano $\delta\beta$-closure of a set A, denoted by $ncl^{\delta\beta}(A)$, is the intersection of nano $\delta\beta$-closed sets including A. The nano $\delta\beta$-interior of a set A, denoted by $nint^{\delta\beta}(A)$, is the union of nano $\delta\beta$-open sets included in A.*

The following theorem presents the main properties of nano $\delta\beta$-closure and nano $\delta\beta$-interior which are required in the sequel to study the properties of nano $\delta\beta$-continuous function.

Theorem 5.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A, B \subseteq U$. Then the following properties hold:*

1. *$ncl^{\delta\beta}(A)$ is a nano $\delta\beta$-closed set and $nint^{\delta\beta}(A)$ is a nano $\delta\beta$-open set.*
2. *$A \subseteq ncl^{\delta\beta}(A)$.*
3. *$A = ncl^{\delta\beta}(A)$ iff A is a nano $\delta\beta$-closed set.*
4. *$nint^{\delta\beta}(A) \subseteq A$.*
5. *$nint^{\delta\beta}(A) = A$ iff A is a nano $\delta\beta$-open set.*
6. *If $A \subseteq B$, then $ncl^{\delta\beta}(A) \subseteq ncl^{\delta\beta}(B)$ and $nint^{\delta\beta}(A) \subseteq nint^{\delta\beta}(B)$.*
7. *$nint^{\delta\beta}(A) \cup nint^{\delta\beta}(B) \subseteq nint^{\delta\beta}(A \cup B)$.*
8. *$ncl^{\delta\beta}(A \cap B) \subseteq ncl^{\delta\beta}(A) \cap ncl^{\delta\beta}(B)$.*

Proof. Obvious.

Remark 5.2. *Example 3.2 shows that*

1. *the inclusion in Theorem 5.1 parts 2, 4, 7 and 8 can not be replaced by equality relation:*
   - **(i)** *for part 2, if $A = \{a, b, c\}, ncl^{\delta\beta}(A) = U$, then $ncl^{\delta\beta}(A) \nsubseteq A$.*
   - **(ii)** *for part 4, if $A = \{d\}, nint^{\delta\beta}(A) = \phi$, then $A \nsubseteq nint^{\delta\beta}(A)$.*
   - **(iii)** *for part 7, if $A = \{a, b\}, B = \{d\}, A \cup B = \{a, b, d\}, nint^{\delta\beta}(A) = A, nint^{\delta\beta}(B) = \phi, nint^{\delta\beta}(A \cup B) = A \cup B$, then $nint^{\delta\beta}(A \cup B) = \{a, b, d\} \nsubseteq \{a, b\} = nint^{\delta\beta}(A) \cup nint^{\delta\beta}(B)$.*
   - **(iii)** *for part 8, if $A = \{d\}, B = \{a, b, c\}, A \cap B = \phi, ncl^{\delta\beta}(A) = A, nclt^{\delta\beta}(B) = U, nint^{\delta\beta}(A \cap B) = \phi$, then $ncl^{\delta\beta}(A) \cap ncl^{\delta\beta}(B) = \{d\} \nsubseteq ncl^{\delta\beta}(A \cap B) = \phi$.*
2. *the converse of part 6 is not necessarily true:*
   - **(i)** *if $A = \{d\}, B = \{a, b, c\}$, then $ncl^{\delta\beta}(A) = A, ncl^{\delta\beta}(B) = U$. Therefore, $ncl^{\delta\beta}(A) \subseteq ncl^{\delta\beta}(B)$, but $A \nsubseteq B$.*

(ii) *if* $A = \{d\}, B = \{b, c\}$, *then* $nint^{\delta\beta}(A) = \phi, nint^{\delta\beta}(B) = B$. *Therefore,* $nint^{\delta\beta}(A) \subseteq nint^{\delta\beta}(B)$, *but* $A \nsubseteq B$.

Theorem 5.2. *Let* $(U, \tau_R(X))$ *and* $(V, \tau^*_{R'}(Y))$ *be nano topological spaces and let* $f : (U, \tau_R(X)) \to (V, \tau^*_{R'}(Y))$ *be a mapping. Then, the following statements are equivalent:*

1. f *is nano* $\delta\beta$*-continuous.*
2. *The inverse image of every nano closed set* G *in* V *is nano* $\delta\beta$*-closed in* U.
3. $f(ncl^{\delta\beta}(A)) \subseteq ncl(f(A)), \forall\, A \subseteq U$.
4. $ncl^{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(ncl(F)), \forall\, F \subseteq V$.
5. $f^{-1}(nint(F)) \subseteq nint^{\delta\beta}(f^{-1}(F)), \forall\, F \subseteq V$.

Proof.

1. (1) $\Rightarrow$ (2) Let f be nano $\delta\beta$-continuous and let G be a nano closed set in V. Then, $V - G$ is nano open in V. Since, f is nano $\delta\beta$-continuous. Then, $f^{-1}(V - G)$ is nano $\delta\beta$-open in U. Then, $f^{-1}(V - G) = U - f^{-1}(G)$ and hence $f^{-1}(G)$ is nano $\delta\beta$-closed set in U.

2. (2) $\Rightarrow$ (1) Let A be a nano open set in V. Then, $f^{-1}(V - A)$ is nano $\delta\beta$-closed in U. Then, $f^{-1}(A)$ is nano $\delta\beta$-open in U. Therefore, f is nano $\delta\beta$-continuous.

3. (1) $\Rightarrow$ (3) Let f be nano $\delta\beta$-continuous and let $A \subseteq U$. Since, f is nano $\delta\beta$-continuous and $ncl(f(A))$ is nano closed in $V, f^{-1}(ncl(f(A)))$ is nano $\delta\beta$-closed in U. Since, $f(A) \subseteq ncl(f(A)), f^{-1}(f(A)) \subseteq f^{-1}(ncl(f(A)))$, then $ncl^{\delta\beta}(A) \subseteq ncl^{\delta\beta}[f^{-1}(ncl(f(A)))] = f^{-1}(ncl(f(A)))$. Thus, $ncl^{\delta\beta}(A) \subseteq f^{-1}(ncl(f(A)))$. Therefore, $f(ncl^{\delta\beta}(f(A))) \subseteq ncl(f(A)), \forall\, A \subseteq U$.

4. (3) $\Rightarrow$ (1) Let $f(ncl^{\delta\beta}(A)) \subseteq ncl(f(A)), \forall\, A \subseteq U$ and let F be nano closed in V. Then, $f^{-1}(F) \subseteq U$. Thus, $f(ncl^{\delta\beta}(f^{-1}(F))) \subseteq ncl(f(f^{-1}(F))) \subseteq ncl(F) = F$ that is $ncl^{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(f(ncl^{\delta\beta}(f^{-1}(F)))) \subseteq f^{-1}(F)$. Thus, $ncl^{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(F)$, but $f^{-1}(F) \subseteq ncl^{\delta\beta}(f^{-1}(F))$. Hence, $ncl^{\delta\beta}(f^{-1}(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is nano $\delta\beta$-closed in U and hence f is nano $\delta\beta$-continuous.

5. (1) $\Rightarrow$ (4) Let f be nano $\delta\beta$-continuous and let $F \subseteq V$. Since, $F \subseteq ncl(F)$, then $f^{-1}(F) \subseteq f^{-1}(ncl(F))$ and hence $ncl^{\delta\beta}(f^{-1}(F)) \subseteq ncl^{\delta\beta}(f^{-1}(ncl(F))) = f^{-1}(ncl(F))$ as $ncl(F)$ is nano closed in V and f is nano $\delta\beta$-continuous. Thus, $ncl^{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(ncl(F))$.

6. (4) $\Rightarrow$ (1) Let $ncl^{\delta\beta}(f^{-1}(F)) \subseteq f^{-1}(ncl(F)), \forall\, F \subseteq V$ and let G be nano closed in V. Then, $ncl^{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}(ncl(G)) = f^{-1}(G)$. Hence, $ncl^{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}(G)$, but $f^{-1}(G) \subseteq ncl^{\delta\beta}(f^{-1}(G))$. Therefore, $ncl^{\delta\beta}(f^{-1}(G)) = f^{-1}(G)$ and hence f is nano $\delta\beta$-continuous.

7. (1) $\Rightarrow$ (5) Let f be nano $\delta\beta$-continuous and let $F \subseteq V$. Since, $nint(F)$ is nano open in V, then $f^{-1}(nint(F))$ is nano $\delta\beta$-open in U. Therefore, $nint^{\delta\beta}[f^{-1}(nint(F))] = f^{-1}(nint(F))$. Also, $nint(F) \subseteq F$ implies that $f^{-1}(nint(F)) \subseteq f^{-1}(F)$. Therefore, $nint^{\delta\beta}(f^{-1}(nint(F))) \subseteq nint^{\delta\beta}(f^{-1}(F))$. That is, $f^{-1}(nint(F)) \subseteq nint^{\delta\beta}(f^{-1}(F))$.

8. (5) $\Rightarrow$ (1) Let $f^{-1}(nint(F)) \subseteq nint^{\delta\beta}(f^{-1}(F))$, for every subset F of V and let G be nano open in V, then $nint(G) = G$. By assumption, $f^{-1}(nint(G)) \subseteq nint^{\delta\beta}(f^{-1}(G))$. Thus, $f^{-1}(G) \subseteq nint^{\delta\beta}(f^{-1}(G))$. But $nint^{\delta\beta}(f^{-1}(G)) \subseteq f^{-1}(G)$. Therefore, $f^{-1}(G) = nint^{\delta\beta}(f^{-1}(G))$ and hence f is nano $\delta\beta$-continuous.

Remark 5.3. *In Theorem 5.2 the equality of parts 3, 4 and 5 does not hold in general as shown in Example 5.1 that:*

1. *for part 3, take* $A = \{b\}$, *then* $f(ncl^{\delta\beta}(A)) = A \neq ncl(f(A)) = V$,
2. *for part 4, take* $F = \{x\}$, *then* $ncl^{\delta\beta}(f^{-1}(F)) = \{a, b\} \neq f^{-1}(ncl(F)) = V$,
3. *for part 5, take* $F = \{z\}$, *then* $f^{-1}(nint(F)) = \phi \neq nint^{\delta\beta}(f^{-1}(F)) = \{c\}$.

6 Conclusions

This paper presented a study of new structure in nano topology which was nano $\delta\beta$-open sets. These sets were generalized the usual notions of nano near open sets [1, 2]. Since, the class of nano $\delta\beta$-open sets was stronger than any type of the previous classes of nano near open sets such as, nano regular open, nano α-open, nano semi-open, nano pre-open, nano γ-open and nano β-open, etc. Several topological characterizations and properties of the current new sort of sets were studied. Additionally, the concepts of nano near continues functions were extended to nano $\delta\beta$-continues functions. It was showed that every nano β-continues function was nano $\delta\beta$-continue functions. Therefore, the nano $\delta\beta$-continues functions were generalization of the other types of nano near continues functions [3–6].

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