Arab J. Nucl. Sci. Appl., Vol. 55, 1, 62-69 (2022)

ISSN 1110-0451

Arab Journal of Nuclear Sciences and Applications

Web site: ajnsa.journals.ekb.eg

(E S N S A)

Generalizing the Liquid Drop Model (LDM) to Assess $\left(Q_{\beta}^{+}-value\right)$ for Nuclei in the Range of $10 \leq Z \leq 98$

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ARTICLE INFO

Article history:

Received: 5th May 2021

Accepted: 1st Sept. 2021

Keywords:

$Q_{\beta}^{+}-value$,

Binding Energy,

Liquid Drop Model,

Beta Decay,

Magic Number.

ABSTRACT

In this research, a new formula of positive beta decay energy $(Q_{\beta}^{+}-value)$ was obtained for light, medium and heavy nuclei (even – even, even – odd, odd – even and odd – odd) for the range of nuclei $(10 \leq Z \leq 98)$. Instead of mass differences values, this formula is expressed in the form of nuclear binding energies. After multiple mathematical derivations, the decay energy is defined using the liquid drop model (LDM) to arrive at a final equation for the positive beta decay energy. The $(Q_{\beta}^{+}-value)$ was calculated using four terms: the first term represents the symmetry energy with the opposite sign as in the liquid drop model, the second term represents the Coulomb repulsion energy, and the other two terms represent the pairing and constant (C) terms, where (C) is provided by the mass difference between the neutron and proton. In the investigation of the proposed novel formula of $(Q_{\beta}^{+}-value)$,the findings showed a good match between experimental and theoretical values, especially for heavy and medium nuclei, but a less match for light nuclei due to the presence of a magic numbers was observed. In the investigation of the novel $Q_{\beta}^{+}-value$ formula, the standard deviation (σ) was used to calculate the dependency of LDM.

1. INTRODUCTION

Beta-decay is one of the weaker reaction equations, but it is significant in nuclear and astrophysics research [1,2]. The decay half-life in nuclear physics is an efficient and effective sensing tool for nuclear composition, as it is mainly influenced by the value of the decay energy (Q) and the elements of the transition matrix, and is determined by the Fermi relationship [3,4]. Many theoretical studies have been centered around the half-life of decay, and this issue became an important topic for research within the past two decades. Many alternative models have been developed to assess the half-life of beta decay in addition to the global quasi-experimental models, and many of these are dubbed micro computations inside the reactive shell model [5-7], while others employ the random quasi-particle approximation. Between the decay energy and the half-life, the spectrum of nuclei located on the Valley of Stability has been closely studied to create an experimental law of beta decay[8]. Several researchers have looked into this extensively and have begun to characterize the energy of beta decay [9,10]. In recent decades, the liquid drop model has gained widespread acceptance in the field of nuclear physics as an optimal model for calculating nuclear binding energy. This idea was first proposed by George Gamow and then developed further by Niles Bohr and John Archibald [11].Within part of a previous work by the authors, employing (LDM) in determining the $(Q_{\alpha}-value)$ of heavy and super heavy nuclei was investigated, as well as proposing the path formation hypothesis [12,13].The investigation was carried out using (LDM) to determine the $(Q_{\alpha}-value)$ of heavy and super heavy nuclei and the path forming hypothesis was proposed as part of our previous work. By using the (LDM) for the $(2 \leq Z \leq 97)$, a new formula for negative beta decay $\left(Q_{\beta}^{-}-value\right)$ has been discovered[14].The Quark-like model was updated in another study to obtain an improved formula for $\left(Q_{\beta}^{-}-value\right)$ in the range $(2 \leq Z \leq 97)$ [15].This study aims to adjust the Liquid Drop model to determine the $\left(Q_{\beta}^{+}-value\right)$of medium, intermediate, and heavy nuclei in the $(10 \leq Z \leq 98)$ range.

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DOI: 10.21608/ajnsa.2021.75297.1471

2. THEORETICAL FRAMEWORK

2.2. The positive beta decay energy in terms of nuclear binding energies

The positive beta energy $(Q_\beta^+ - value)$ process can be determined from mathematical formula in terms of mass difference between parent and daughter nuclei[12]:

$$Q_\beta^+ = [M({}_Z^AX_N) - M({}_{Z-1}^{\;\;A}X_{N+1})]C^2 - 2m_eC^2 \quad (\mathbf{1})$$

where:

$M({}_Z^AX_N)$, $M({}_{Z-1}^{\;\;A}X_{N+1}) -$ represents the masses for parent and daughter nuclei, respectively

$m_e -$ represents the electron mass

$C^2 -$ represents the conversion factor $= 931.15\, MeV/amu$.

The atomic mass of the parent nuclei can be calculated as follows:

$$M(\ Z, A) = Zm_p + Nm_n\ - B(A,Z)C^2 \quad (\mathbf{2})$$

Moreover, for the daughter nuclei, the number of protons will decrease by one unit (Z-1) while the number of neutrons will increase by one unit, so the mass of the daughter nuclei becomes:

$$M(A, Z-1) = (Z-1)m_p + (N+1)m_n - B(A, Z\ \ -1)/C^2 \quad (\mathbf{3})$$

Where $B(A,Z)$, $B(A, Z-1)$ represent the nuclear binding energy of parent and daughter nuclei, respectively.

To obtain a new formula to determine the positive beta decay energy $Q_\beta^+ - value$ in terms of the nuclear binding energies, Eqs. (2) and (3) will be substituted in Eq. (1) , thus we obtain :

$$Q_\beta^+ = (Z-1)m_p + (N+1)m_n - B(A, Z-1)/C^2 - Zm_p + Nm_n - B(A,Z)/C^2$$

$$Q_\beta^+ = Zm_pC^2 + Nm_nC^2 - B(A,Z) - (Z-1)m_pC^2 - (N+1)m_nC^2 + B(A, Z-1) - 2m_eC^2$$

$$Q_\beta^+ = B(A, Z-1) - B(A,Z) - C \quad (\mathbf{4})$$

where

$$C = (m_p - m_n)C^2 - 2m_eC^2$$

$$(m_p - m_n)C^2 = -0.78246$$

$$2m_eC^2 = 1.022\, MeV$$

When the experimental values of the nuclear binding energies of parent and daughter nuclei were substituted, the final formula to calculate the positive beta decay energy in terms of nuclear binding energies of parent and daughter nuclei yielded a perfect result.

2. 3. Utilization of The Liquid Drop Model (LDM) to Calculate the $(\boldsymbol{Q_\beta^+ - value})$:

The liquid drop model is a well-known and effective nuclear model for evaluating a variety of nuclear properties (the most important of which is the nuclear binding energy). Thus, this model will be used to measure the positive beta decay energy $Q_\beta^+ - value)$ using the semi-empirical approximation for one of the shapes of the liquid drop model for the (even – even, even – odd, odd-even, odd – odd nuclei [16], and the final formula to calculate the positive beta decay energy $Q_\beta^+ - value)$ $could\ be\ obtained\ as\ followsw$:

$$B(A,Z) = c_vA - c_aA^{\frac{2}{3}} - a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}} - c_s\frac{(N-Z)^2}{A} \mp \delta \quad (\mathbf{5})$$

Where

$c_v -$ represents the volume constant which is equal to 15.835 MeV

$c_a -$ represents the surface constant which is equal to 16.8 MeV

$a_c -$ represents the coulomb constant which is equal to 0.703 MeV

$c_s -$ represents the symmetry constant which is equal to 24.8 MeV

$\delta -$ represents the parity term which is equal to [17] :

$$\delta = a_p A^{\frac{-1}{2}} = \begin{cases} +12\ MeV\ A^{\frac{-1}{2}} & even - even\ nuclei \\ 0 & odd - A\ nuclei \\ -12\ MeV\ A^{\frac{-1}{2}} & odd - odd\ nuclei \end{cases}$$

To derive an equation to determine the positive beta decay energy, Eq. (5) represents the binding energy of parent nuclei, while the binding energy of daughter nuclei will be written as:

$$B(A, Z-1) = c_v A - c_a A^{\frac{2}{3}} - a_c \frac{Z-1(Z-2)}{A^{\frac{1}{3}}} - c_s \frac{(N-Z+2)^2}{A} \pm \delta \quad (6)$$

By substituting the nuclear binding energies of parent and daughter nuclei according to the liquid drop model in Eq. (4) the following relation will be obtained:

$$Q_\beta^+ = c_v A - c_a A^{\frac{2}{3}} - a_c \frac{Z-1(Z-2)}{A^{\frac{1}{3}}} - c_s \frac{(N-Z+2)^2}{A} \pm \delta - \left[c_v A - c_a A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - c_s \frac{(N-Z)^2}{A} \mp \delta\right] - C$$

To find the (Q_β^+), mathematical sequential steps should be performed:

$$Q_\beta^+ = c_v A - c_a A^{\frac{2}{3}} - a_c \frac{Z-1(Z-2)}{A^{\frac{1}{3}}} - c_s \frac{(N-Z+2)^2}{A} \pm \delta - c_v A + c_a A^{\frac{2}{3}} + a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} + c_s \frac{(N-Z)^2}{A} \pm \delta - C$$

$$Q_\beta^+ = a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_c \frac{Z-1(Z-2)}{A^{\frac{1}{3}}} + c_s \frac{(N-Z)^2}{A} - c_s \frac{(N-Z+2)^2}{A} \pm 2\delta - C$$

$$Q_\beta^+ = a_c \frac{Z^2-Z-(Z^2-2Z-Z+2)}{A^{\frac{1}{3}}} + c_s \frac{(N-Z)^2}{A} - c_s \frac{(N-Z+2)^2}{A} \pm 2\delta - C$$

$$Q_\beta^+ = a_c \frac{Z^2 - Z - Z^2 + 2Z + Z - 2}{A^{\frac{1}{3}}} + c_s \frac{N^2 - 2NZ + Z^2}{A} - c_s \frac{(N-Z+2)(N-Z+2)}{A} \pm 2\delta - C$$

$$Q_\beta^+ = a_c \frac{2Z-2}{A^{\frac{1}{3}}} + c_s \frac{N^2-2NZ+Z^2}{A} - c_s \frac{N^2-NZ+2N-ZN+Z^2-2Z+2N-2Z+4}{A} \pm 2\delta - C$$

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} + c_s \frac{N^2-2NZ+Z^2}{A} - c_s \frac{N^2-2NZ+4N-4Z+Z^2+4}{A} \pm 2\delta - C$$

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} + c_s \frac{N^2-2NZ+Z^2-N^2+2NZ-4N+4Z-Z^2-4}{A} \pm 2\delta - C$$

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} + c_s \frac{-4N+4Z-4}{A} \pm 2\delta - C$$

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} + c_s \frac{-4(N-Z+1)}{A} \pm 2\delta - C$$

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} - 4c_s \frac{N-Z+1}{A} \pm 2\delta - C$$

The last equation can be also written as:

$$Q_\beta^+ = 2a_c \frac{Z-1}{A^{\frac{1}{3}}} - 4c_s \frac{N-Z+1}{A} \pm 2a_p A^{\frac{-1}{2}} - C \quad (7)$$

Equation (7) represents the final formula to determine the positive beta decay energy $(Q_\beta^+ - value)$ according to the liquid drop model, and the pairing energy of the last formula will be written as follows:

$$\delta = a_p A^{\frac{-1}{2}} = \begin{cases} -12\ MeV\ A^{\frac{-1}{2}} & even - even\ nuclei \\ 0 & odd - A\ nuclei \\ +12\ MeV\ A^{\frac{-1}{2}} & odd - odd\ nuclei \end{cases}$$

If we substitute the values of $a_c = 0.71\ MeV$, $c_s = 23.21\ MeV$, $a_p = 12\ MeV$, $C = 1.80446\ MeV$ in Eq. (7) we will obtain:

$$Q_\beta^+ = 1.42\ \frac{Z-1}{A^{\frac{1}{3}}} - 92.84\ \frac{N-Z+1}{A} \pm 24\ A^{\frac{-1}{2}} - 1.80446 \quad (8)$$

When we applied the above equation on all studied nuclei, it showed an acceptable match between the experimental and theoretical values with acceptable standard deviation. The equation (8) we will be denoted as modified liquid drop model (MLDM).

2.4. Determining the Standard Deviation for the Proposed Model:

The standard deviation was calculated in order to determine the accuracy of equation (8) in estimating the $(Q_\beta^+ - value)$[2].

$$\delta = \sum_{i=1}^{N} \frac{\left|Q_{\beta}^{+}exp.-Q_{\beta}^{+}theo.\right|}{N} \qquad (9)$$

where:

$Q_{\beta}^{+}exp.-$ represent the experimental value

$Q_{\beta}^{+}theo.-$ represent the theoretical value

3. RESULTS AND DISCUSSION

Figure 1 illustrates the disparity between experimental and theoretical positive beta decay energy values with a mass number according to the Liquid Drop Model (LDM) for all nuclei tested.

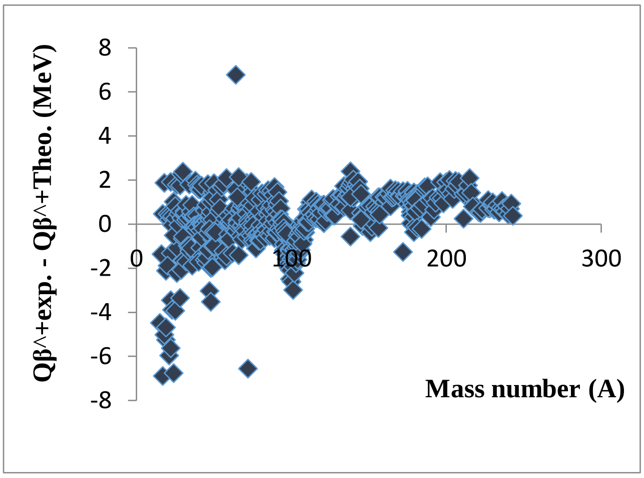

Fig. (1): The difference between experimental and theoretical positive beta decay energy values versus the mass number (A) according to the (LDM).

It should be noted that as the difference between the experimental and theoretical values of positive beta decay was close to zero, this would make the model closer to be adopted. We noted in the (MLDM) that all the differences between experimental and theoretical values were centered around zero, especially in the range $(A \geq 99, Z = 45 - 98)$, i.e. for the medium and heavy nucleus, while for the light nuclei the difference between experimental and theoretical values will be increased, and this is due to the presence of nuclei possessing magic numbers (2, 8, 20, 28, 50) for protons, neutrons, or both. These nuclei have closed nuclear envelopes that make them more stable in comparison with adjacent nuclei; this increases their nuclear binding energy, and thus the values of the positive beta decay (Q_{β}^{+}) will drop in, where the nuclei that have a magic numbers will consume high energy to overcome its high nuclear binding energy in order to decay. It's also worth noting that the term representing symmetry energy in the liquid drop model is negatively signaled since it lowers nuclear binding energy, but it retains its negative sign even after derivation as in equation (8), which lowers positive beta decay energy. Coulomb energy, on the other hand, has a positive signal in the denoted equation; as a result, it would have a large effect for light nuclei, and its effect decreases with the increase in the number of protons rather than the number of neutrons, which makes the experimental and theoretical values more similar, leading to the adoption of the model in determining positive energy decay. Figure (2) illustrates the positive beta decay energy relationship to the mass number (A) according to (MLDM) for the studied nuclei in the range$10 \leq Z \leq 98$.

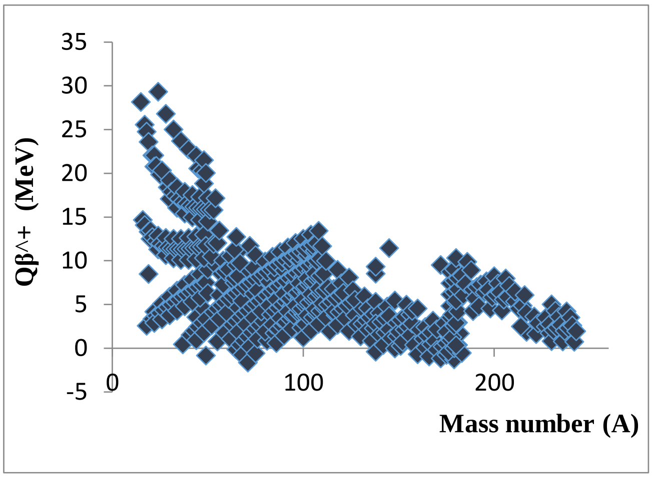

Fig. (2): The positive beta decay energy versus the mass number (A) according to the liquid drop model (LDM).

It was impossible to explain the decay energy as the mass number increased because the spectrum of positive beta decay energy is continuous. We can see that the positive beta decay energy increases as the mass number (A) increase for the same isotope, and then decreases as the mass number increases, as seen in Figure (2). Our findings contradict those of [18.19] , who investigated the half-lives of several isotopes that decay by negative beta particle and found that the half-life decreases as the mass number (A) increases, and because the relationship between half-life and decay energy is inverse (the fifth power law of $(T_{\frac{1}{2}} \propto Q_{\beta}^{-5})$) [20], it became clear that the positive decay behavior was the polar opposite. When we look at the two isotopes (Z=21,33), for example, we can see in Figures (3) and (4) that the decay energy varies depending on the type of isotope and the number of neutrons. Despite the rise in mass number, the decay energy decreases by moving from (o – o) isotope to (e – o) isotope. This is attributed to an increase in the nuclear binding energy of (e – o) isotopes, which reduces the decay energy of the isotope, allowing it to deplete a considerable portion of its energy to decay. This activity can be applied to all isotopes as well.

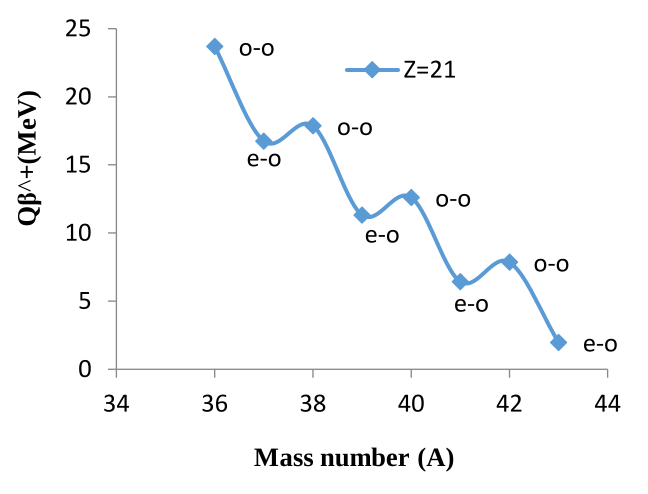

Fig. (3): The relationship of the positive beta decay energy versus the mass number (A) for the isotope (Z=21).

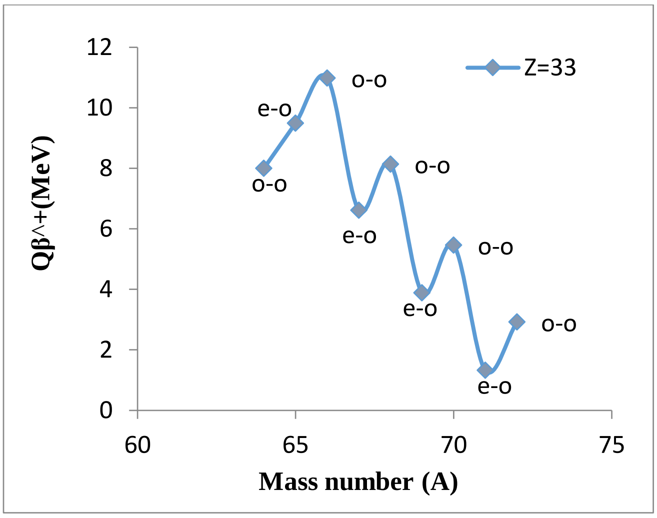

Fig. (4): The relationship of the positive beta decay energy versus the mass number (A) for the isotope (Z=33).

Figure 5 illustrates the general behavior of the positive beta decay energy with the number of neutrons, which indicates an exponential decrease in the number of neutrons for all studied nuclei, and this behavior is due to the same reason as above. It's worth noting that this behavior varies depending on the isotone form, but it's the opposite of what's described in the isotope behavior section. Figures 1 and 2 show this action (6 and 7).

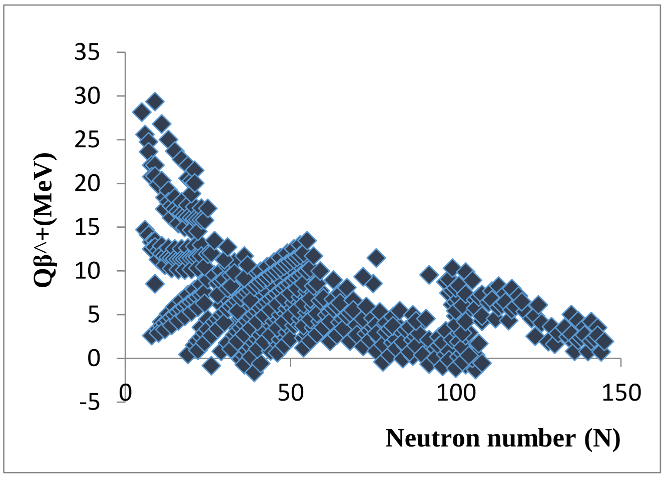

Fig. (5): The relationship of positive beta decay energy versus the number of neutrons(N).

Our findings are following those of [21],who obtained similar results from measurements of the half-life of beta decay for some isotones, showing that the half-life of beta decay increases with increasing mass number, corresponding to the inverse relationship between decay energy and half-life. According to equations (9) of the (MLDM), the decrease in decay energy for any isotone with the mass number is due to an increase in repulsive Coulomb energy, which influences the decay energy of the isotone in terms of its reduction, affecting the stability of that nucleus. As the isotone is converted from (o – o) to (o – e), as seen in Figures (6) and (7), the decay energy decreases marginally amid the rise in mass number since the nuclear binding energy of the (o - o) type element is less than that of the (o – e) type element, which is expressed by decreasing the energy of positive beta decay.

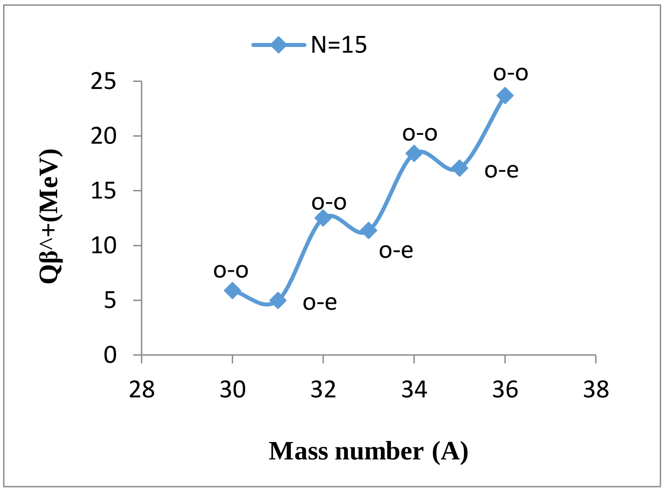

Fig. (6): Positive beta decay energy relationship versus the mass number (A) for isotone (N=15).

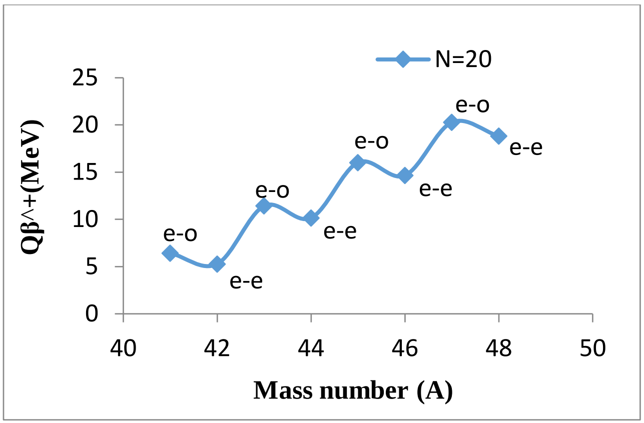

Fig. (7): Positive beta decay energy relationship versus the mass number (A) for isotone (N=20).

In order to discuss the decay energy $(Q_{\beta}^{+} - value)$ and its significant effect on the half-life we take, for example, the isotope of ${}^{64}_{31}Ga$ and ${}^{67}_{31}Ga$ with half-lives of $(2.6\,m)$ and $(4698\,m)$ respectively, also that decay energy for the first nucleus $(7.17\,MeV)$ and for the second nucleus $(1\,MeV)$. In other words, decreasing the energy decay Q_{β}^{+} will increase the half-live by a factor of 1800 times, which would clearly demonstrate the importance of the decay energy value to determine the half-life with an extreme accuracy. This suggests the nuclei that emit high-energy beta particles have brief half-lives, and vice versa. Based on the topic above, accurately determining the beta decay energy leads to an accurate estimation of the half-life, which justifies the significant scientific feasibility of performing these types of studies, especially given the difficulties of obtaining a mathematical model that calculates the beta decay energy with a small standard deviation.

Table (1), shows the standard deviation (δ) of the $(Q_{\beta}^{+} - value)$ according to the (MLDM) for all types of studied nuclei, in order to know the validity, accuracy and compatibility of the experimental and theoretical values results of the beta decay energy for the studied nuclei. Whenever the standard deviation is lower and closer to zero, the model becomes more adoptable to calculate the positive beta decay.

Table (1): the standard deviation of the positive beta decay energy values for light, medium, and heavy nuclei, respectively, for the (LDM).

Standard deviation	$10 \le Z \le 98$	$10 \le Z \le 40$	$41 \le Z \le 98$
δ	0.99	1.11	0.89

The standard deviation values indicate the possibility of adopting the (LDM) in determining the$(Q_{\beta}^{+} - value)$, especially in medium and heavy nuclei. The findings are acceptable, as seen in the table above since they can be depended upon to find the positive beta decay energy. Because of the difficulty of the continuous beta continuum, where beta particles share between a neutrino and daughter nuclei, getting stronger results of (δ) less than 0.99 is extremely difficult. The beta decay energy differs from the alpha decay energy in that the distribution is linear, as well as the fact that the alpha particle itself carries 98 % of the decay energy. Figure 8 depicts agreed compatibility between the average nuclear binding energy of the experimental and theoretical nuclear binding energy of the (LDM).

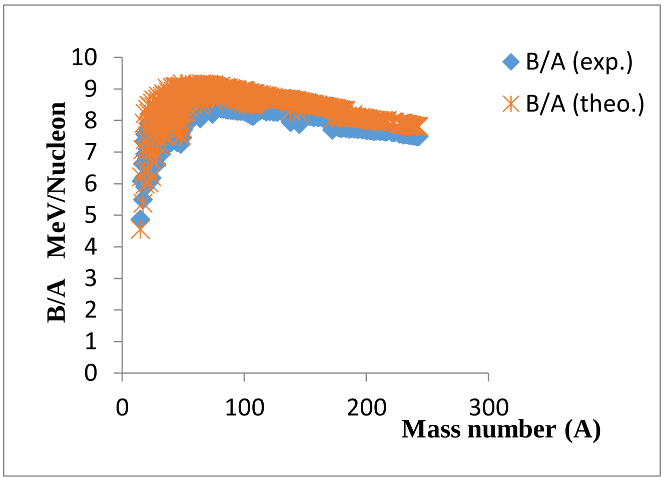

Fig. (8): Average nuclear binding energy versus mass number (A) for experimental and theoretical values of the (LDM).

It is clear that there is a substantial possibility of adopting it to determine the nuclear binding energy for all studied nuclei regardless of its type and their locations from the stability line, especially for medium and heavy nuclei. Table (2), shows the standard deviation (δ) for the average nuclear binding energy in the range of $(10 \le Z \le 98)$ according to the (LDM).

Table (2): shows standard deviation values for the average nuclear bonding in the range $(10 \le Z \le 98)$ for the (LDM).

The model	Standard deviation(δ)
LDM	0.38

The results indicated the accuracy of the new equation (8) in determining the $(Q_{\beta}^{+} - value)$ in terms of nuclear binding energy instead of masses difference as it is usual, thus, indicates high possibility of (LDM) in determining the nuclear binding energy for all studied nuclei.

4. CONCLUSION

- The results showed that there are acceptable discrepancies between the experimental and theoretical values of $(Q_{\beta}^{+} - value)$, especially in the range of medium and heavy nuclei, while these discrepancies increase in light nuclei, especially those that have closed shells or near them.
- The results of $(Q_{\beta}^{+} - value)$ showed that they depend on Z, N, and A except for the closed shells.
- The (MLDM) can be adopted and relied upon due to the wide range of studied nuclei of all kinds, with numbers exceeding 500 nuclei.
- The results showed that the positive beta decay energy decreases with the increase in the mass number of isotopes for the same element.
- The positive beta decay energy increased for isotones that have an equal number of neutrons (N).
- In general, the positive beta decay energy decreases with the increase in the number of neutrons (N) and the mass number (A).

Acknowledgements
We offer our thanks to the Department of Physics, College of Science, University of Mosul, for their continuous support in carrying out the research.

Declaration of conflicting Interests
The author(s) declared no potential conflicts of interest concerning the research, authorship, and/or
publication of this article.

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