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Original Article

Statistical analysis and performance comparison of improved and optimized CSC using different chaotic maps under different fading channels

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Abstract

Spread spectrum technique becomes one of the most important secure communication techniques in the recent decades. One of the most important parameter in spread spectrum is the spreading sequence, especially in the direct sequence spread spectrum. Due to the huge increase in the applications and the demanded users, the traditional spreading codes become not sufficient to fulfill the development requirements. As an alternative solution, the chaotic codes appeared to solve most problems of the traditional spreading codes. Recently, many researches focused on the chaotic codes, due to its attractive security properties, in addition to its availability to generate a huge number of spreading codes,which is very useful in the multi-access applications. However, as the number of the multiple access interference is increased, the performance of the chaotic codes is degraded. One of the degradation reasons is the assigning of the initial condition of the chaotic map. This paper presents a comprehensive statistical analysis of an improved and optimized chaotic sequence code. In addition to the traditional chaotic code; the analysis is performed over improved codes such as the self-balanced, the zero mean, and the self-balanced zero mean chaotic code. The spreading chaotic codes are generated from optimized maps with optimum initial conditions. The analysis consists of the balance, orthogonality, the normalized maximum autocorrelation side lobe, and the normalized average cross correlation property. In addition, the performance of the mentioned code is evaluated and compared with the Gold code for different code lengths over different fading channel.

I. INTRODUCTION

Since the main Federal Communication Commission (FCC)has been established, its basis in managing the spectrum allocations was depending on the request-by-request basis. As the communications applications are hugely increased, it has realized that it had no more spectra to allocate[1]. One of the available solutions of this problem is to make multiple applications share the same spectrum throughout the spread spectrum. The spread spectrum simply can be defined as the transmission way in which the signal occupies a bandwidth much wider than its original band width. This wider bandwidth is obtained by using certain spreading code completely independent on the message signal, and has much higher bit rate than the original signal. This spreading code is synchronized in the receiver to retrieve the original signal throughout the despreading process[2].

In direct-sequence spread spectrum (DS-SS) systems, the information signal is modulated by the spreading code prior to transmission, resulting in a wideband signal resistant to narrowband jamming, multipath, interception, in addition to providing the multiple-access capability. Typically, there are many codes types that can be used for the spreading process. The pseudo-noise (PN) sequences which are generally produced by the linear feedback shift register are the most popular codes used in the multiusers application. Although PN code has attractive properties, however it has also many drawbacks such as the limitation of the number of such sequences, in addition to its periodicity feature, which make the intercepted signal is predictable and is reconstructed by linear regression which leads to security limitations.

One of the alternative solutions is the chaotic code signals. Chaotic codes have a lot of attractive properties over the conventional PN sequences, especially from the security point of view. It is completely non-periodic, wide band, and more difficult to predict and to reconstruct. Moreover, Chaos is a deterministic, random-like process found in non-linear dynamical system, which is non-converging and bounded[3]. It has also a very sensitive dependence upon its initial condition and parameters, which gives the availability to generate an infinite number of spreading codes[4]. These properties make chaotic sequences more difficult to intercept, and more secured to decode the information spreaded upon them[5]. In recent decades, many researchers have studied and discussed the properties of the chaotic codes from different point of views. The correlation properties is one of the most important point of views that has been studied and discussed in detail specially the auto-correlation and the cross-correlation properties, since it has a direct effect on the performance of the multi-access systems. These studies lead to create many types of maps that are used to generate the chaotic sequences, such as logistic map, tent map; Gauss iterated map, Gingerbread man map, and Henon map[6]. All these chaotic maps are depending on the same principle which is assign an initial value to generate the chaotic sequence. Since not all of the chaotic maps used in generating the chaotic sequence can outperform the PN performance, it was necessary to study and analyses the different properties of the different chaotic maps under different conditions to clarify which maps are suitable to generate a spreading code with good properties and which map is not suitable. One of the most important parameter that has aneffect on the performance of the chaotic code is the initial value.

This paper introduces a statistical study and analysis of different optimized chaotic maps; with different improvement stages over different code lengths, and under effect different fading channel types. The initial values of the generated codes are optimized according to the rule of minimizing the mean bit error rate (BER) of the used code under certain conditions.

The study presents the optimum initial value for the expressed codes in case of the additive wide Gaussian noise(AWGN), Raleigh fading, and frequency selective fading channels. The applied chaotic maps in this paper are the logistic map and the tent map. The study consists of the traditional code generated directly from the chaotic map, in addition to improvement stages, which are the self-balanced (SB) version, the zero mean (ZM) versions, and the self-balanced zero mean (SBZM) version. The studied properties are the balance property, the orthogonality property, the autocorrelation side lobe, and the cross correlation. The analysis is performed over 7 different lengths starting from length 32 up to 2048 according to the relation2n. In addition; a performance comparison of the presented chaotic codes is evaluated and compared with the Gold code under differentmultiple access interference(MAI) over Raleigh fading channel, and frequency selective fading channel.

The paper is organized as follows; section 2 discusses the study the statistical analysis of the presented chaotic codes with the different improvement stages. Section 3 presents the performance evaluation and simulation comparison of the presented codes with the Gold Code for selected code lengths. Finally, the conclusion is presented in section 4.

STATISTICAL ANALYSIS

In this section, the characteristics of the chaotic spreading codes generated from the logistic chaotic map and tent map are analyzed, discussed, compared and tabulated throughout the different stages under the same conditions. The characteristics to be studied and analyzed are the optimum initial value, balance property, the orthogonality property, the normalized maximum autocorrelation side lobe (NMACSL) property, and the normalized average cross correlation (NACC) property. The statistical study is performed between the traditional raw code, the ZM code, the SB code, and the ZMSB code. The analysis is performed in presence of the AWGN; relight fading and frequency selective fading channels. The relight fading channel is simulated with 5 paths has the following gain [0.8321 0.2774 0.2774 0.2774 0.2774], with the corresponding paths delays [0 20 40 60 80] μsec respectively. The frequency selective fading channel simulation is performed using also the same 5 different paths with the same gain of the flat fading channel, but the paths delays are adjusted to be [0 1.2 1.4 1.6 1.8] *msec*, respectively. The simulation is performed using Monte Carlo method with 105 symbols, data rate 104 bit/sec, and 100 independent trials. The statisticalresults areaveraged over 100 different codes, while the performance comparison isperformed in presence of 10 MAI for all the stages and code lengths to simplify the simulation and decreasing the parameter variation.

Generally, the logistic map can be represented by the following function represented in (1):

$$F(x,r) = rx(1-x) \tag{1}$$

Or, can be rewritten in its recursive form as:

$$x_{n+1} = rx_n(1-x_n)\,, \quad 0 \le x_n \le 1\;, 0 \le r \le 4 \tag{2}$$

Where, F is the transformation mapping function, r is called the bifurcation parameter.

Regarding the tent map, the state space description of the first order can be represented in (3) as:

$$x_{n+1} = a - b|x_n - c| \equiv F(x_n) \tag{3}$$

Where a,b, and c are constants, such that $a \ge 1$, $1.5 \le b \le 2$, and $c \le 1$. The analysis is performed over 7 different code lengths starting with 32, and ends with 2048. Although the analysis can be performed over any code length, however it is constrained with the rule 2^n to make the comparison with the Gold code faire, since the length of the Gold code is restricted to this length. As mentioned before, the results are averaged over 100 different codes, with optimized initial value, whereas the value of r is set to be 3.99.

Since the output of any chaotic map is real values, a transformation method is needed to map the real values into binary values. Actually, there are various methods can be used to map the real values of the chaotic code into the binary values such as digitization method, threshold sequence, and the zero mean method[7]. In the case of the balance property, the second method which is the threshold sequence is suitable. To represent this method mathematically, let w be the real valued chaotic sequence, for transforming this real valued sequence to binary sequence the threshold function θ_t (w) can be defined as

$$\theta_t(w) = \begin{cases} 1 \ , & w \geq t \\ 0 \ , & w < t \end{cases} \quad (4)$$

Where t is the threshold value. By using these functions, it can be obtained a binary sequence which is referred as a chaotic threshold sequence. The output of these digital values is unbalanced, which mean that the number of ones does not equal the number of zeroes. The self-balancing process is performed, in which the process is explained in detail in[8]. The process of making the number of ones exactly equal to the number of zeros, enhance the statistical properties in some cases. Its idea is briefly depends on four main steps which are, inversion, all upside down, radix-S block upside down, and finally the shift combination.

Regarding to the ZM, the zero mean method which can be represented by (5), can be used,

$$X_1 = sign(g\{x(t) - mean[x(t)]\}) \quad (5)$$

This method depends on shifting the real value for the basic chaotic code by its mean value and generates a new basic sequence g(x) has a zero mean. The binary values will be obtained by taking the sign function of the new zero mean sequence g(x).

A. Optimum Initial Value:

Tables I and II represent the optimum initial values for both the logistic and tent maps. As shown in the tables, the values are optimized for code lengths from 32 to 2048, for AWGN, relight fading, and frequency selective fading channels. The optimization process is performed based on the criteria of minimum mean bit error rate (BER) under the same conditions with initial step value 0.01. Regarding to the logistic map, the following conclusion can be obtained from table 1:
(I) The results obtained from the AWGN channel shows that all the logistic stages (traditional, ZM, SB, ZMSB) have the same optimum initial value for the same code length, due to the channel simplicity.
(II) Considering the code stages of the AWGN channel, the optimum initial values have nearly identical or close values for all the lengths (0.12, 0.16, and 0.17) except for the length 127.
(III) Regarding to the Relight fading channels, it can be noted that both the traditional and the ZM code have nearly close initial values, especially for long code lengths.
(IV) The results obtained from the FSF channel is varying severely due to the channel complexity.

Table 1: Optimum initial value for the logistic map

Code type/code length		32	64	128	256	512	1024	2048
AAWGN	TRAD.	0.12	0.12	0.56	0.12	0.16	0.12	0.17
	ZM.	0.12	0.12	0.56	0.12	0.16	0.12	0.17
	SB	0.12	0.12	0.56	0.12	0.16	0.12	0.17
	ZMSB	0.12	0.12	0.56	0.12	0.16	0.12	0.17
RELIGHT FAD.	TRAD.	0.06	0.55	0.21	0.64	0.15	0.44	0.06
	ZM	0.40	0.55	0.32	0.73	0.15	0.44	0.05
	SB	0.66	0.18	0.69	0.25	0.3	0.1	0.16
	ZMSB	0.73	0.45	0.89	0.34	0.32	0.68	0.16
FSF	TRAD.	0.40	0.56	0.21	0.81	0.26	0.12	0.9
	ZM	0.40	0.01	0.21	0.82	0.41	0.37	0.36
	. SB	0.66	0.20	0.39	0.31	0.13	0.68	0.3
	.ZMSB	0.66	0.20	0.22	0.22	0.33	0.63	0.45

Regarding to the tent map, table 2 shows the following results:
(I) for the AWGN case, the optimum initial values have the same trend as in the logistic map, in which all the tent stages have the same optimum initial value for the same code length, also due to the channel simplicity.
(II) Considering the code stages, the optimum initial values have nearly identical or close values for all the lengths (0.74, 0.62 and 0.61), except for the length 127 and 255.
(III) Regarding to the fading channels, the optimum initial values have a sever variation due to the effect of the fading channel gain.

Generally, it is clear that the results obtained from both the logistic and tent maps have the same attitude for the same channel type, code stage, and code length. This gives an indication that the simulation is accurate and convincing.

Table 2: Optimum initial vlaue for the tent map

Code type/code length		32	64	128	256	512	1024	2048
AAWGN	TRAD.	0.74	0.74	0.23	0.48	0.61	0.62	0.62
	ZM.	0.74	0.74	0.23	0.48	0.61	0.62	0.62
	SB	0.74	0.74	0.23	0.48	0.61	0.62	0.62
	ZMSB	0.74	0.74	0.23	0.48	0.61	0.62	0.62
RELIGHT FAD.	TRAD.	0.24	0.86	0.76	0.48	0.89	0.79	0.14
	ZM	0.39	0.39	0.88	0.74	0.19	0.09	0.09
	SB	0.57	0.31	0.78	0.46	0.84	0.79	0.24
	ZMSB	0.37	0.33	0.02	0.46	0.78	0.55	0.49
FSF	TRAD.	0.77	0.02	0.23	0.23	0.17	0.62	0.84
	ZM	0.23	0.18	0.36	0.7	0.19	0.57	0.55
	. SB	0.87	0.31	0.73	0.46	0.84	0.79	0.24
	.ZMSB	0.44	0.45	0.49	0.46	0.03	0.48	0.03

B. Balance Property:

The balance property gives an indication about the ratio between the number of ones (or zeros) to the code length. Ideally, it's preferred to obtain this value as 0.5. Table 3 illustrates the balance property of the different stages along the discussed code lengths of the proposed chaotic codes. It can be concluded that: (I) Generally, the SB, and the ZMSB codes have perfect balancing all over the code lengths for all the chaotic code types. (II) Regarding to the logistic map, the increase of the code length has a negative effect on the balance of the traditional ZM logistic codes. (III) Regarding to the Tent map, as the code length of the traditional code increased, the balance property slightly enhanced until the length 128, and then it degrades until reach to the length 2048. (IV) Also, as the code length increased, the balance property of the ZM tent code is improved.

Table 3: Balance property

Code ype/code Length		32	64	128	256	512	1024	2048
LOGISTIC	TRAD.	0.5097	0.5254	0.5284	0.5271	0.5319	0.5362	0.5334
	ZM.	0.5	0.4794	0.4866	0.4937	0.4871	0.4843	0.4844
	SB	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	ZMSB	0.5	0.5	0.5	0.5	0.5	0.5	0.5
TENT	TRAD.	0.6613	0.6571	0.6519	0.662	0.6716	0.6712	0.6695
	ZM	0.4839	0.4563	0.4811	0.5071	0.5016	0.5015	0.5019
	SB	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	ZMSB	0.5	0.5	0.5	0.5	0.5	0.5	0.5

The orthogonality property is considered as one of the most important properties that should be studied due to its effective influenceon the performance, especially in presence of multiple accesses. This property measures how much the codes are orthogonal to each other. It is desired to have zero values between all the sequences of the mentioned codes, and can be calculated and averaged over 100 different sequences according to (6).

$$\sum_{k=1}^{K}\sum_{i=1}^{N} x_{ki}.x_{li}\ , \quad l=1,2,\dots,K, \quad k\neq l \qquad (6)$$

Where K represents the number of the total spreading sequences, which is set to 100. From table 4, it is easy to notice the following conclusions: (I) regarding to the logistic map, for the same code length, it's clear that traditional code has the best orthogonality values over the rest of the enhanced logistic codes. This gives indication that the enhanced processes applied to the logistic code have negative affect on the orthogonality property. (II) as the code length increased, the orthogonality of all the logistic code is improved, which mean that MAI effect is decreased as the code length increased, but this will be on the account of the complexity. (III) Regarding to the tent map, it is clear that the enhancement applied to the traditional code improves the orthogonality property, as the ZMSB has the best values. (IV) as mentioned in the case of the logistic codes, as the code length increased generally, the orthogonality of all the tent code is improved, which mean that MAI effect is also decreased as the code length increased, and this will be on the account of the complexity.

Table 4: Orthogonality property

Code ype/code Length		32	64	128	256	512	1024	2048
LOGISTIC	TRAD.	0.0753	0.0571	0.0411	0.0170	0.0154	0.0260	0.0089
	ZM.	0.0896	0.0476	0.0394	0.0301	0.0268	0.0190	0.0164
	SB	0.1111	0.0625	0.0590	0.0310	0.0226	0.0273	0.0132
	ZMSB	0.1111	0.0694	0.0451	0.0278	0.0217	0.0356	0.0163
TENT	TRAD.	0.0896	0.0829	0.0656	0.0711	0.0850	0.0849	0.0896
	ZM	0.0609	0.0494	0.0376	0.0353	0.0276	0.0164	0.0111
	SB	0.1528	0.0833	0.0868	0.0833	0.0859	0.0803	0.0816
	ZMSB	0.0833	0.0417	0.0451	0.0278	0.0252	0.0169	0.0217

Correlation Properties:

In this subsection, the auto-correlation, and the cross-correlation of the proposed maps will be analyzed and discussed. Ideally, it is desired to produce spreading code have the δ-like autocorrelation function, i.e. $C_a(\tau) = \sum_{i=1}^{N} x_i . x_{i+\tau} = \delta(\tau)$ and zero cross correlation function i.e. $C_c(\tau_i) = \sum_{i=1}^{N} x_i . z_{i+\tau_i} = 0$ where N is the code length. However, due to the imperfection of the generated codes, these two targets are not ideally achieved. For this reason most of the researches aim to minimize the autocorrelation side lobes regarding to the main lobe, as well as the cross correlation peaks. TableV illustrates the maximum autocorrelation side lobe normalized over the main lobe (NMACSL), which is desired to be minimum as much as possible. In addition, tableVIdiscusses the normalized average cross correlation (NACC) function.

Starting with table 5, the following conclusions about the NMACSL can be obtained:

(I) Regarding to the logistic map, as the code length increased, the values of the NMACSL improved, except for the 2048 length. Also, it is clear that the SB and the ZM processes have negative effects of the NMACSL.

(II) Regarding to the tent map, as the code length increased, the attitude of the NMACSL improved. In addition, the ZM stage has better values than the other tent codes.

(III) The ZM process in the tent map significantly enhances the NMACSL property as the code length increased. This can be cleared as the ZM, and the ZMSB have better values than the traditional and the SB code.

Table 5: Nomralized maximum autocorrelation side lobe

Code ype/code length		32	64	128	256	512	1024	2048
LOGISTIC	TRAD.	0.3323	0.2873	0.231	0.1635	0.1360	0.1110	0.1177
	ZM.	0.4097	0.2873	0.222	0.1847	0.1301	0.1440	0.185
	SB	0.4125	0.3313	0.272	0.2078	0.1742	0.1328	0.1289
	ZMSB	0.4875	0.3688	0.272	0.2078	0.1547	0.1445	0.1502
TENT	TRAD.	0.3968	0.3794	0.3693	0.3314	0.2992	0.2904	0.3065
	ZM	0.3258	0.2254	0.1976	0.1675	0.1207	0.0916	0.074
	SB	0.5	0.4686	0.3312	0.3282	0.3016	0.3012	0.3029
	ZMSB	0.425	0.3001	0.2188	0.1969	0.1578	0.1539	0.1273

Concerning the NACC, Table 6 clears that (I) The ZM code in the logistic map shows a good performance all over the length, and it is clear that generally the enhancement applied to the logistic codes has slightly positive effect. Also, it is clear that the performance is improved generally with the increase of the code length. (II) The tent map generally shows worse results compared with the logistic map, however it has the same behavior, in which the enhancement applied has slightly positive effect. (III) The ZMSB has the nearly the better values than the other codes. The attitude is improved generally with the increase of the code length

Table 6: Normalized average cross correlation

Code ype/code Length		32	64	128	256	512	1024	2048
LOGISTIC	TRAD.	0.3323	0.2873	0.231	0.1635	0.1360	0.1110	0.1177
	ZM.	0.4097	0.2873	0.222	0.1847	0.1301	0.1440	0.185
	SB	0.4125	0.3313	0.272	0.2078	0.1742	0.1328	0.1289
	ZMSB	0.4875	0.3688	0.272	0.2078	0.1547	0.1445	0.1502
TENT	TRAD.	0.3968	0.3794	0.3693	0.3314	0.2992	0.2904	0.3065
	ZM	0.3258	0.2254	0.1976	0.1675	0.1207	0.0916	0.074
	SB	0.5	0.4686	0.3312	0.3282	0.3016	0.3012	0.3029
	ZMSB	0.425	0.3001	0.2188	0.1969	0.1578	0.1539	0.1273

SIMULATION RESULTS

In this section, the performance comparison and the simulation results of the logistic map and the tent map is presented for selected code lengths. The expressed Figs. compare the performance of the mentioned code stages with the Gold code as an example of traditional code under the same conditions.

Fig. 1 shows the performance comparison of the traditional and different enhanced stages of the logistic chaotic code after the optimization process, and the Gold code with length 63. The simulation is performed in presence of frequency selective fading channel under effect of 10 MAI. The results show that the traditional logistic code and the logistic ZM code outperform the performance of the Gold code by about 1.5 dB and 0.5 dB respectively at BER 10-2. This means that the SB process degrades the performance of the logistic code, and this result matches the results obtained in tables IV, V, and VI.

Fig. 2 shows the initial values against the minimum mean error for all the logistic stages for the same code length 63 under the FSF channel. It is clear from the Fig. that the traditional code has its optimized value at 0.56, whereas the ZM at 0.01, the SB at 0.2, and the ZMSB at 0.2. These values are cleared in table 1.

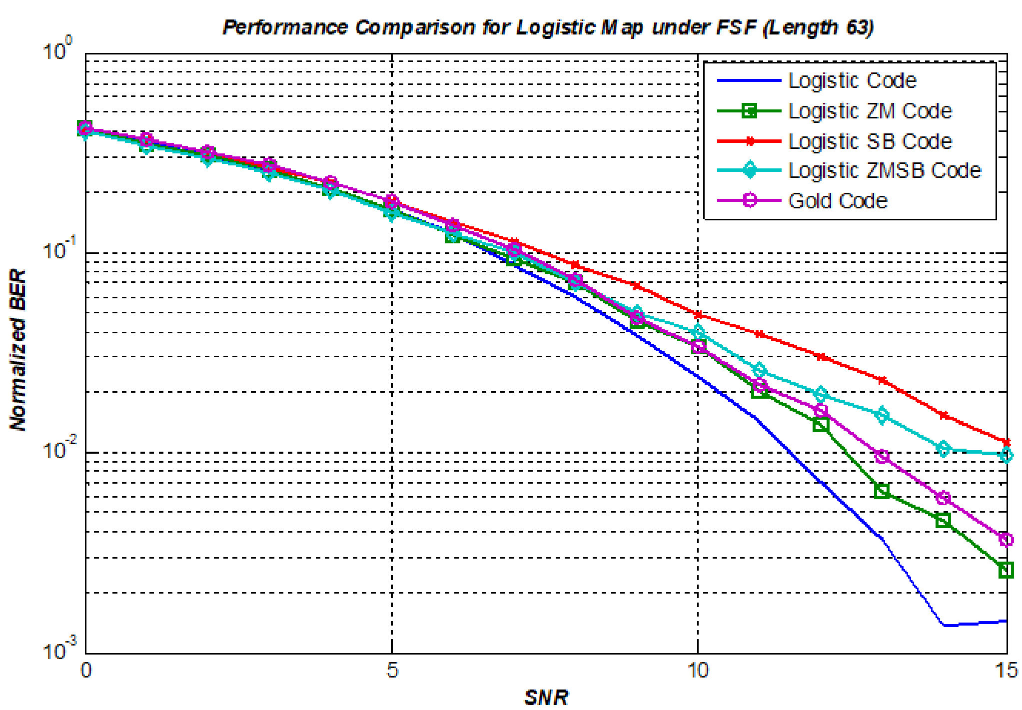

Fig. 1: Logistic map performance comparison (FSF, Length 63)

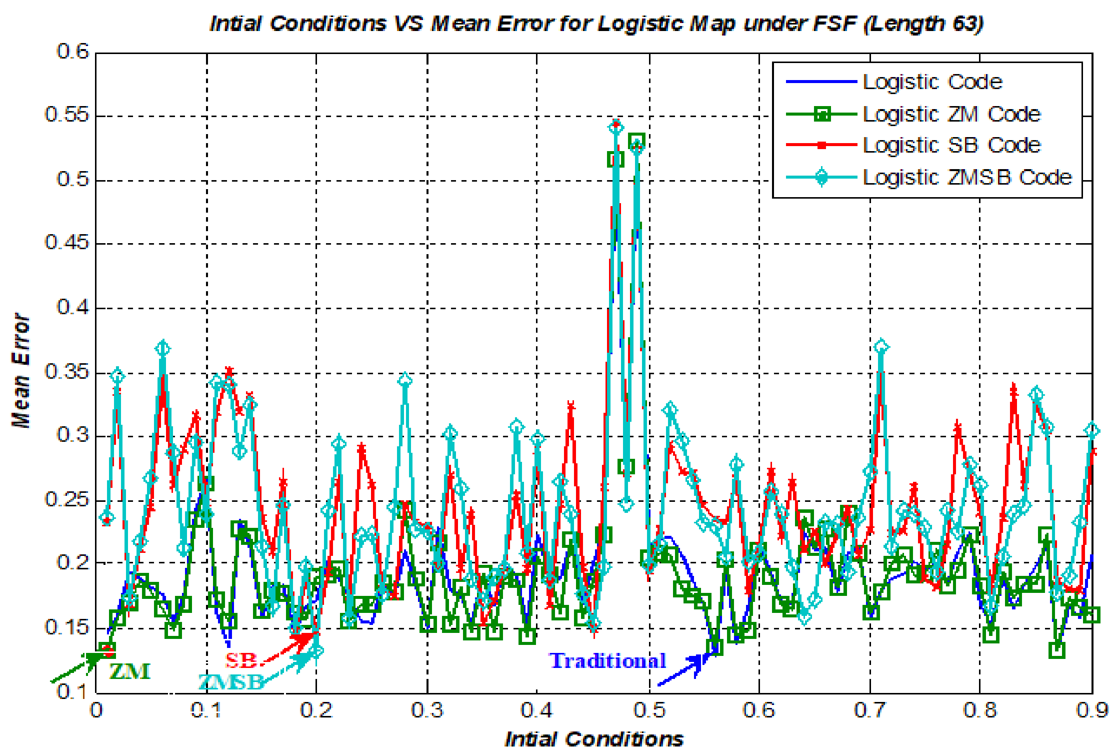

Fig. 2: Initial values vs the mean error for logistic codes (FSF, Length 63)

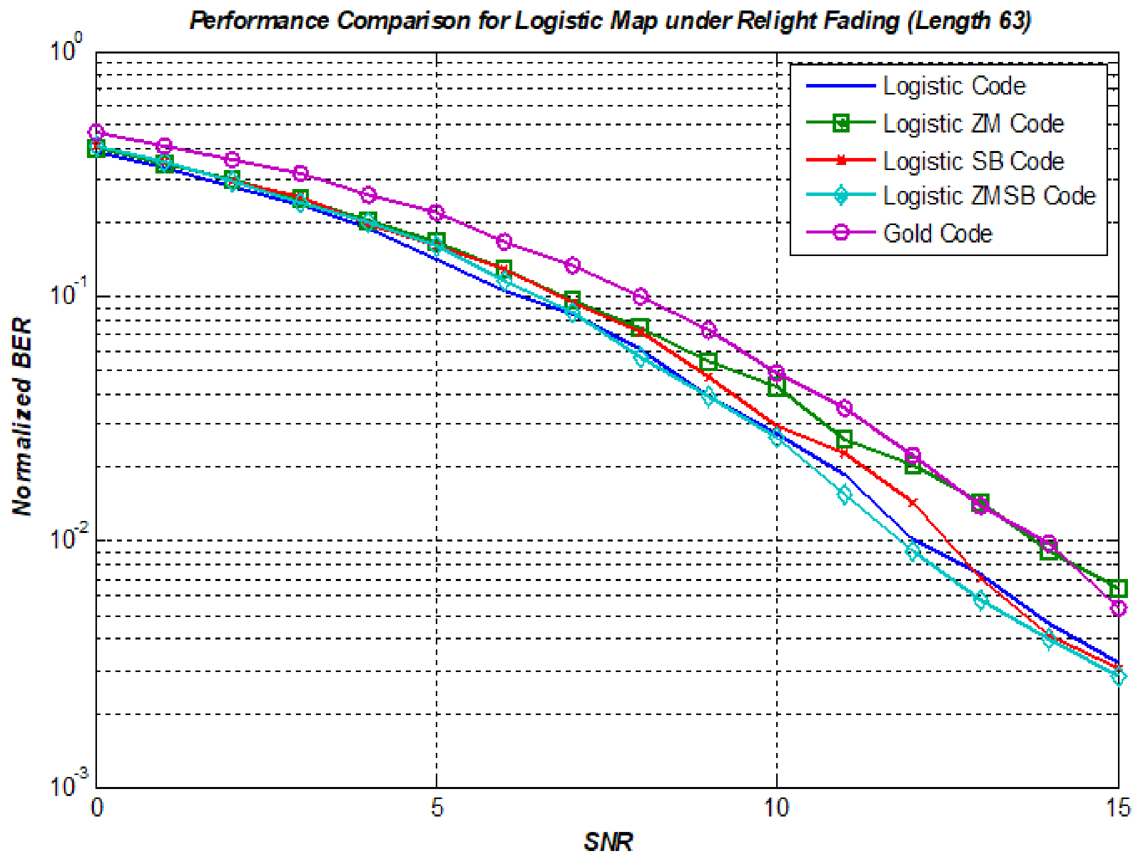

Fig. 3: Logistic map performance comparison (Relight Fad, Length 63)

Regarding to the Relight fading channel, Fig. 3 expresses the performance comparison for the logistic map with the same length. The Fig. shows that most of the logistic codes have better performance than the performance of the Gold code, except the ZM which has nearly the same performance of the Gold code. The performance enhancement is about 2 dB at BER 10^{-2}.

In Fig. 4 the initial values regarding to the length the same length in presence of the Relight fading channel is shown. Also as stated in table 1, the minimum mean error values of the different stages are 0.55, 0.55, 0.18, and 0.45, respectively.

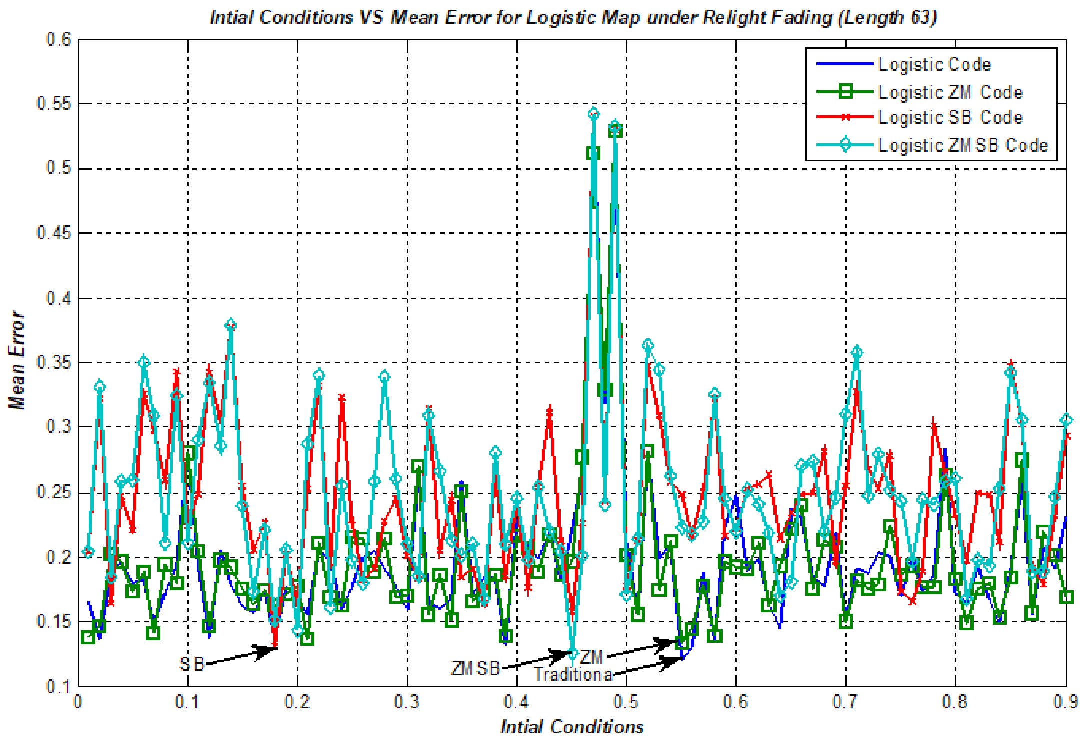

Fig. 4: Initial values vs the mean error for logistic codes (Relight Fad, Length 63)

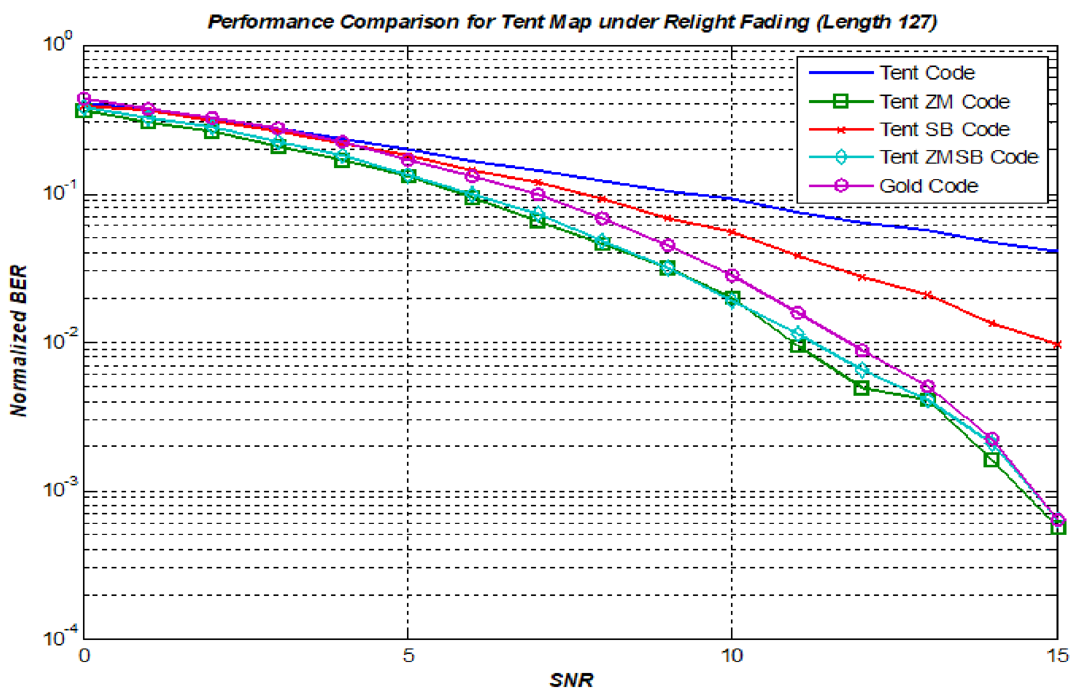

Fig. 5: Tent map performance comparison (Relight Fad, Length 127)

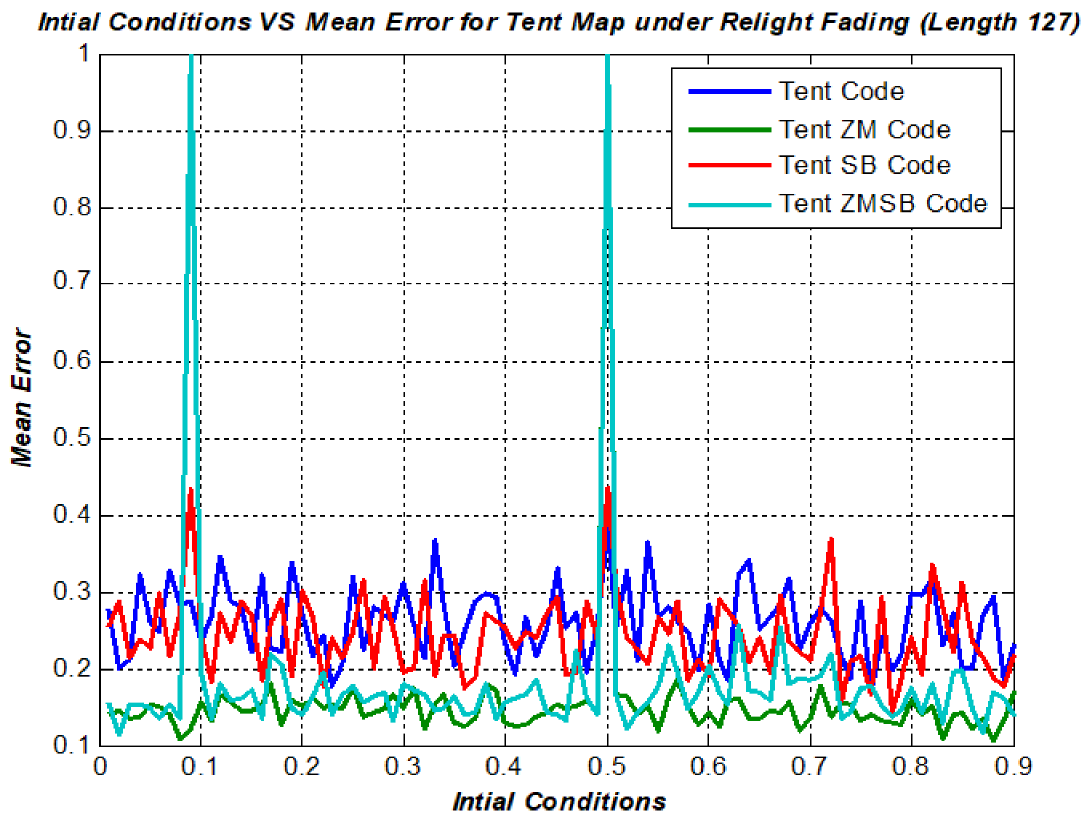

Fig. 6: Initial values vs the mean error for tent codes (Relight Fad, Length 127)

Regarding to the tent map, Fig. 5 shows the performance comparison of the different stages with Gold sequence for length 127 under relight fading channel. The Fig. shows that ZM and the ZMSB nearly have the same performance, and slightly improved than that of the Gold code, whereas both the traditional and SB code have degraded performance due to the fading channel effect.

Fig. 6 represents the corresponding initial values against the minimum mean error for all the tent stages for the code length 127 under effect of relight fading channel. The shown results ensure the values stated in table 1, in which the optimized initial values are 0.76, 0.88, 0.76, and 0.02, respectively.

CONCLUSION

This paper introduced a statistical study and analysis of different chaotic maps with optimized initial values. The study includes the optimization process for different improvement stages over different code lengths, and under effect of different fading channel types. The initial values of the generated codes are optimized according to the rule of minimizing the mean bit error rate (BER).The number of codes that the average is taken over is 100 different codes under the same conditions. The analysis is performed over the balance property, the orthogonality property, the autocorrelation side lobe, and the cross correlationproperty for code lengths ranged from 32 to 2048. The results show that both the logistic and tent maps have nearly close initial values in presence of the AWGN channel, whereas in fading channels the initials have different values. The results show also that most of the enhancement operations have positive effect on the statistical properties when the initial values are optimized. The performance evaluation show that the traditional and ZM logistic codes has better performance compared with the Gold code in presence of FSF channel. In Relight fading channel the most of the logistic codes outperformed the performance of the Gold code. For the tent map, the ZM process significantly improves the performance of the ZM, and SBZM codes over the Gold code. Regarding to the future work, this analysis can be performed over different types of chaotic codes with different dimensions to evaluateand compare its performance with the other codes.

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