



## Effect of design parameters on a two-stage launch vehicle performance

### Original Article

Assem M. F. Sallam<sup>1</sup>, Aly M. Elzahaby<sup>2</sup>, Ahmad E. Makled<sup>1</sup>, Mohamed K. Khalil<sup>3</sup>

<sup>1</sup>Space Technology Center, Cairo, Egypt.

<sup>2</sup>Department of Mechanical Engineering, Faculty of Engineering, Tanta university, Tanta, Egypt.

<sup>3</sup>Department of Aircraft Mechanics, Military Technical College, Cairo, Egypt

### Keywords:

Launch pad location, launch vehicle azimuth, launch vehicle payload, launch vehicle trajectory.

### Corresponding Author:

Assem M. F. Sallam, Space Technology Center, Cairo, Egypt, **Tel:** 22718708.

**Email:** assemsallam@yahoo.com

### Abstract

Change in design parameters of Launch Vehicle affects its overall flight path trajectory. In this paper, several design parameters are introduced to study their effect. The selected parameters include the Launch Vehicle mass, which is presented in the form of payload mass, the maximum allowable angle of attack the Launch Vehicle can withstand, the flight path angle that is predefined for the Launch Vehicle second stage, the required inclination and its effect on the launch azimuth and finally by changing the launch pad coordinate. Selected design parameters are studied for their effect on the variation of altitude, ground range, absolute velocity and the flight path angle. Study gives a general way of adjusting the design parameters to reach the required Launch vehicle performance.

## I. INTRODUCTION

Adjusting the separation of a Spacecraft (SC) from a Launch Vehicle (LV) according to the required orbit conditions is a very critical part of the design of LV.

Separation conditions are mainly concerned by velocity of SC to be equivalent to the related orbit and to be in the direction of rotation without any radial component, separation should also satisfy the required orbit altitude. Different design parameters have great effect on the SC separation conditions, knowing the effect of each parameter on the overall LV trajectory and how they affect the SC separation is the goal of study. LV mathematical model used in this study is based on a Russian medium lift LV<sup>[1]</sup> Zenit-2 model with a maximum payload lift-mass of 14000 kg<sup>[2]</sup> and approved for its consistency with the actual model performance within an error less than 10%. When studying the effect of certain design parameter, only the parameter under study will be varied and the rest of parameters will be kept constant at its initial design values.

Basic LV equations of motion of the center of mass over an oblate rotating Earth, LV mass variation as a result of propellant consumption is taken into consideration, the

related equations are taken from<sup>[3, 4 and 5]</sup>. The state variables are mainly the velocity components ( $V_R$ ,  $V_\lambda$  and  $V_\delta$ ), see Fig. 1 and position variables ( $R$ ,  $\lambda$  and  $\delta$ ) as in (1) and (2).

$$\frac{d}{dt} \begin{bmatrix} R \\ \lambda \\ \delta \end{bmatrix} = \begin{bmatrix} V_R \\ \frac{V_\lambda}{R \cos \delta} - \omega_e \\ \frac{V_\delta}{R} \end{bmatrix} \quad (1)$$

$$\frac{d}{dt} \begin{bmatrix} V_R \\ V_\lambda \\ V_\delta \end{bmatrix} = \frac{1}{R} \begin{bmatrix} V_\lambda^2 + V_\delta^2 \\ V_\lambda(V_\delta \tan \delta - V_R) \\ -V_\lambda^2 \tan \delta - V_R V_\delta \end{bmatrix} + \frac{\bar{F}}{m} \quad (2)$$

Where:

- $R, \lambda, \delta$  Inertial position components, m.
- $V_R, V_\lambda$  and  $V_\delta$  Inertial velocity components, m/s.
- $\omega_e$  Earth's angular velocity, rad/s.
- $\bar{F}$  Vectorial sum of all external forces, N.
- $L$  Subscript indicating variables in the local coordinate system.

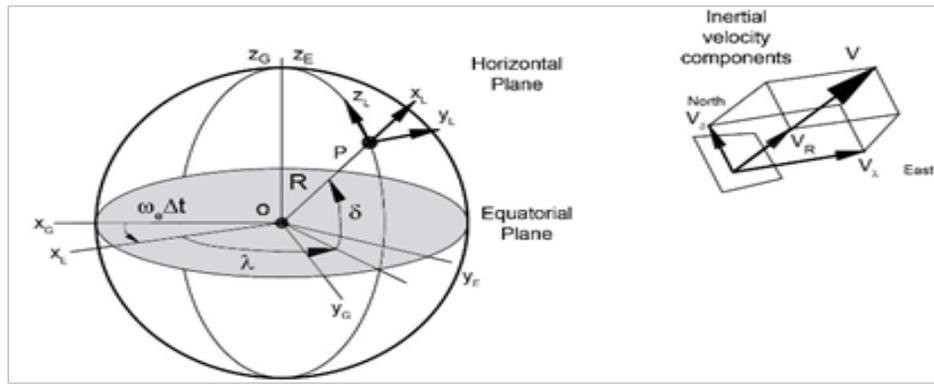


Fig. 1: Definition of Earth and local coordinate systems with parameters relating both coordinates<sup>[4]</sup>.

Three acting forces are taken into account in the mathematical model stated from (3) to (6), which are the LV thrusting force, aerodynamic drag force ; both are multiplied by transformation matrix X (transfer from body to local coordinate system), while the drag force is also multiplied by matrix Z (to compensate for the angle of attack effect), third acting force is the Earth's gravitational force-acting on the  $\vec{R}$  and  $\vec{\delta}$  directions

$$\vec{F} = X\vec{T} + X Z\vec{F}_{aero} + \vec{G} \quad (3)$$

$$\vec{T} = \begin{bmatrix} g_o I_{spvac} \dot{m}_b(t) - A_e \rho \\ 0 \\ 0 \end{bmatrix}_B \quad (4)$$

$$\vec{F}_{aero} = q A_{ref} \begin{bmatrix} -C_D(M) \\ \left(\frac{\partial C_L}{\partial \alpha}(M)\right) \alpha \\ -\left(\frac{\partial C_N}{\partial \alpha}(M)\right) \alpha \end{bmatrix}_B \quad (5)$$

$$\vec{G} = -\frac{\mu}{R^2} \begin{bmatrix} 1 + \frac{3J_2}{2} \left(\frac{R_E}{R}\right)^2 (1 - 3 \sin^2 \delta) + 2J_3 \left(\frac{R_E}{R}\right)^3 (3 - 5 \sin^2 \delta) \sin \delta \\ 0 \\ 3J_2 \left(\frac{R_E}{R}\right)^2 \sin \delta \cos \delta + \frac{3J_3}{2} \left(\frac{R_E}{R}\right)^3 (5 \sin^2 \delta - 1) \cos \delta \end{bmatrix}_L \quad (6)$$

Where:

- $\alpha$  Angle of attack, rad.
- $A_{ref}$  LV reference area subjected to aerodynamic force, m<sup>2</sup>.
- $C_D$  Drag Coefficient.
- $C_L$  Lift Coefficient.
- $C_N$  Normal Force Coefficient.
- $I_{spvac}$  Propellant vacuum specific impulse, s.
- $J_2$  Second Jeffery's constant of gravity.
- $M$  LV Mach number.
- $q$  Dynamic pressure, Pa.
- $\beta$  Side slip angle, rad.
- $\mu$  Earth's gravitational constant, km<sup>3</sup>/s<sup>2</sup>.
- $g_o$  Sea level gravitational acceleration, m/s<sup>2</sup>.

### III. ZENIT-2 PARAMETRIC STUDY

In this section, different design and control parameters are altered from their normal values, to study the effect of changing of each variable on the overall performance of the LV. Generally, the variable under study is only changed and the rest of design parameters taken from the Zenit-2 manual<sup>[6]</sup> are kept constant.

#### A. Effect of variation of payload mass

For this study, three payload masses are chosen with a step difference 5000 kg, selected masses are 4000 kg, 9000 kg and the maximum payload mass stated for that LV which is 14000 kg. LV performance is mainly examined by the variation of its altitude and velocity over the whole flight time.

##### 1) Effect on LV altitude

Decreasing the payload mass increases the final achieved altitude of LV as shown in Fig. 2. The previous statement related to payload mass that inversely proportional to the maximum altitude is very obvious, in addition the flight time becomes shorter as well as the corresponding ground range, a fact which is clear in Fig. 3 and 4. As the LV is lighter in weight, the thrusting force has a greater effect that leads to increasing the LV velocity faster, so the LV will reach a higher altitude with its relative required velocity over a shorter range and flight time.

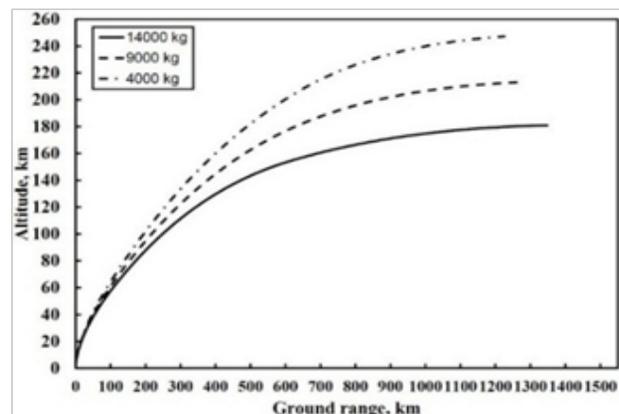


Fig. 2: LV altitude variation with ground range for different payload masses

LV altitude with the corresponding ground range and total flight time for the three used payload masses are stated in Table 1. It is found that for a reduction in payload mass from the maximum payload mass (14000 kg) by 5000 km will result in an increase in altitude by 17.9% as well as for the decrement by 10000 results in an increase by 36.5%. It may conclude that the effect of changing payload mass is approximately linear.

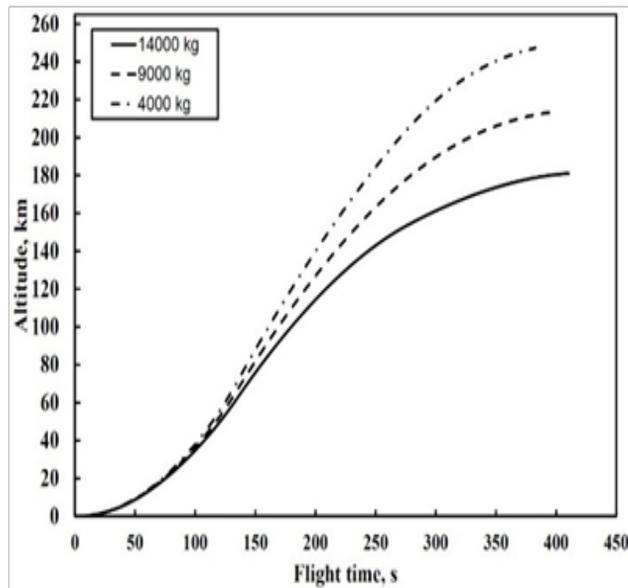


Fig. 3: LV altitude variation with flight time for different payload masses

Table 1: Payload parametric study output

Payload mass, kg	14000	9000	4000
Altitude, km	181	213.4	247.2
Ground range, km	1348.5	1285.7	1225.6
Flight time, s	410.3	396.2	382.7

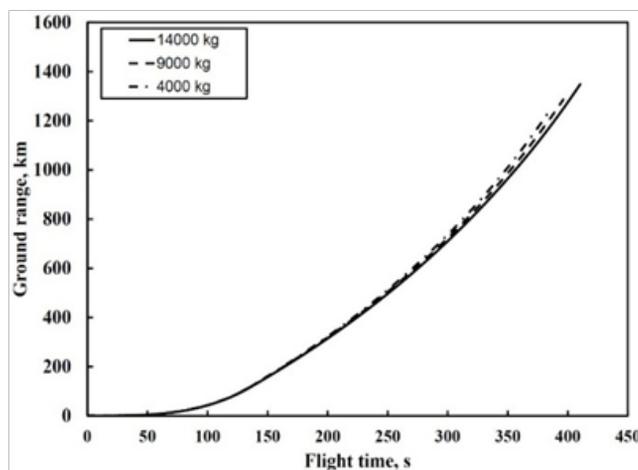


Fig. 4: LV ground range variation with flight time for different payload masses

Payload mass affects the ground range in a proportional manner, increasing the payload mass increases the covered ground range ; this is because as stated before increasing the payload mass increases the time to reach the corresponding speed at lower altitude, which results in increasing the ground range. A decrease in mass by 5000 kg results in shortening the ground range by 4.7% ; while for the reduction by 10, 000 kg, the shortening is 9.1%. These shortening values also give the indication of approximately linear change under the effect of payload mass change.

## 2) Effect on LV velocity

For studying the effect of payload mass on the LV velocity, also three masses are selected starting by the maximum payload mass 14000 kg and reducing the mass by 5000 kg to have the other two masses 9000 kg and 4000 kg.

Adjusting the LV insertion or burn out point is achieved by reaching predesigned values of altitude and velocity. Carrying a lighter payload mass makes the LV capable of reaching higher altitudes faster. To go to higher altitudes needs slower velocities than at lower altitudes (under the effect of gravitational force), which describes why carrying a lighter payload mass leads to a faster reaching of burn-out point at higher altitude. Figure. 5 shows also an interpretation of what is described, for lighter payload masses, burn-out point is reached faster with lower velocities.

When using the maximum payload mass (14000 kg), burnout velocity is 7794.9 m/s after 410.3 s ; and for the payload mass 9000 kg, burnout velocity is 7776 m/s after 396.1 s and for the payload mass 4000 kg, burnout velocity is 7754.8 m/s after 382.7 s.

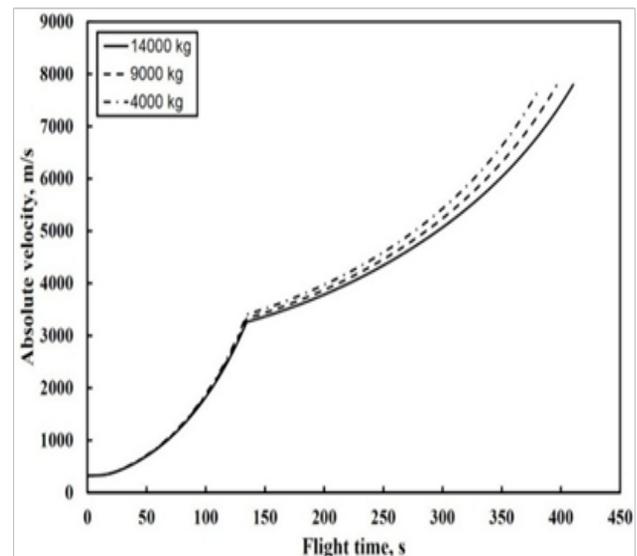


Fig. 5: Velocity variation for different payload masses over flight time

Percentages of change in LV velocity and flight time at the burn-out point are stated in Table II.

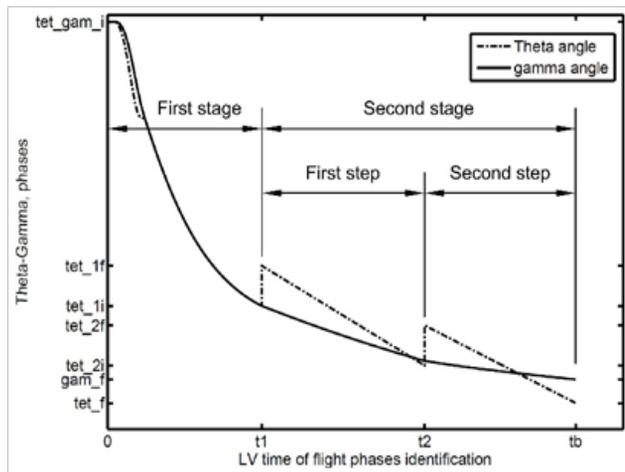
**Table 2:** Payload mass effect on LV velocity

Payload mass, kg	14000	9000	4000
Burn-out velocity, m/s	7794.9	7776	7754.8
Burn-out time, s	410.3	396.2	382.7
% change of B-O velocity	baseline	0.25%	0.51%

**B. Effect of variation of allowable maximum angle of attack**

The maximum angle of attack effect takes place over the first stage flight, it is a major parameter that affects the flight path angle, as well as the final altitude of LV.

LV flight path angle ( $\theta$ ) variation over flight time is divided into first and second stages as shown in Fig. 6. In the first stage ( $\theta$ ), angle variation is left uncontrolled under the effect of gravitational force ; while for the second stage, it is divided into two sections, each section has its intentional induced change to ( $\theta$ ) at its start as well as its rate of change during the whole step duration. The control of ( $\theta$ ) during the second stage should ensure the required separation conditions.



**Fig. 6:** Change of flight path and pitch angles ( $\theta$ ) and ( $\gamma$ ) over whole flight of LV

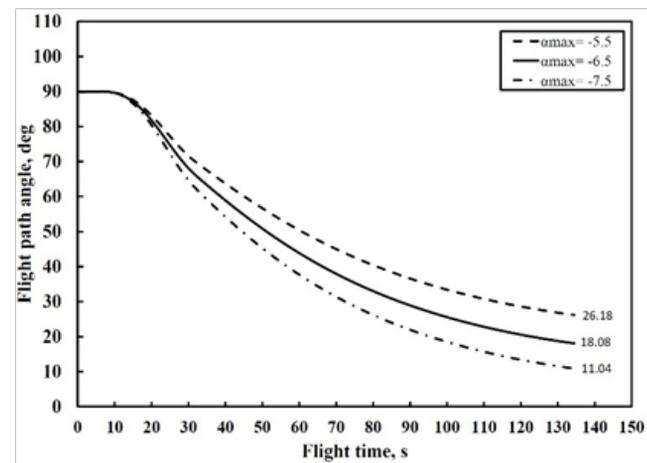
SC separation should satisfy the following conditions:

- A zero-radial velocity with a zero-pitch angle or as close as possible to zero.
- Separation velocity must match the velocity of required parking orbit.

As stated in Zenit-2 manual<sup>[6]</sup>, the maximum allowable angle of attack ( $\alpha_{max}$ ) for Zenit-2 launcher is in the range  $5 \div 10^\circ$ , analysis will take three different values for  $\alpha_{max}$  and study its effect on flight path angle and the final altitude of

LV. The selected payload mass is the maximum allowable payload mass (14000 kg). The first and second flight pitch angle changes (jump) are constant with value  $10^\circ$  and the first section of flight pitch change rate value is taken 4.383 o/s and the final angle at the end of second section of flight pitch angle (at the end of flight) is  $-6.5^\circ$ .

The chosen values for maximum allowable angle of attack are  $6.5 \pm 1^\circ$ , these values are within mid-range of allowable Zenit-2  $\alpha_{max}$  stated,  $\alpha_{max}$  reaches up to  $20^\circ$  in small LV's<sup>[7]</sup>. The first stage operation is an uncontrolled stage from point of view of flight path angle change, therefore the effect of allowable  $\alpha_{max}$  is studied over the first stage, as its effect will be studied clearly with no external influence on LV flight path. During first stage of flight, the LV is tending to turn downwards to the direction of Earth under the effect of gravitational turn, this tendency makes the LV flight path angle ( $\theta$ ) less than the pitch angle ( $\gamma$ ), that is why the angle of attack at this phase is negative and shows a change of angle ( $\theta$ ) lower than angle ( $\gamma$ ). Figure 7 shows the effect of  $\alpha_{max}$  on the flight path angle at the end of first stage ( $\theta_{1i}$ ), an increase by 15% of  $\alpha_{max}$  will lead to a decrease in the final reached flight path angle by approximately 38.8%. It can be described that by increasing the allowable angle of attack (in the negative direction) will result in a greater decrease of the final reached ( $\theta_{1i}$ ) and on contrary, the decrease of allowable  $\alpha_{max}$  by 15% will increase the final reached ( $\theta_{1i}$ ) (smaller change) by 44.8%.



**Fig. 7:** Flight path angle variation for different angles of attack for the LV first stage period

In our case, allowing the angle of attack to reach higher values (in the negative direction), will make the LV fly with smaller flight path angle decreasing radial velocity component, Fig. 8, represents the effect of varying  $\alpha_{max}$  on the final reached altitude and ground range.

Increasing the allowable  $\alpha_{max}$  will show a decreased flight path angle profile, as a result, the final reached altitude is decreased, and the covered ground range is increased (Fig. 9 and Fig. 10).

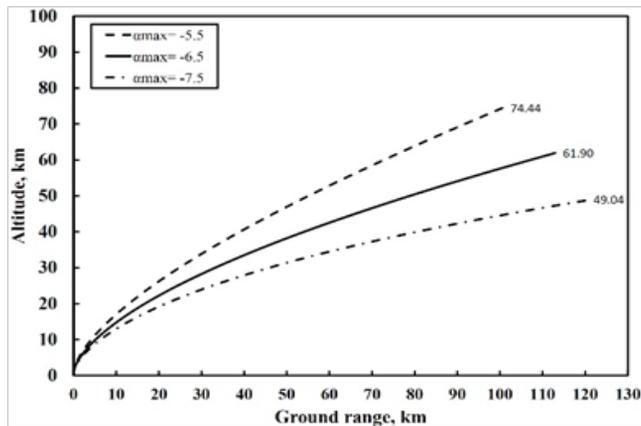


Fig. 8: Altitude variation over ground range for different angles of attack for the LV first stage period

For LV to reach a higher altitude, it must have smaller change to its flight path angle during ascent. While, for a ground to ground missile, it has to have greater change (to fly with small flight path angle) to its flight path angle to increase its ground range. As a summary, an increase in allowable  $\alpha_{max}$  by 15% decreases the final reached flight path angle by approximately 40% and as a result decreases the final reached altitude by 20.7% and increases the covered ground range by 8%, and the effect is vice versa for decreasing the allowable  $\alpha_{max}$ .

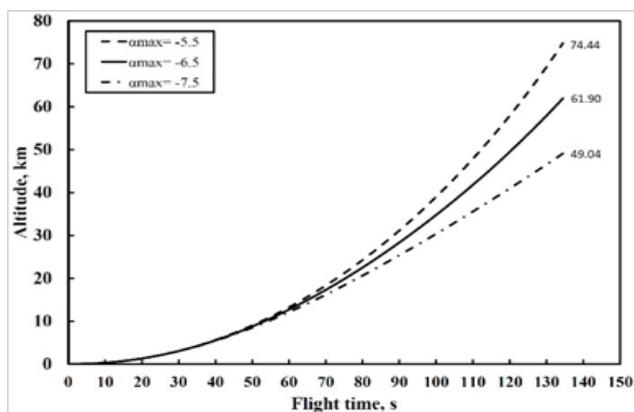


Fig. 9: LV altitude variation for different angles of attack for the LV first stage period

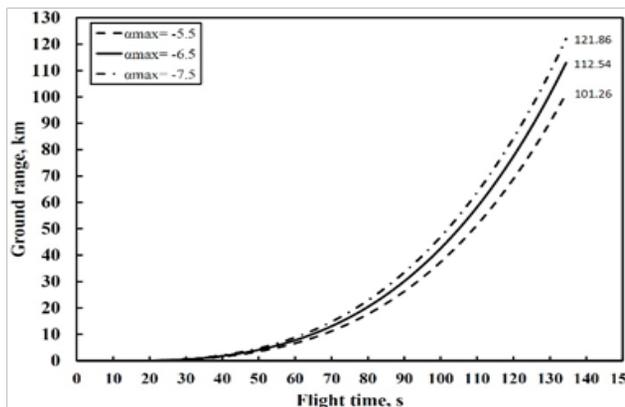


Fig. 10: Ground range variation for different angles of attack for the LV first stage period

It is important to consider the effect of allowable  $\alpha_{max}$  on the final reached velocity, as the velocity with altitude controls the final reached orbit requirements. It is found that, Fig. 11, the obvious advantage of increasing the allowable  $\alpha_{max}$ , is to have higher reached velocity (as a result of decreased opposing gravitational force) which is beneficial in adjusting velocities for lower required altitudes.

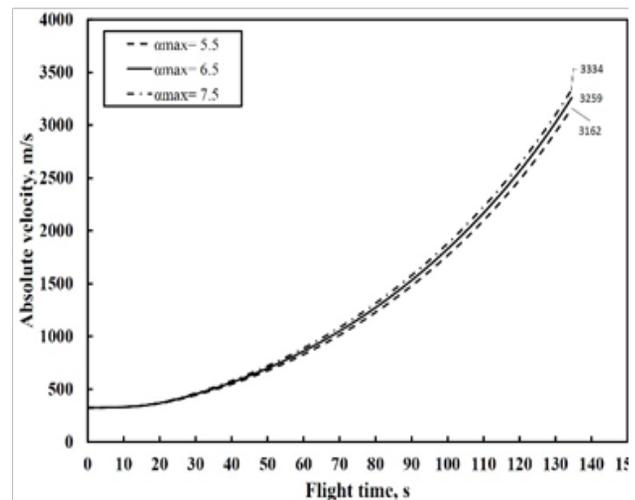


Fig. 11. Absolute velocity variation for different angles of attack for the LV first stage period

An increase in allowable  $\alpha_{max}$  by 15% will lead to increase in final absolute velocity by nearly 2.4%, also it shows that by decreasing allowable  $\alpha_{max}$  by 15% decreases the final reached absolute velocity by 3%.

### C. Effect of path angle on LV second stage

Zenit-2 LV second-stage flight path angle is controlled during the flight time of second stage. Controlling this angle affects the altitude and velocity of the final separation point. The second stage is divided to two sections with equal periods; the initial and final flight path angles of second step are studied to see their effect, the study is carried out using the LV maximum payload mass (14000 kg) and a maximum allowable angle of attack 6.5°.

The default control values of second flight path angle (jump) ( $\theta_{2i}$ ) is 10° and final angle ( $\theta_f$ ) is -6.5°. Initial path angle of second step variation is taken  $\pm 5^\circ$  from the 10° initial default value.

#### 1) Effect of ( $\theta_{2i}$ ) on LV altitude

Second stage of LV is the controlled stage for the flight path angle, to investigate the effect of induced change at the mid of second stage, three changes (jump) values are selected, 5°, 10° and 15°, it is also reserved the final reached flight path angle to -6.5°. The final reached conditions affect mainly the shape of the satellite's orbit. Figure. 12 shows the effect of changing the value of second jump on the final reached altitude, an increase in the jump value means an increase of the LV center line or tip towards higher altitude, which means that the LV will reach higher altitude using

higher flight path angle. The increase in flight path angle at mid time of second stage leads to increase in altitude by 5.3%, while for decreasing the angle by 50% will decrease final reached altitude by 5.2%.

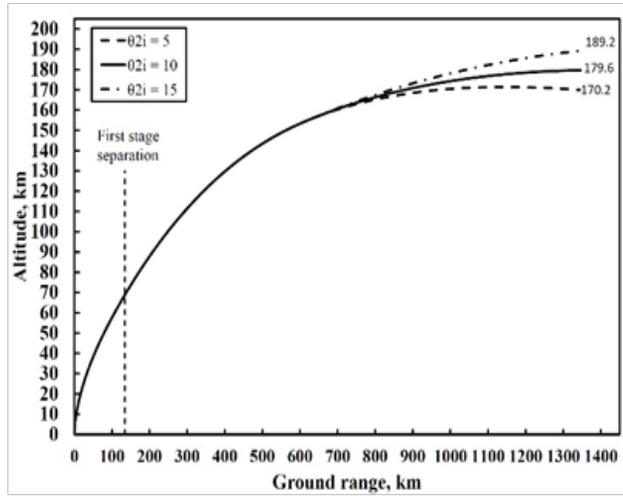


Fig. 12: Altitude-ground range variation for different flight path angles ( $\theta_{2i}$ )

2) Effect of ( $\theta_{2i}$ ) on final radial velocity of LV

Increasing the induced flight path angle ( $\theta_{2i}$ ) will increase the LV flight path angle, which increases the angle of attack and consequently increases the final reached radial velocity, Fig. 13. Increasing ( $\theta_{2i}$ ) by 50% will result in increasing the final reached radial velocity by 122.7 m/s, while decreasing the angle ( $\theta_{2i}$ ) by 50% decrease the radial velocity by 128.5 m/s.

Variation effect of ( $\theta_{2i}$ ) is opposite to the effect of maximum allowable angle of attack in the LV first stage. This difference is due to that the angle of attack direction during the LV second stage (positive) is different from that at the first stage (negative).

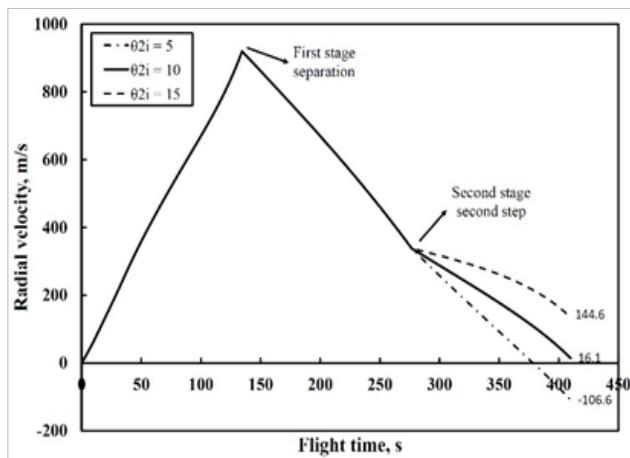


Fig. 13: Radial velocity variation for different flight path angles ( $\theta_{2i}$ )

D. Effect of required inclination on trajectory azimuth

During the design phase of a LV, an input for the required inclination ( $i$ ) is required. For each required inclination, a change in the flight trajectory azimuth takes place. To study this change in azimuth, three different inclinations are chosen; these inclinations are actually used for launch from the Baikonur launch pad. The three used inclinations are  $51.6^\circ$  (International Space Station orbit),  $63.4^\circ$  (critically inclined orbit) and  $98^\circ$  (Sun synchronous orbit).

Increasing the required inclination from a fixed launch pad location (Baikonur), decreases the launch azimuth. As decreasing the azimuth means the more LV head will point towards the north during flight, Fig. 14 shows an increase in inclination by 23% will lead to a decrease in azimuth by approximately 35%.

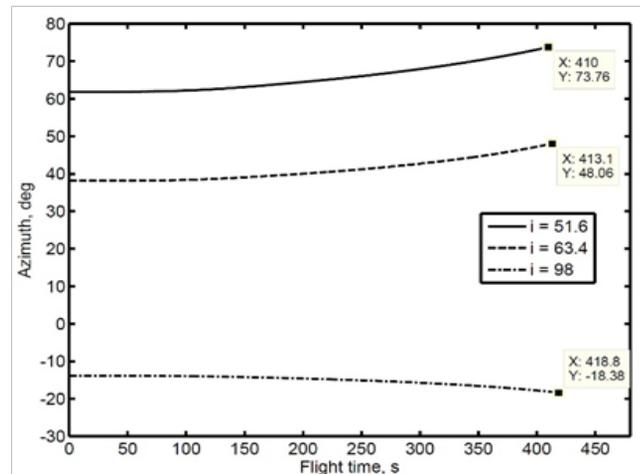


Fig. 14: LV azimuth variation with flight time for different required orbit inclinations

The change of inclination during the flight time takes place mainly during the uncontrolled period of LV flight (first stage) as shown in Fig. 15. A 2-D plot of how the trajectory azimuth will look like is also presented on Fig. 16.

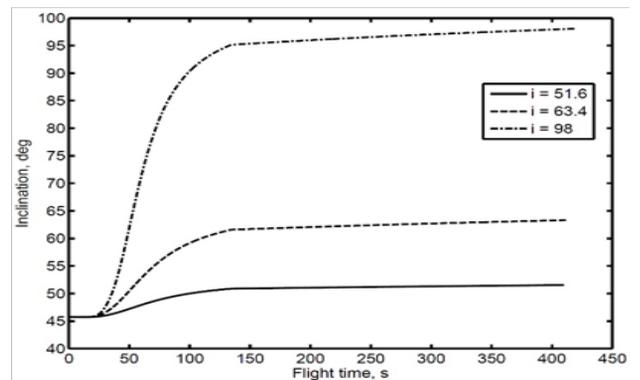


Fig. 15: LV inclination variation with flight time for different required orbit inclinations

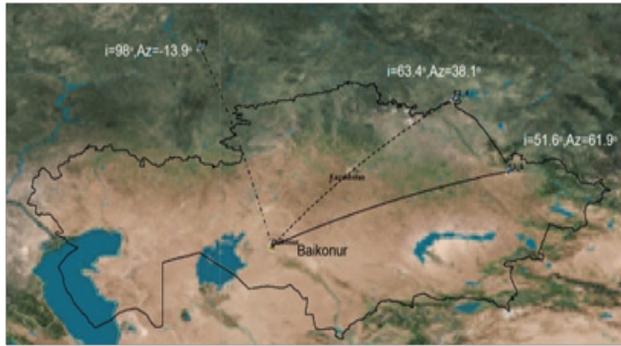


Fig. 16: 2-D drawing of Baikonur launch pad showing three azimuth trajectories for the three inclinations under study

### E. Effect of changing launch site location on LV

It is not clear whether changing the launch site location has an effect on the final LV insertion parameters. Different locations are chosen, as shown in TABLE III. A study is carried out to see effect of changing the launch pad location and the final LV altitude and velocity.

Table 3: Different Launch pad locations

Country / city	Latitude, deg	Longitude, deg
Kazakhstan / Baikonur	45.963	63.653
Egypt / Cairo high	30	30
Egypt / Matrouh high	30	27
Egypt / Matrouh Low	23.5	27
Egypt / Cairo Low	23.5	30

The locations are chosen in Egypt as in Fig. 17, to understand thoroughly the effect of changing location of launch pads over Egypt, which may help to select the better suitable location to propose for launch in Egypt. Another location is selected, Baikonur, which is the main launch pad for the Zenit-2 launcher.

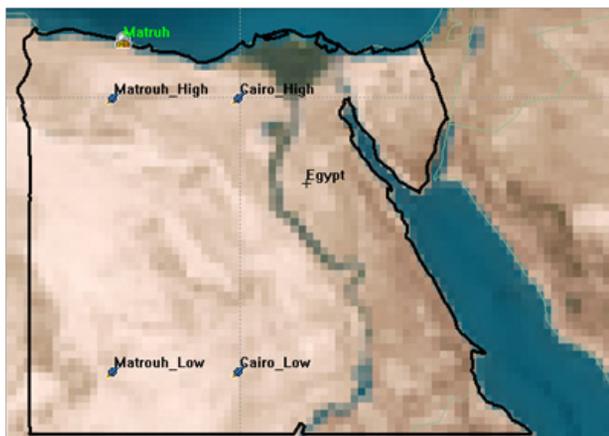


Fig. 17: Different proposed LV launch pads over Egypt territory

Basic Zenit-2 launch designed parameters of maximum payload mass and flight path angles are used.

In Fig. 18, it is found that placing launch pad at “Cairo high” and “Matrouh high” results in the same launch profile, where altitude over ground range is typical for both locations.

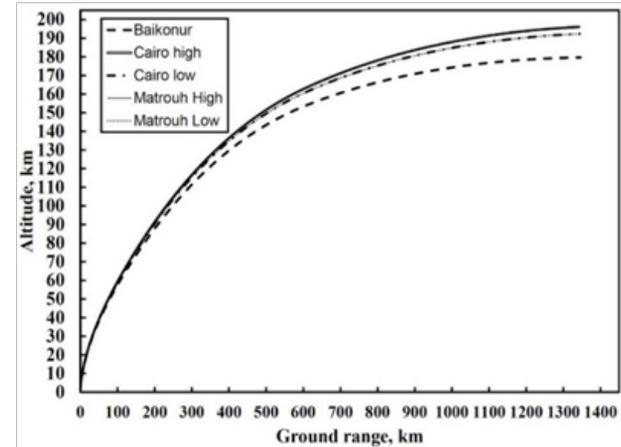


Fig. 18: Altitude variation over ground range for Zenit-2

### LV for different launch pad locations

Similar results occur with the two locations located at lower Egypt “Cairo low” and “Matrouh low”. The results show that locations at same latitude produce the same flight profile and the point with higher latitude reaches a higher altitude. Previous fact is contradicted for the location at Baikonur, where launch pad is at higher latitude and the final reached altitude is lower than for lower latitude locations chosen inside Egypt territory.

The only difference between locations located inside Egypt’s territory and the location of Baikonur is that LV launch azimuth differs greater at Baikonur. Fig. 19 shows also matching between azimuth variations for launch pads with the same latitude, difference in azimuth between the “High” Egypt’s locations and the “Low” is approximately 4°. While, at Baikonur, difference of azimuth with the “High” Egypt’s launch pads is approximately 20°.

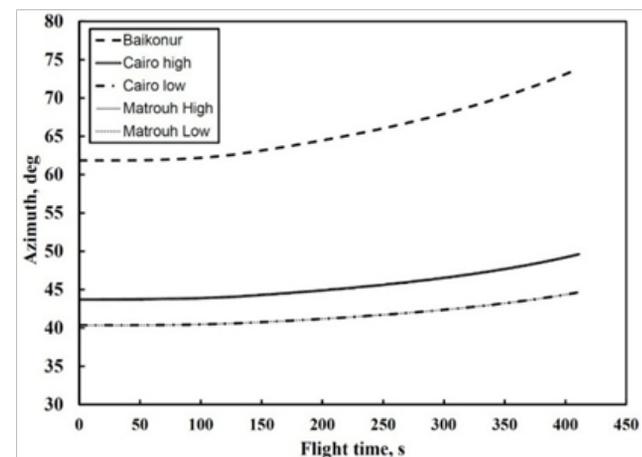


Fig. 19: Azimuth variation over LV flight time for different launch pad locations

From Fig. 18 and Fig. 19, azimuth has a greater effect than the effect of changing latitude on the final altitude of LV. In other words, increasing launch pad latitude with the same launch azimuth will increase the final reached altitude, but if latitude is increased with the same required inclination (which means azimuth must be increased to guarantee same inclination) will lead to a decrease of final LV altitude, where launch azimuth has greater effect than the launch pad latitude.

#### IV. CONCLUSION

Parametric study is carried out on a two-stage lv, parameters taken in study are payload mass, maximum allowable angle of attack, flight path angle, orbit inclination and the launch site location. Results of analysis are summarized as follows:

- For a two-stage LV with gross lift-off mass 460 tons, the decrease in payload mass by 1000 kg will approximately increase the altitude by 3.65% and the final absolute velocity by 0.05%.

- Approximately increasing allowable  $\alpha_{\max}$  by 15% decreases the final reached flight path angle by approximately 38.8% and consequently decreases the final reached altitude by 20.7% and increases the covered ground range by 8%. While, the effect is vice versa for decreasing the allowable  $\alpha_{\max}$ . Also, an increase in allowable  $\alpha_{\max}$  by 15% will lead to an increase in final absolute velocity by nearly 2.5%. It also shows that by decreasing allowable  $\alpha_{\max}$  by 15% decreases the final reached absolute velocity by 2.9%.

- The increase in flight path angle at mid time of second

stage leads to an increase in altitude by 5.3% ; while for decreasing the angle by 50% will decrease the final reached altitude by 5.2%. Increasing the same angle by 50% will result in increasing the final reached radial velocity by 122.7 m/s ; while decreasing the angle by 50% decreases the radial velocity by 128.5m/s.

- An increase in inclination by 10% will lead to a decrease in azimuth by approximately 17.4% and vice versa.

- Increasing launch pad latitude with the same launch azimuth will increase the final reached altitude. But, if latitude is increased with the same required inclination will lead to a decrease of final LV altitude, where launch azimuth is of greater effect than the launch pad latitude.

#### V. REFERENCES

- [1] P. K. McConnaughey, M. G. Femminino, S. J. Koelfgen, R. A. Lepsch, R. M. Ryan and S. A. Taylor, "NASA Space Technology Roadmaps-Launch Propulsion Systems," National Aeronautics and Space Administration, Washington, DC 20546, 2010.
- [2] A. M. F. Sallam and . A. E.-S. Makled, "Two-Stage Launch Vehicle Trajectory Modeling for Low Earth Orbit Applications," International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering, vol. 11, no. 1, pp. 86-93, 2017.
- [3] Buhl W., Ebert K. and Wolff H., Modelling: Advanced Launcher Trajectory Optimization Software, Nordwijk, Netherlands: European Space Technology and Research Center, 1992.
- [4] K. H. WELL, "Neighboring Vehicle Design for a Two-Stage," 2003.
- [5] J. R. F. Michael D. Griffin, Space Vehicle Design, Virginia: AIAA, 2004.
- [6] G. V. Kirsanov, УЧЕБНОЕ ПОСОБИЕ ПО БАЛИСТИКЕ ДЛЯ ВЫПОЛНЕНИЯ КУРСОВОЙ РАБОТЫ СТУДЕНТАМИ ВЕЧЕРНЕГО ФАКУЛЬТЕТА ПО СПЕЦИАЛЬНОСТИ "Zenit-2 manual" 0543, 1981.
- [7] B. N. Pamadi, P. V. Tartabini and B. R. Starr, "Ascent, stage separation and glide-back performance of a partially reusable small launch vehicle," in AIAA, Reno, Nevada, 2004.