



A Review on Motion Control of Unmanned Ground and Aerial Vehicles Based on Model Predictive Control Techniques

Review Article

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Key words:

Model predictive control (MPC), motion control, path following, trajectory tracking, unmanned aerial vehicles (UAVs), unmanned ground vehicles (UGVs).

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Abstract

Recently, unmanned vehicles have attracted a great deal of attention in academic, civilian and military communities as prospective solutions to a wide variety of applications. With this growing interest, there has been a great development of unmanned systems control techniques. One of the promising approaches in the field of unmanned systems is model predictive control (MPC) due to its ability to handle the multi-variable constrained systems. Therefore, the goal of this paper is to present a comprehensive literature of applying MPC for motion control of both unmanned ground vehicles (UGVs) and unmanned aerial vehicles (UAVs). First, an overview of motion control principles is presented. Next, an overview of MPC including its concept, formulation, types, and its stability is provided. Then, a comprehensive literature review of applying MPC to both UGVs and UAVs is introduced, including the basic motion tasks such as path planning, point stabilization, and trajectory tracking. Finally, open problems, challenges, and future directions are highlighted.

I. INTRODUCTION

From the literal meaning, mobile robots can move autonomously from one place to another without assistance of external human operators. Mobile robots have the special feature of moving around freely within a predefined workspace to achieve their goals. This mobile capability makes them prospective in both civilian and military applications, including surveillance [1, 2, 3], search and exploration [4, 5, 6, 7], cooperative reconnaissance [8], environmental monitoring [9, 10], and cooperative manipulation [11, 12], respectively.

According to the environment in which they move, mobile robots can be classified into unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs), and unmanned ground vehicles (UGVs), respectively. Ground vehicles are further distinguished in wheeled mobile robots (WMRs) and legged mobile robots (LMRs), respectively. WMRs are appropriate for typical applications with relatively low mechanical complexity and energy consumption. LMRs are suitable for tasks in

non-standard environments, stairs, heaps of rubble, etc. Note that mobile manipulators (wheeled or legged robots equipped with one or more light manipulators to perform various tasks) are also considered as mobile robots [13].

Robot control deals with the problem of determining the forces and torques that are generated by the robotic actuators to reach a desired position and track a desired trajectory. In general, robot control is aimed at performing a specific task with desired performance requirements [13]. In well-structured and fixed environments (e.g. factories and laboratories), the configuration of the environment is known *a priori*. While in uncertain environments, the control algorithms must involve intelligence. Therefore, the basic tasks of unmanned vehicle controller are: mapping, localization, path planning, and tracking. To be more specific, the vehicle has to create the map of the surrounding environment, by which the localization is achieved. Then, the path is planned and the control inputs are generated to ensure that the vehicle tracks planned path and eventually accomplishes the task.

Path planning focuses on determining how a robot moves in a workspace to achieve its goals. The path

planning problem involves computing a collision-free path between the start and destination points. The robot-surrounded environment may be fully known, partially known, or fully unknown. In most practical cases, the environment is only partially known, where the robot, prior to path planning and motion, has partial knowledge of the workspace. Path planning may be either local or global. Local path planning is performed while the robot is moving and receiving data from local sensors. In this case, the robot has the ability to generate a new path in response to environment changes. On the other hand, global path planning can be performed only if the environment is static and perfectly known to the robot. In this case, the path planning algorithm produces a complete path from the start point to the goal at the initial stage of the motion.

In order to enable an efficient, smooth, and continuous movement of vehicles along the desired path, a trajectory tracking is of great importance. Most of UGVs and UAVs are nonholonomic and with highly nonlinear dynamics. As a result, the control of unmanned vehicles must overcome vehicle nonlinearities as well as the nonholonomic constraint, so that the vehicle can be stabilized with sound robustness. According to Brockett [14], a smooth, time-invariant, static state feedback control law cannot be used to stabilize a nonholonomic system at a given configuration. To tackle this limitation, a variety of nonlinear control strategies have been presented for the trajectory tracking and stabilization. Though numerous control algorithms are found in the literature, the controller design is still challenging due to the nonholonomic nature and the nonlinearity nature. Many control algorithms have been developed to solve the UGVs control problem, including Lyapunov based method [15, 16, 17], dynamic feedback linearization [18, 19, 20], sliding mode control [21, 22], and model predictive control (MPC) [23, 24, 25].

On the other hand, control of UAVs is much more challenging due to the complexity of the flight conditions. Several control algorithms are found in literature to solve the UAVs control problem, including proportional–integral–derivative (PID) [26, 27], fuzzy control [28,29], adaptive control [30,31], neural network [32,33], genetic algorithm (GA) [34], and MPC [35, 36].

Recently, unmanned vehicles control problem is formulated as an optimal control problem, where optimization-based techniques can be applied. One of these approaches is MPC. The past decade has witnessed the development of MPC to be applied to unmanned systems. Its ability to handle constraints makes it promise for single vehicle control and cooperative control of a team of unmanned vehicles.

This paper is intended to review and highlight the existing work of applying MPC for motion control of both UGVs and UAVs. This survey can be divided into three

sections: 1) an overview of MPC, including the principle, types, formulation, and summary; 2) a review of the existing work of controlling UGVs based on MPC. This review indicates how to apply MPC to address the main issues of UGVs motion control: trajectory tracking, point stabilization, collision avoidance, and path planning; and 3) a review of the existing work of applying MPC to UAVs' motion control including the following issues: path planning, disturbance rejection, trajectory tracking, and fault tolerant control (FTC).

The rest of this paper is organized as follows. Section II presents an overview of MPC formulation and types. Section III gives a comprehensive review of applying MPC to UGVs, In Section IV, a comprehensive review of controlling UAVs based on MPC is investigated. Challenging issues and main future directions are presented in Section V. Finally, concluding remarks are presented in Section VI.

II. MODEL PREDICTIVE CONTROL

MPC, which is also known as receding horizon control (RHC), has received a great deal of attention in the control community, due to its ability to solve multi-variable constrained problems. Although it has been used for a long time in some industrial processes such as oil refinery, biomedical industry, and chemical plants [37], MPC recently start being applied for UAVs [35, 36, 38] and WMRs [23, 24].

The importance of applying MPC in the control community arises from its ability to handle the states and inputs constraints, and real-time predication, optimizing and correcting the feedback. Compared to the conventional control methods that use pre-computed control laws, MPC is based on iterative, finite horizon optimization of a plant model to obtain an estimate of its future behavior. An optimization problem based on a cost function is then solved to choose an optimal sequence of controls from all feasible sequences. The first control input of this optimal sequence is then applied to the feedback control loop, while the whole procedure is repeated at each subsequent step. Fig. 1 shows the basic structure of MPC. The main principle of building an MPC controller can be summarized as follows [39]:

- Calculate the predictions of the future system behavior based on the explicit use of plant model;
- Optimize the objective function subject to constraints, resulting in the optimal sequence of controls; and
- Use the receding horizon strategy, in which only the first element of the optimal sequence of controls is applied on-line.

To reduce the computational burden, MPC uses both a control horizon and a prediction horizon. The control horizon determines the number of actuation signals to find. On the other hand, the prediction horizon determines how far the behavior of the system is predicted.

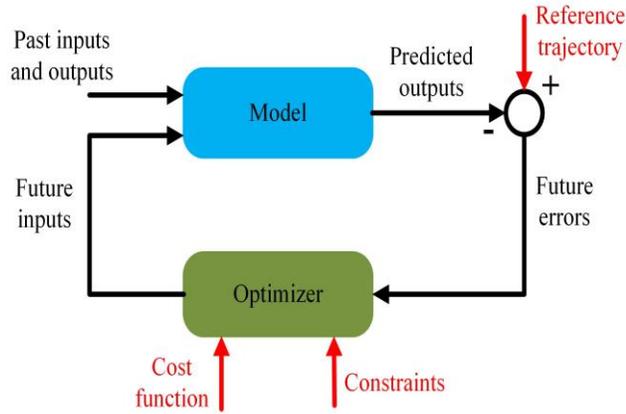


Fig. 1: Illustration of MPC scheme [40]

The MPC methodology attempts to solve an on-line open loop finite horizon optimal control problem subject to input, state, and/or output constraints. As shown in Fig. 2, at a time t , the system model and the measured variables (outputs) are used to predict the future behavior of the controlled plant over the prediction horizon N_p . Usually, the system's future response is expected to return to a desired set point by following a reference trajectory from the current states. The difference between the predicted output and the reference trajectory is called predicted error. A finite horizon optimal control problem with a performance index (usually be minimizing the predicted control input and the predicted error) is solved online. In consequence, an optimal control input $u^*(t)$ over a control horizon N_c (usually $N_c \leq N_p$), which minimizes the predicted error, is obtained. Only the first element of $u^*(t)$ is implemented to the plant. All the other elements are discarded. Then, at the next time interval, the whole procedure is repeated.

The advantages of MPC can be summarized as [40, 41]:

- It can deal with multi-variable and nonlinear systems;
- It is very useful when future references are known;
- Higher efficiency based on the minimization of the cost function;
- It allows operation within constraints; and
- It can handle multiple systems easily by merging them into the objective function.

However, the main disadvantages of MPC are [41]:

- As the system complexity increases, the on-line calculation burden is substantially increased;
- In case of closed-loop systems, it is difficult to predict the controller performance; and
- Theoretical results regarding stability and robustness are not easily applied to general cases.

One should notice that there are two main types of MPC,

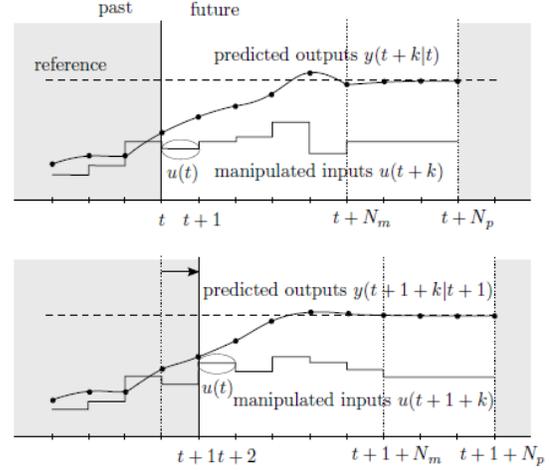


Fig. 2: The principle of MPC

Linear MPC (LMPC) and Nonlinear one (NMPC). However, researchers started to introduce new approaches depend mainly on combining MPC with learning techniques. These new approaches such as Learning Based model predictive control (LBMPC) and neural predictive control guarantee stability and improve the performance of the systems in the presence of uncertainties. In the coming sub-sections, these types will be discussed briefly. Finally, in the end of this section, MPC stability will be briefly outlined.

A. Linear Model Predictive Control (LMPC)

Consider the following discrete-time linear system:

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$, n is the number of states and m is the number of control inputs. $A \in \mathbb{R}^{n \times n}$, and $B \in \mathbb{R}^{n \times m}$. Then, at each time interval k , MPC can be formulated as the following optimization problem

$$\min_{u(\cdot)} J_{(N_p, N_c)}(x_k) \quad (2)$$

subject to

$$\begin{aligned} x(k+i|k) &= Ax(k+i-1|k) + Bu(k+i-1|k), \\ x(k+i|k) &\in \mathcal{X}, \\ u(k+i|k) &\in \mathcal{U}, \end{aligned} \quad (3)$$

The performance index J can be defined as:

$$\begin{aligned} J_{(N_p, N_c)}(x_k) &= x^T(k+N_p)Px(k+N_p) \\ &+ \sum_{i=1}^{N_p-1} x^T(k+i|k)Qx(k+i|k) \\ &+ \sum_{i=0}^{N_c} u^T(k+i|k)Ru(k+i|k) \end{aligned} \quad (4)$$

where $P \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{n \times n}$, and $R \in \mathbb{R}^{m \times m}$ are the three positive semi-definite weighting matrices with $P > 0$, $Q >$

0, and $R > 0$. The coefficients of P , Q , and R reflect the relative importance of the final state error cost, the intermediate state error cost, and the control input error cost, respectively. $X \subset \mathbb{R}^n$ are the state constraints, while $U \subset \mathbb{R}^m$ are the input constraints. Usually, $U = \{u \in \mathbb{R}^m : u_{min} \leq u \leq u_{max}\}$, where u_{min} and u_{max} are known constants in \mathbb{R}^m . The first term on the right-hand side of Eq. (4) is called the terminal state penalty, the second term is called the state penalty, and the last term is called the control penalty.

B. Nonlinear Model Predictive Control (NMPC)

Consider the following continuous time nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)) \quad (5)$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$, n is the number of states and m is the number of control inputs. MPC can be formulated as the following optimization problem [42]:

$$\min_{u(\cdot)} J(x(t), u(\cdot)) \quad (6)$$

subject to

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)), \\ x_{min} &\leq x(s; x(t), t) \leq x_{max}, \quad t \leq s \leq t + T_h, \\ u_{min} &\leq u(s) \leq u_{max}, \quad t \leq s \leq t + T_h, \end{aligned} \quad (7)$$

The performance index J can be defined as:

$$J_{T_h}(x(t), u(\cdot)) = \int_t^{t+T_h} (\|\bar{x}(s; x(t), t)\|_Q^2 + \|u(s)\|_R^2) ds + \|\bar{x}(t+T_h; x(t), t)\|_P^2 \quad (8)$$

where T_h represent both the prediction and the control horizons.

C. Learning Based Model Predictive Control (LBMPC)

LBMPC is constructed by combining MPC and learning control techniques. It combines both statistical learning and control engineering, allowing the system model to change gradually over time. This feature can lead to a decrease of the computation time and improvement of the decision quality rather than conventional MPC, results in improving the system performance and guarantees safety, robustness and convergence [43, 44]. In practice, the following factors have to be considered at the control design stage:

- 1) The states and the inputs constraints;
- 2) The ability of the control system to optimize the performance with respect to the cost function;
- 3) The ability to use statistical identification tools to learn about the uncertainties of the model; and
- 4) The ability to converge to the required results in an obvious and provable manner (provably converge).

MPC only considers (1) and (2). It is necessary to cooperate with a learning based control technique to cover (3) and (4). LBMPC is an integration between two control techniques; MPC combines with a learning control technique to fulfill the requirements (1–4). The main concept of LBMPC is to return the measured data from the model at specific points to a time varying oracle. Then, by using this stored data, the system learns how to behave, i.e. by joining both the MPC technique and the learning based control, a time varying oracle is created. The oracle can learn the performance of the model guaranteeing the improvement in system performance, safety and robustness when different states, inputs, and control constraints are present [43].

LBMPC is not fully investigated in the literature, in [107] LBMPC has been implemented on-board a quadrotor helicopter to control its flight in real time experiments. The quadrotor succeeded to learn how to fly in a certain arena and catch a thrown ball.

D. MPC Stability

One of the main concerns of MPC is its stability. Although MPC formulation seems not difficult, the stability may not be guaranteed since the control sequence is generated from a finite optimal control problem. Without proper selection of the weighting matrices, the MPC algorithm may lead to divergent responses [42]. Therefore, much effort has been devoted to obtain the sufficient conditions for MPC stability. As a powerful analysis tool, Lyapunov methods are frequently used in MPC stability. According to [45], the performance index is monotonic and it can be used as a Lyapunov function. For linear systems, the key idea of the monotonicity is to adopt the performance index function J as a Lyapunov function, then the following inequality of the performance index function should be achieved. Let $N_p = N_c = N$, then

$$J_N(x(k)) - J_N(x(k+1)) \geq 0 \quad \text{for } x \neq 0 \quad (9)$$

Substitute in Eq. (4) to find the value of $J_N(x(k)) - J_N(x(k+1))$, one can obtain:

$$\begin{aligned} J_N(x(k)) - J_N(x(k+1)) &= x^T(k)P x(k) + \\ &u_{N-1}^{*T}(x(k))R u_{N-1}^{*T}(x(k)) + J_{N-1}(x(k+1)) - \\ &J_N(x(k+1)) \end{aligned} \quad (10)$$

According to the assumption $Q > 0$ and $R > 0$, the second line of Eq. (10) is always positive. However, it is difficult to guarantee that the third line is nonnegative. The positivity of the third line assures that the performance index is decreased. Several approaches are proposed to guarantee the constant decreasing of the performance index J_N . In [46, 47], the idea of zero terminal constraints is proposed. To guarantee stability, a global optimum solution must be found at each time step. Although



optimization problem with terminal equality constraint can be solved, the computational effort for finding the global optimum solution is too high. Even when a feasible solution exists, the convergence to that solution is not guaranteed. A dual-mode MPC algorithm is developed in [48] to handle both the global optimality and the feasibility problems. A terminal region is introduced to relax the terminal equality constraint, while the terminal region must be reached at the end of the prediction horizon, and the stabilizing controller is employed. With this algorithm, a feasible solution at the initial time guarantees the feasibility at all future time steps. However, for higher order systems, it is difficult to define the terminal region. A quasi-infinite MPC proposed in [49] can overcome both the global optimization and the feasibility problem without using controller switching. As the algorithm presented in [48], a feasible control sequence solution at time t leads to feasible solutions in the future, and stability of the closed-loop system is guaranteed. However, the terminal region calculation is still difficult. A contractive MPC is introduced in [50, 51]. A constraint is added to the performance index to enforce both the actual and predicted state to contract. Within this context, the stability can be proven.

III. UGVs MOTION CONTROL BASED ON MPC

Although MPC is not a new control approach, a few works deal with control of UGVs by means of MPC are found in the literature. In this section, a review of applying MPC for motion control of UGVs is presented, while a summary of this detailed review is summarized in Table I.

An NMPC with time varying weights is designed in [52] for both trajectory tracking and posture stabilization of a unicycle mobile robot. A stabilizing NMPC is adopted in [53] to achieve simultaneous tracking a pre-defined trajectory. Stability is addressed in this work by forcing the terminal state to move into a terminal state region through adding a stability term to the cost function. An NMPC algorithm considering side slip and tangential wheel slip is presented in [54]. Predicted future position errors are minimized by numerical computation of an optimal sequence of control inputs using pre-specified shape functions based on a Gauss-Newton algorithm NMPC and LMPC approaches for trajectory tracking of a differentially-driven WMR are exploited in [55]. The results show that the computational effort of NMPC is much higher than that in the case of LMPC. An NMPC of a unicycle mobile robot based on Taylor approximation is proposed in [56]. The main advantage of this approach is that it doesn't require an on-line optimization, inducing a

less computational effort. NMPC for an omni-directional UGV trajectory tracking is presented in [57] based on the conjugate gradients. Experimental results have shown the good performance of the strategy of the control proposed. A reactive algorithm based on NMPC for trajectory tracking and obstacle avoidance is proposed in [58]. In the presence of obstacles, the controller deviates from the reference trajectory by incorporating into the optimization obstacle-distance information delivered from the sensors. A first-state contractive MPC for trajectory tracking and point stabilization is developed in [59]. Stability is guaranteed by adding a first-state contractive constraint at the beginning of the prediction horizon. An NMPC approach is proposed in [23] to be applied for trajectory tracking and obstacle avoidance of a single UGV. A nonlinear-programming problem is solved on-line an NMPC approach for trajectory tracking of a differentially-driven WMR is applied experimentally in [60]. A new algorithm for trajectory generation of a car-like UGV in a cluttered environment is proposed in [61]. An optimal tracking problem with obstacle avoidance is solved on-line with nonlinear programming. Information of obstacles is incorporated on-line in the NMPC framework as they are sensed. With this algorithm, both the local and the global path planning problems can be solved. An NMPC approach for trajectory tracking of an Omni-directional UGV is presented in [62]. The closed-loop system stability is ensured by deriving the linear matrix inequalities (LMI) constraints for the monotonicity of the upper bound of the cost function. An NMPC approach based on both dynamics and kinematics of a unicycle mobile robot is presented in [63]. Stability is guaranteed by adding a terminal state penalty to the cost function and constraining the terminal state to a terminal region. The terminal region and its corresponding local controller are developed based on T-S fuzzy model. The point stabilization problem of a unicycle WMR subject to wheel slippage is investigated in [64]. When slippage occurs, the robot is modeled as hybrid systems, while NMPC is applied to find the optimal solution. A nonlinear predictive control strategy based on an extreme learning machine (ELM) is proposed in [65]. The ELM-based identifier is used to learn the robot dynamics and provide the output signal for the nonlinear optimizer. An NMPC for a car-like UGV to control its velocity and steering simultaneously is exploited in [66]. The optimization solver is based on the genetic algorithm (GA). Real-time experiments show the effectiveness of the proposed algorithm. An NMPC algorithm to automatically steer the vehicle along a desired trajectory is presented in [67] considering the vehicle dynamics and the tire-ground contact nonlinearities. The developed strategy can guarantee the stability of the closed-loop control system.

To tackle the computational efforts problem associated

TABLE I: Summary of applying MPC to UGVs motion control

Ref.	UGV Type			UGV Model		Objective			MPC Model		Results	
	Uni-cycle	Car-like	Omni-dir.	Kin.	Dyn.	Traj. Tracking	Point Stab.	Path planning	LMPC	NMPC	Sim.	Exp.
[52]	✓			✓		✓	✓			✓	✓	
[53]	✓			✓		✓	✓			✓	✓	
[54]	✓			✓		✓				✓	✓	
[55]	✓			✓		✓			✓	✓	✓	
[56]	✓			✓		✓				✓	✓	
[57]			✓		✓	✓				✓		✓
[58]	✓			✓		✓		✓		✓	✓	
[59]	✓			✓		✓	✓			✓	✓	
[23]	✓			✓		✓		✓		✓	✓	
[60]	✓			✓		✓				✓	✓	✓
[61]		✓			✓			✓		✓	✓	
[62]			✓		✓	✓				✓	✓	
[63]	✓			✓		✓	✓			✓	✓	
[64]	✓				✓		✓			✓	✓	
[65]	✓			✓		✓				✓	✓	
[66]		✓		✓		✓				✓		✓
[67]		✓			✓	✓				✓	✓	
[25]	✓			✓		✓			✓			✓
[68]	✓			✓		✓			✓			✓
[69]	✓			✓		✓			✓			✓
[70]		✓			✓	✓			✓	✓	✓	✓
[24]	✓			✓		✓			✓			✓
[71]		✓			✓	✓			✓		✓	✓
[72]			✓	✓		✓			✓			✓
[73]	✓			✓		✓			✓		✓	✓
[74]			✓		✓	✓			✓			✓
[75]	✓			✓		✓			✓		✓	✓
[76]		✓		✓		✓			✓			✓
[77]		✓		✓		✓			✓		✓	
[78]		✓		✓				✓	✓		✓	

Notes: **Omni-dir.** Omni-directional, **Kin.** Kinematics, **Dyn.** Dynamics, **Traj.** Trajectory, **Stab.** Stabilization, **Sim.** Simulation, **Exp.** Experimental

with NMPC, LMPC approach is proposed. In [25] and [68], a linear time varying description of the robot model based on linearizing the error dynamics between the reference trajectory and the actual one of a unicycle mobile robot is presented. The states of the linearized system are the errors in the position in x and y directions and the error in the orientation angle φ . The control law is derived by

the optimization of a quadratic cost function. In [69] a combination between LMPC and a fuzzy control is presented. LMPC is used to predict the position and the orientation of the robot and the fuzzy control is used to deal with the nonlinear characteristics of the system. In [70], both LMPC and NMPC for controlling the front steering of a car-like UGV is presented. As in [55], the



results show that the computational effort of LMPC is much lower than of NMPC. In [24], an LMPC for a unicycle UGV trajectory tracking is exploited based on linearizing the error dynamics model between the reference trajectory and the actual one. The objective function is to minimize the difference between the future trajectory-following errors of the robot and the reference trajectory. The control law is explicitly obtained without using any optimization solver, while the bounded velocity and acceleration constraints are considered in the low-level controller. Experimental results illustrate the effectiveness of the proposed control law compared to a state-tracking controller presented in [15]. An LMPC for trajectory tracking of a car-like UGV is developed in [71] with consideration of the kinematics and dynamics in a cascade manner. Two approaches are used: the first one is based on linearizing the error dynamics between the reference and actual trajectories; while the second one is based on the local linear model of the UGV. When comparing between the two approaches, the second one has less computational effort and better response. An LMPC for trajectory tracking of an omnidirectional robot is presented in [72] based on linearizing the error dynamics of the robot. An explicit LMPC scheme is developed in [73], where the solution of the minimization problem can be calculated off-line and expressed as a piecewise affine function of the current state of the robot, thus avoiding the need for on-line minimization. By obtaining such optimal controller, which has a form of a look-up table, there is no need for expensive and large computational infrastructure. However, since all the computations are calculated off-line, this method cannot guarantee that the robot continues tracking the desired trajectory in case of sudden situations such as fault occurrence and facing any obstacles. A predictive controller for an omnidirectional mobile robot is exploited in [74] considering the robot dynamics and the friction compensation. Experimental results are presented showing the effectiveness of the proposed algorithm. An integrated approach combining dynamic feedback linearization and LMPC is developed in [75]. The linearized model of the robot with nonlinear dynamics is found through feedback linearization, while LMPC is applied to the linear model. By virtue of this approach, the computational effort problem associated with MPC can be avoided. An LMPC based on linearizing the tracking error model of a car-like UGV is adopted in [76]. Experimental results on an unmanned tractor-trailer are presented. Considering the road friction and lane changing situations, an LMPC for trajectory tracking of a car-like UGV is developed in [77]. An LMPC algorithm is applied in [78] for path planning and obstacle avoidance of a car-like UGV. The authors adopted two models, a fourteen degree-of-freedom (DOF) car

model and a two DOF reduced model.

From the existing literature, compared to NMPC, using LMPC based on a linearized model can be implemented since its computational effort is less than the computational effort of NMPC.

IV. UAVs MOTION CONTROL BASED ON MPC

In this section, a survey for the application of MPC in solving the control problem of UAVs is conducted. MPC, with its different types, plays a central role in solving the control problem of UAVs guaranteeing stability, robustness and success of desired mission. Most of the existing MPC work applied to UAVs is indicated in Table II.

The path planning problem for a rotor UAV is investigated in [79] using an NMPC in the presence of states and input constraints. The controller can track the generated position and heading trajectories. Moreover, the introduced controller succeeds to handle input constraints and system parameters uncertainties. In [80, 81], an NLMPC approach is developed to a pursuit/evasion game as a higher order controller of a fixed wing UAV. The controller allows the UAV to change its mode according to the current and future state of the vehicle with respect to the adversarial aircraft. In [82], NMPC is adopted to generate suitable trajectories in the case of obstacles. The designed controller can guarantee that the best trajectory is chosen avoiding threats. This algorithm is applied to both fixed-wing and quadrotor UAVs. A theoretical study to reduce the computational time of the onboard NMPC is presented in [83] using a new set of design tools to compute finite horizon optimal controls. The proposed approach enables a fixed-wing UAV to track a pre-determined trajectory and reduce the mathematical complexity. Hence the capability of UAVs can be enhanced. Furthermore, in [84], NMPC is applied for an autonomous UAV to track a pre-defined trajectory. The presented controller depends mainly on solving a group of Taylor series allowing real-time implementation. An NMPC algorithm is proposed in [85] as high-level controller for a fixed-wing UAV tracking a pre-determined path. The low-level controller is applied on the UAV while the NMPC reduces the tracking error from the desired trajectory providing better performance with faster and smoother convergence. An NMPC is applied in [86] to control a rotary-wing UAV during autonomous hovering and forward flight with low speed. The designed NMPC can decrease the control effort with the help of servo dynamics during the prediction phase. In [87], an NMPC approach is applied to control a fixed-wing UAV to track a desired path quickly and smoothly. Moreover, the controller ensures the ability of the UAV to track adjoined multiple line segments. Stability analysis for the system is performed to provide the conditions guaranteeing the

TABLE II: Summary of applying MPC to UAVs motion control

Ref.	UAV Type		Objective					MPC Model		Results	
	Quad-rotor	Fixed-wing	Traj. tracking	Point stab.	Dis. rejection	Path planning	Others	LMPC	NMPC	Sim.	Exp.
[79]	✓					✓			✓	✓	✓
[80]		✓					evasion		✓	✓	✓
[81]		✓					evasion		✓	✓	✓
[82]	✓	✓				✓			✓	✓	
[83]	✓	✓	✓						✓	✓	
[84]		✓	✓						✓	✓	
[85]		✓	✓						✓	✓	
[86]	✓			✓					✓	✓	
[87]		✓	✓						✓	✓	
[88]	✓		✓						✓	✓	✓
[89]	✓					✓			✓	✓	
[90]		✓					Endurance		✓	✓	
[91]		✓					Endurance		✓	✓	
[92]	✓				✓				✓	✓	
[93]	✓				✓				✓	✓	
[94]	✓			✓				✓		✓	✓
[95]		✓	✓					✓		✓	
[96]	✓		✓					✓		✓	
[97]	✓					✓		✓		✓	
[35]	✓		✓					✓		✓	✓
[98]	✓					✓		✓			✓
[99]	✓		✓					✓			✓
[100]	✓					✓		✓			✓
[101]	✓	✓					Time delay	✓		✓	
[102]	✓				✓			✓		✓	✓
[103]	✓				✓			✓		✓	✓
[104]	✓						FTC	✓		✓	
[105]	✓						FTC	✓		✓	
[106]	✓			✓				✓		✓	✓
[107]	✓						Catch a ball	✓		✓	✓
[108]	✓						Encirclement	✓		✓	✓
[109]	✓						Encirclement	✓		✓	✓
[110]	✓						Encirclement	✓		✓	✓

Notes: **Dis.** Disturbances, **Traj.** Trajectory, **Stab.** Stabilization, **Sim.** Simulation, **Exp.** Experimental

stability of a fixed wing UAV during executing the desired mission. A visual NMPC is exploited in [88] to ensure the stability of a quadrotor UAV executing a spiral tracking. The controller succeeds to decrease the control effort, guarantees stability of the system, and provides robustness to model mismatch and external disturbances. In [89], an NMPC is applied to control an

UAV through a determined trajectory in the presence of random up-draft distributions. The main goal of the controller is to choose the optimal path of flight toward high energy locations through the atmosphere respecting these random up-draft distributions leading to improve the flight endurance, reduce fuel consumption, and increase battery life.



One of the main challenges in UAVs is the limited running time. MPC is exploited in [90, 91] to enhance the UAVs' running time. In [90], NMPC is applied to glider-type UAV. The proposed algorithm aims to allow the UAV to extract the maximum amount of energy from the surrounding environments updrafts (vertical winds) along a pre-determined trajectory. In addition, the proposed algorithm increases the fuel efficiency and loiter time. Also, in [91], NMPC is applied to a soaring UAV allowing harvest of the energy form the surrounding environment updrafts. The designed controller can guarantee the optimal trajectory while success in energy extraction in a challenging dynamic environment. Environmental disturbance and noise rejection are of great importance in the UAV control. In [92], an NMPC is developed to solve the problem of the environmental disturbance and measurement noise that affect the quadrotor's propellers. In [93], NMPC and PID are combined to achieve better performance of a quadrotor UAV under different environmental noises and disturbance conditions. The proposed technique can handle different sort of uncertainties, guarantee the stabilization of quadrotor UAV under different perturbed and unperturbed conditions.

As discussed previously, applying NMPC in real-time applications of UAVs is still difficult due to the computational effort problem associated with NMPC. To avoid this demerit, LMPC is applied based on the UAVs' linearized models. In [94], a combination of neural network and a state-dependent Riccati equation is proposed to guarantee the stability of a six DOF autonomous helicopter model. LMPC is presented in [95] to control a fixed-wing UAV during passing through pre-determined way-points. The designed controller succeeds to solve the optimization problem in the presences of constraints and random disturbance proving its robustness. In [96], LMPC is exploited for trajectory tracking to solve the convex optimization problem for a small unmanned helicopter tracking pre-determined waypoint trajectories. The proposed approach generates a substantially less control effort to track the desired trajectories under different flight conditions. In [97], LMPC is employed to address the problem of obstacle avoidance for a small-scale helicopter. The designed controller determines the optimal trajectory from the starting point to the desired target position avoiding different obstacles in the track. An efficient LMPC to reduce the computational burden is developed in [35]. A closed-loop prediction algorithm is presented to calculate the future behavior of quadrotor UAV based on the vehicle linear internal model. In [98], an LMPC approach for path planning and obstacle avoidance of a quadrotor UAV is presented. MPC is used as a position controller. The collision avoidance is achieved in the

MPC based on a sigmoid function. In [99], an embedded system for stabilization and control of a micro UAV is proposed. The control algorithm used a disturbance estimator and LMPC to find optimal control inputs that allow the vehicle to track the pre-defined trajectory. A combination of a flatness-based controller and LMPC is proposed in [100] for trajectory planning of a quadrotor UAV. A flatness-based approach is applied to get the linear input-output nonlinear quadrotor dynamics. Feasible reference trajectories are generated using LMPC.

An LMPC algorithm is developed in [101] to solve the problem of time delay in man-in-the loop UAVs. This time delay affects the performance of the UAV and has a great impact on executing the UAV mission causing system instability. The MPC controller compensated the time delay decreasing its effect on the performance of the man-in-the loop UAVs.

LMPC is applied on UAVs to investigate the problem of disturbance and noise rejection. In [102], a switching MPC algorithm is presented to solve the problem of disturbance rejection for a quadrotor helicopter flying in the presence of wind gusts. The proposed controller is computed based on a piecewise affine (PWA) model of the quadrotors attitude dynamics considering the effect of the atmospheric turbulence. The influence of disturbances on the stability of a team of UAVs during flight based on LMPC is studied in [103]. The disturbance information is stored and shared with the other vehicles on the same track. This allows UAVs to predict the disturbances which improve their stability during the flight. Proposed controller succeeded to suppress the influence of disturbances affecting the UAVs stability.

For reliable and safe operations of UAVs, MPC is used for FTC of UAVs. In [104], MPC based FTC integrated with a moving horizon estimation (MHE) and/or unscented Kalman filter (UKF) is presented. Moreover, in [105], a comparison between two MPC algorithms for controlling the height of a quadrotor helicopter (Qball-X4) in fault-free and actuator fault cases. The proposed algorithms succeeded to manage an acceptable performance in both cases.

Recently, the combination of a learning algorithm and MPC has become a central of research focus. Applying the learning algorithm to the MPC improves the performance of the system and guarantees safety, robustness and convergence in the presence of states and control inputs constraints [106, 107]. In [106], LBMPC is applied to approximate quadrotor UAV dynamics and stabilize in a desired altitude. A dual extended Kalman filter (DEKF) is used for learning the quadrotor un-certainties, while an MPC is adopted to solve the optimization control problem. In [107], LBMPC is used on a quadrotor UAV to learn it catching a ball during flight. Experimental results are presented in both references.

One of the main applications of UAVs is the dynamic encirclement, where UAV is used to encircle a target. MPC is recently applied in such application. In [108], the problem of dynamic encirclement of a quadrotor around an invading target using LMPC is investigated. Moreover, in [109] LMPC solve the encirclement problem. Taylor series linearization technique is developed to linearize the nonlinear UAV dynamics considering the dynamics uncertainties. The circular path around the stationary target is divided into eight regions where the controller is switched from one to another according to the quadrotor position. In [110], a combination of linear MPC and feedback linearization is proposed to solve the encirclement problem of a quadrotor UAV. System identification technique, based on least-square algorithm, and the linearization of the Cartesian to polar transformation via feedback linearization are used successfully to identify the required model. The proposed algorithm can ensure the circular motion of the UAV around the invading target, stability of the system and the convergence to the desired radius of encirclement. This work is extended and applied to dynamic encirclement using a team of UAVs [111].

V. CHALLENGES AND FUTURE DIRECTIONS

Although tremendous effort has been dedicated to implement MPC in unmanned systems, there still exist significant challenges. As mentioned in this review, applying MPC is still limited in real-time applications especially for nonlinear systems such as UGVs and UAVs. MPC is regarded as a predicting approach, while an optimization problem has to be solved on-line. The main problem associated with MPC is the computational time. Therefore, the main research direction in this field is to reduce the computational burden.

When applying NMPC, a nonlinear programming problem to be solved on-line is usually non-convex. As a result, finding an optimal solution may be difficult. LMPC is a good option to address this problem. However, many challenges still exist when applying LMPC. To apply LMPC, a linearized model is needed. Linearizing UGV model using Taylor series approximation may result in a linear time varying (LTV) model as proposed in [25, 68]. Despite the computational time is reduced, the problem still exists as the system states change at every sample. Another solution is to simplify the nonlinear model as reported in [112] for the quadrotor model. However, some of these assumptions may be difficult to apply in UAV outdoor testing.

Many future directions can be proceeded towards overcoming the previously mentioned challenges. Most of them are focus on doing most of the required computations off-line, leaving only the rest of computation to be performed on-line. The future directions can be summarized as follows.

1) The first future direction is to integrate MPC with

dynamic feedback linearization as proposed in [110, 113]. Feedback linearization is a common approach used with nonlinear systems. The concept is to make use of algebraic transformation of nonlinear system dynamics to an equivalent linear system with new control inputs. The resulting model becomes linear time invariant (LTI), therefore LMPC can be applied and the computational time can be reduced. However, the major drawback of applying feedback linearization is that the actual control inputs have to be replaced by new ones. Thus, the actual control inputs' constraints cannot be directly applied in this approach;

- 2) The second direction is to do some of the computations off-line. This idea can be performed by applying the multi-parametric quadratic programming (MP-QP) technique to the linear models as presented in [114, 115]. With this idea, MPC solution turns out to be a piecewise affine controller;
- 3) The third direction is to apply the approximation functions such as the artificial neural networks (ANN), that can learn the nonlinear functions or solve specific problems where massive parallel computation is required [40]. Another approximation function can be employed is the hinging hyperplanes; and
- 4) The fourth direction is to utilize the LBMPC. As mentioned in this review, LBMPC has been applied to UAVs control but not applied yet to the UGVs. The idea is that the vehicle can learn its unmodeled dynamics using the learning algorithm at every time step and the updated system is controlled by the MPC to achieve the desired objective.

VI. CONCLUSION

MPC is one of the promising approaches for motion control of unmanned systems. Although MPC has several challenges, but its advantages and benefits in control of constrained systems have inspired tremendous studies in UGVs and UAVs. This paper has presented a technical review on applying MPC to motion control of UGVs and UAVs. The basic definitions, classification, and stability of MPC are presented. The application of MPC to UGVs and UAVs in the literature are categorized and summarized. The challenges and future directions have also been mentioned to facilitate the research progress in this field of research.

VII. REFERENCES

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